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Adaptive Attenuation of Disturbance Formed as a Sum of Sinusoidal Signals Applied to a Benchmark Problem*

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Abstract— **The problem of adaptive attenuation of a disturbance formed as a finite sum of unknown sinusoidal signals is solved for a discrete-time plant with unstable zeros. It is assumed that a reliable model of the plant is known and the system is internally stable. We propose to construct a control signal as a linear combination of outputs of carefully chosen filters. The coefficients of the combination are tuned via an on-line identification based on the plant model. However, our approach avoids constructing an inverse of the plant model. The technique is applied to a case study on a challenging benchmark example in the field of active vibration control. Attenuation of a disturbance formed as a sum of up to three unknown/time-varying sinusoidal signals is demonstrated via simulation and experimental studies.**

I. INTRODUCTION

The problem of disturbance compensation is one of the key problems of control theory and engineering practice. Vibration is a common type of disturbances that can be easily found in many applications and can be modeled as a finite sum of sinusoidal signals [1]. As parameters of disturbance are assumed to be unknown and/or time-varying, adaptive disturbance attenuation is considered.

Most of the approaches dealing with this problem utilize the internal model principle, [2]. The direct approach is based on the on-line tuning of the parameters of the controller to force the output error to zero. It is a common approach in active noise control field [3] or in the active vibration control [4], [5]. With the indirect approach the parameters of the disturbance are identified and then a controller is re-tuned; it can be performed at the each step of operation or be triggered by some conditions. An adaptive disturbance observer can be designed [6], [7] with the following feedforward compensation [8], [9]; it usually requires an identification of the frequency of the disturbance [10]–[12]. However, the exact number of unknown frequencies (order of the disturbance model) is often assumed known. The model of the plant affected by the disturbance is often assumed to be known as well. A recent result related to the problem of the adaptive disturbance compensation for a plant-model mismatch case can be found in [13].

In this paper we present a direct adaptive disturbance attenuation method based on a representation of a disturbance as a linear combination of outputs of carefully chosen filters. The proposed method can deal with discrete-time systems with unstable zeros; the parameters of the plant are assumed to be known (identified). However, the knowledge of the exact number of sinusoidal signals in the disturbance is not required, only the upper bound should be provided. In the ideal case when noise is absent, the proposed method provides exact disturbance compensation. For noisy signals, a good attenuation level is obtained as it can be seen from experimental results. It is also shown that transients are fast and allow to deal with time-varying frequencies.

This paper is organized as follows. In Section II the problem is formulated, a representation of disturbances is discussed, and the controller design and implementation are presented. Section III describes experimental studies. These experiments are done according to the protocols established in [14]. Results of a real-time experiment are provided for the case of one, two, and three sinusoidal signals with constant and time-varying frequencies. Finally, in the Conclusion section, the key ideas of the method are summarized and some important remarks are given.

II. MAIN RESULT

A. Problem statement

Consider a linear time-invariant plant

$$y(t) = \frac{q^{-d}B(q^{-1})}{A(q^{-1})}u(t) + \rho(t) \tag{1}$$

where $y(t)$ is a measured output signal, $u(t)$ is an input signal, $A(q^{-1})$ and $B(q^{-1})$ are known polynomials in the delay (shift) operator q^{-1} $(y(t) = q^{-1}y(t+1))$, $A(q^{-1})$ is stable. Signal $\rho(t)$ represents an unmeasured disturbance formed as a finite sum of unknown sinusoidal signals:

$$\rho(t) = \sum_{i=1}^{N} A_i \sin(\omega_i t + \phi_i). \tag{2}$$

The parameters of the disturbance $A_i > 0$, $\omega_i > 0$, ϕ_i and the exact number of sinusoidal signals N are unknown, but we know the upper bound N_{max}, $N \leq N_{max}$. The goal is to design a control law $u(t)$ that allows disturbance compensation, i.e. $y(t) \to 0$ as $t \to \infty$.

We suggest below a design based on an accurately identified model of (1):

$$y(t) = \frac{q^{-d}\hat{B}(q^{-1})}{\hat{A}(q^{-1})}u(t) + \rho(t).$$

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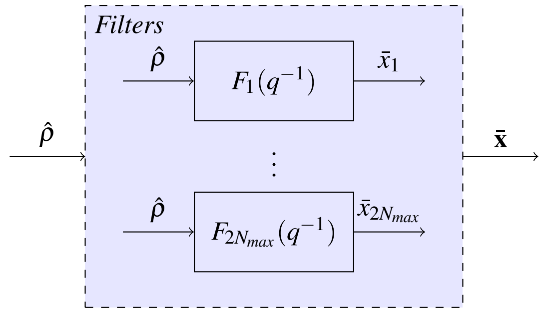

Fig. 1. Filters scheme.

B. Control signal design

We suppose that $\hat{A}(q^{-1}) = A(q^{-1})$, $\hat{B}(q^{-1}) = B(q^{-1})$ and d is known. Then, we can estimate the disturbance from (1):

$$\hat{\rho}(t) = y(t) - \frac{q^{-d}\hat{B}(q^{-1})}{\hat{A}(q^{-1})}u(t).$$

Since the plant is internally stable, all the terms caused by initial conditions decay in the steady state, therefore, it holds:

$$\hat{\rho}(t) \to \rho(t) \text{ as } t \to \infty. \tag{3}$$

Then, it seems that a compensating control signal can be taken as

$$u(t) = -\left(\frac{q^{-d}\hat{B}(q^{-1})}{\hat{A}(q^{-1})}\right)^{-1}\hat{\rho}(t). \tag{4}$$

However, in typical cases, the ideal inversion on the plant can not be stably implemented for systems with unstable zeros. We present now an idea how to avoid the inversion of the plant.

Suppose, our estimate of the disturbance can be represented as a linear combination of signals $x_j(t)$ (here and below $j = 1, \ldots, 2N_{max}$):

$$\hat{\rho}(t) = \sum_{j=1}^{2N_{max}} k_j x_j(t), \tag{5}$$

where k_j are constants and the signals $x_j(t)$ are constructed from avaliable signals $\bar{x}_j(t)$:

$$x_j(t) = \frac{q^{-d}\hat{B}(q^{-1})}{\hat{A}(q^{-1})}\bar{x}_j(t). \tag{6}$$

Then control signal

$$u(t) = -\sum_{j=1}^{2N_{max}} k_j \bar{x}_j(t) \tag{7}$$

is asymptotically equivalent to (4). Substituting of (3), (5) – (7) to (1) leads to $y(t) \to 0$ as $t \to \infty$.

C. Representation of disturbances

The key problem to use control law (7) is to find such signals $\bar{x}_j(t)$ that disturbance estimate $\hat{\rho}(t)$ can be represented as (5), (6). Possible solution is to form a set of $2N_{max}$ signals $\bar{x}_j(t)$ as a finite sum of sinusoidal signals of the same frequencies as in (2):

$$\begin{aligned}\bar{x}_j(t) &= \sum_{i=1}^{N} B_{i,j}\sin(\omega_i t + \psi_{i,j}) \\ &= \sum_{i=1}^{N} B^s_{i,j}\sin(\omega_i t) + B^c_{i,j}\cos(\omega_i t),\end{aligned} \tag{8}$$

where $B^s_{i,j} = B_{i,j}\cos(\psi_{i,j})$, $B^c_{i,j} = B_{i,j}\sin(\psi_{i,j})$. since plant is stable, signals $x_j(t)$ formed as (6) are the finite sums of sinusoidal signals of the same frequencies as (8) neglecting exponentially decaying terms. Next two lemmas show when there exists $\mathbf{k}$, such that (5) holds with (3).

Lemma 1: For the signals (2) and (8) exist k_j, such that

$$\rho(t) = \sum_{j=1}^{2N_{max}} k_j \bar{x}_j(t) \tag{9}$$

if the $2N \times 2N_{max}$ matrix

$$X = \begin{bmatrix} B^s_{1,1} & B^s_{1,2} & \cdots & B^s_{1,2N_{max}} \\ B^c_{1,1} & B^c_{1,2} & \cdots & B^c_{1,2N_{max}} \\ \cdots & \cdots & \cdots & \cdots \\ B^c_{N,1} & B^c_{N,2} & \cdots & B^c_{N,2N_{max}} \end{bmatrix} \tag{10}$$

has rank $(X) = 2N$, and this solution is unique *iff rank* $(X) = 2N_{max}$.

Proof: The finite sum of sinusoidal signals (2) can be represented as

$$\rho(t) = \sum_{i=1}^{N} A^s_i \sin(\omega_i t) + A^c_i \cos(\omega_i t).$$

where $A^s_i = A_i\cos(\phi_i)$, $A^c_i = A_i\sin(\phi_i)$. Then, the equation (9) can be reformulated in a matrix form as

$$\begin{aligned} &\Upsilon = XK, \\ &\Upsilon = \begin{bmatrix} A^s_1 & A^c_1 & A^s_2 & A^c_2 & \cdots & A^s_N & A^c_N \end{bmatrix}^T, \\ &K = \begin{bmatrix} k_1 & k_2 & \cdots & k_{2N_{max}} \end{bmatrix}^T, \end{aligned}$$

the dimension of Υ is $2N \times 1$ and the dimension of K is $2N_{max} \times 1$. From the rank $(X) = 2N$ it follows that the rank $(\begin{bmatrix} X & \Upsilon \end{bmatrix}]) = 2N$ and the Lemma 1 follows from the Kronecker-Capelli theorem. ■

Lemma 2: If the signals $\bar{x}_j(t)$ satisfy Lemma 1 and the amplitude gain $L(\omega_i)$ of $q^{-d}\hat{B}(q^{-1})/\hat{A}(q^{-1})$ satisfies

$$0 \ < \ L(\omega_i) \ < \ \infty \tag{11}$$

for all frequencies ω_i, $\quad i = 1, \ldots, N$, then for the signals $x_j(t)$ formed as (6) there exists $\mathbf{k}$, such that (5) holds for $t \to \infty$.

Proof: Let us show that for the signals $x_j(t)$ formed as (6) asymptotically holds

$$\rho(t) = \sum_{j=1}^{2N_{max}} k_j x_j(t). \tag{12}$$

Then the lemma follows from (3). Let us denote matrix (10) formed by signals $\bar{x}_j(t)$ as X_1 and the same matrix formed by signals $x_j(t)$ as X_2. Then, if (11) holds, the matrices X_2 and X_1 are related with the set of elementary raw operations. Then, rank (X_2) = rank (X_1) and (12) holds according to Lemma 1. ■

D. Filters design

Signals $\bar{x}_j(t)$ can be obtained by applying the signal $\hat{\rho}(t)$ to the bank of stable filters $F_j(q^{-1})$ (see Fig. 1):

$$\bar{x}_j(t) = F_j(q^{-1})\hat{\rho}(t).$$

If these filters do not vanish harmonic signals of any frequency ω_i, $i = 1,\dots,N$ then after transients decay signals $\bar{x}_j(t)$ have form (8). There are multiple suitable choices for the bank of filters from which the signals $\bar{x}_j$ satisfy the Lemma 1 (however, not any stable choice is suitable). One possible solution is to design filters

$$\begin{aligned} &F_1(q^{-1}) = F_0(q^{-1}), \\ &F_k(q^{-1}) = q^{-1} \cdot F_{k-1}(q^{-1}), \\ &\qquad k = 2,\dots,2N_{max}, \end{aligned} \tag{13}$$

that provides

$$\begin{aligned} &\bar{x}_1(t) = F_0(q^{-1})\hat{\rho}(t), \\ &\bar{x}_k(t) = q^{-1}\bar{x}_{k-1}(t) = \bar{x}_{k-1}(t-1), \\ &\quad k = 2,\dots,2N_{max}. \end{aligned}$$

Then the bank of filters becomes a cascade of a FIR filter and a base filter $F_0(q^{-1})$. The last one can be chosen arbitrary. For example, if the range of frequencies is a priory known, which is a common in engineering practice, then the base filter can be chosen as a band-pass filter with the band equal to the known range of frequencies. Such a choice allows to reduce the noise in the signals x_j and provides a better disturbance attenuation. Other filters $F_{2,\dots,2N_{max}}$ are constructed with the shift operators. An advantage of this approach is an extreme reduction of computational cost comparing to a usage of $2N_{max}$ filters. The admissibility for using the filters (13) is stated in the next lemma.

Lemma 3: For $N \le N_{max}$, if the signals $\bar{x}_j(t)$ are formed as

$$\begin{aligned} &\bar{x}_1(t) = \sum_{i=1}^{N} B_i \sin(\omega_i t + \psi_i), \\ &\bar{x}_k(t) = \bar{x}_{k-1}(t-1), \\ &\quad k = 2,\dots,2N_{max}, \end{aligned}$$

then Lemma 1 is satisfied and the matrix (10) has rank equal to $2N$.
Proof of the Lemma 3 is based on a complex generalization of the Vandermonde matrix and is omitted here because of the space limitations.

It is important to notice here that with such a choice of the bank of filters, the base filter $F_0(q^{-1})$ directly affects the input sensitivity function of the closed-loop system. As a result a choice of the base filter allows to shape the input sensitivity function in a straightforward manner. Particularly, the choice of a bandpass filter ensures a very low level of the input sensitivity function outside the band of the filter that implies robustness with respect to possible uncertainties.

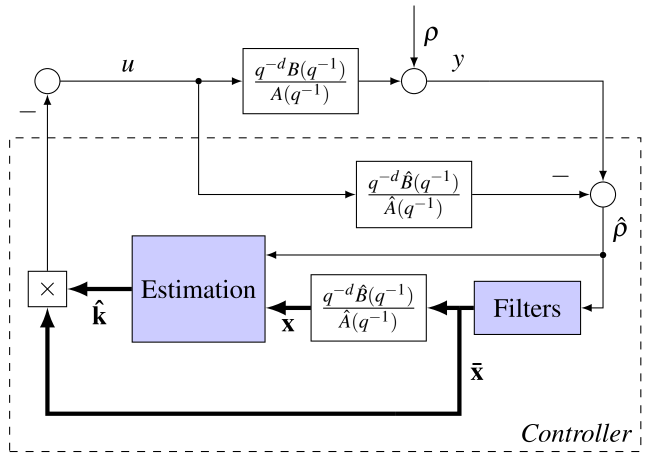

Fig. 2. Control scheme.

E. Controller implementation

The control scheme is presented in Fig. 2. First, the estimation of the plant is used to obtain the estimation of the disturbance $\hat{\rho}(t)$. Then, the signal $\hat{\rho}(t)$ passes through the bank of linear time-invariant filters and produces a set of signals $\bar{x}_j$. Signals $x_j(t)$ are formed by the plant model according to (6). Block ”Estimation” computes parameters of (5) solving the estimation problem formulated as

$$\hat{\mathbf{k}} = \arg\min_{\hat{\mathbf{k}}} \sum_t \left(\hat{\rho}(t) - \sum_{j=1}^{2N_{max}} x_j(t)\,\hat{k}_j \right)^2, \tag{14}$$

where $\hat{\rho}(t)$, $x_j(t)$ are known signals. It should be noticed that signals $x_j(t)$ are sums of sinusoidal signals, therefore, the “persistent excitation” condition holds. Finally, the control signal is formed as (7). Assuming that estimation problem (14) is solved, and $\hat{k}_j$ are constants, then, the disturbance is compensated.

Note that any estimation method is suitable as long as it can deal with (14). Nevertheless, in practice the signals are noisy, and the exact choice for the estimation routine can significantly affect attenuation performance. We do not analyze efficiently of different estimation routines here; the routine used for experimental studies is described in Section III.

Remark 1: If the computational cost of the bank of filters is smaller than the computational cost of the plant model multiplied by $2N_{max} - 1$ then a more computationally effective scheme can be proposed, see Fig. 3.

III. RESULTS OF EXPERIMENTS

Experimental studies are carried out with respect to the benchmark problem, that is described in [14], [15]. This benchmark considers a testbench that is modeled as a linear time-invariant plant of order 26 and is perturbed by an external disturbance formed as a finite sum of sinusoidal signals. The model of the plant is assumed to be known (provided via identification). The plant is stable, but has unstable zeros. System operates at a sampling frequency 800 Hz. Three *levels* of the benchmark are considered, where

TABLE I

TABLE OF REAL-TIME RESULTS FOR THE SIMPLE STEP TEST. LEVELS 1-3.

LEVEL 1 - SIMPLE STEP TEST							
Frequency (Hz)	**Global (dB)**	**Dist. Atte. (dB)**	**Max. Amp. (dB@Hz)**	**Norm^2 Trans.** ($\times 10^{-3}$)	**Norm^2 Res.** ($\times 10^{-3}$)	**Max. Val.** ($\times 10^{-3}$)	**Trans. ratio**
50	34.7	41.7	9.3@54.7	99.2	6.4	31.0	1.141
55	34.6	45.9	11.7@128.1	40.4	5.1	34.3	1.040
60	32.8	47.8	9.6@46.9	26.7	5.5	39.6	0.879
65	33.1	50.5	8.4@90.6	25.3	4.5	36.4	0.993
70	30.8	47.6	8.0@64.1	17.8	5.0	33.4	1.055
75	30.2	46.4	9.0@281.3	16.5	4.9	32.2	0.878
80	30.1	46.8	7.7@115.6	21.9	4.5	31.5	1.005
85	29.2	40.5	10.6@73.4	24.4	4.8	41.2	0.963
90	28.0	41.5	10.1@67.2	25.7	4.9	38.4	1.019
95	25.8	40.0	10.9@129.7	25.4	4.9	26.2	1.092
LEVEL 2 - SIMPLE STEP TEST							
Frequency (Hz)	**Global (dB)**	**Dist. Atte. (dB)-(dB)**	**Max. Amp. (dB@Hz)**	**Norm^2 Trans.** ($\times 10^{-3}$)	**Norm^2 Res.** ($\times 10^{-3}$)	**Max. Val.** ($\times 10^{-3}$)	**Trans. ratio**
50-70	33.1	39.4 - 35.6	15.6@90.6	179.4	12.7	77.7	0.946
55-75	35.1	45.0 - 45.4	11.4@92.2	90.0	7.9	56.8	1.008
60-80	37.7	49.0 - 47.6	9.6@70.3	52.5	5.1	63.0	1.019
65-85	34.5	49.0 - 39.9	10.5@106.3	63.5	6.6	70.6	0.961
70-90	33.8	51.4 - 40.6	12.2@59.4	64.4	6.4	58.3	0.955
75-95	31.7	44.9 - 42.1	14.1@103.1	68.2	7.4	49.1	0.981
LEVEL 3 - SIMPLE STEP TEST							
Frequency (Hz)	**Global (dB)**	**Dist. Atte. (dB)-(dB)-(dB)**	**Max. Amp. (dB@Hz)**	**Norm^2 Trans.** ($\times 10^{-3}$)	**Norm^2 Res.** ($\times 10^{-3}$)	**Max. Val.** ($\times 10^{-3}$)	**Trans. ratio**
50-65-80	38.0	39.4 - 42.8 - 39.5	14.3@93.8	302.6	9.6	116.8	0.977
55-70-85	36.8	45.7 - 51.4 - 42.9	10.8@40.6	294.0	9.1	140.5	0.947
60-75-90	37.1	45.6 - 48.7 - 44.4	12.8@68.8	312.8	7.8	146.2	1.107
65-80-95	34.7	44.4 - 44.6 - 42.5	13.9@51.6	105.1	9.1	62.9	0.970

the disturbance consists of one, two, and three sinusoidal signals respectively. The frequencies are located in the range from 50 Hz to 95 Hz and frequency response of the plant satisfy Lemma 2 for this range. For each level three *protocols* of experiment are considered: frequencies are constant (the simple step test), frequencies change step-wise several times (step changes), frequencies linearly vary (chirp changes).

The scheme presented in Fig. 3 was used. The base filter $F_0(q^{-1})$ was chosen as an elliptic bandpass filter of order 14 with the passband edges 50 Hz and 95 Hz. To solve the estimation problem (14) an iterative least-square method with the forgetting factor was used [16, p. 53]:

$$\begin{aligned}
\hat{\mathbf{k}}(t) &= \hat{\mathbf{k}}(t-1) + G(t)\,\varepsilon(t), \\
\varepsilon(t) &= \hat{\rho}(t) - \mathbf{x}^T(t)\,\hat{\mathbf{k}}(t-1), \\
G(t) &= P(t)\,\mathbf{x}(t) = \frac{P(t-1)\,\mathbf{x}(t)}{\lambda + \mathbf{x}^T(t)\,P(t-1)\,\mathbf{x}(t)}, \\
P(t) &= \frac{1}{\lambda}\left(P(t-1) - G(t)\,\mathbf{x}^T(t)\,P(t-1)\right),
\end{aligned} \tag{15}$$

where $\mathbf{x}(t) = \{x_j\}$, $j = 1, \ldots, 2N_{max}$, and the forgetting factor is $\lambda = 0.98$.

All experiments were done both for the case of known number of frequencies, i.e. $N_{max} = N$, and for the case of unknown N, i.e. $N_{max} = 3$ for all three levels. It was found that with the known N the real-time results are slightly better then with the unknown. However, for practical applications a small deterioration of the performance can be preferable because of the capability to deal with the unknown number of sinusoidal signals in the disturbance.

Due to the space restrictions only real-time results for the case of known N are presented in this paper. All simulation results and the real-time results for the case of unknown N can be found at the benchmark website [15].

Evaluations

The next values are evaluated (for more details see [14], [15]):

- For a steady state performance: Global Attenuation (*GA*), Disturbance Attenuation (*DA*), Maximum Amplification (*MA*), Square of the two norm of the residual force (N^2R);
- For a transient performance: Square of the truncated two norm of the transient (N^2T), Maximum value of the transient (MV), Transient ratio;
- For a *Chrip* protocol performance: Mean Square of the residual force, Maximum Value.

The results of the "Simple Step" protocol are presented in Table I for all three levels. One can see that a good global attenuation is provided. The only experiment where *GA* is below benchmark specifications is Level 1 at 85 Hz. Very good *DA* is provided as well. Particularly, for Level 1, the benchmark specifications are met for all frequencies, and for Levels 2 and Level 3 *DA* is only slightly below specifications for several frequencies. Unfortunately, *MA* requirements are not satisfied for all experiments; the proposed approach does allow to tune this criterion. The benchmark specification on the transient time is satisfied (all *Trans. ratio* are below 1.21 that implies that all transients do not exceed 2 seconds).

The results for the "Frequency Step Changes" protocol are presented in Table II and the results for the "Chrip Changes" protocol are presented in Table III.

The output signal for Level 2 is presented in Fig. 4 in the time domain for all three protocols and the Power Spectral Density estimation is presented in Fig. 5 for "Simple Step" protocol for frequencies 60 Hz and 80 Hz. The results for Level 3 are presented in Fig. 6 in time domain and the Power Spectral Density estimation is presented in Fig. 7 for frequencies 60 Hz, 75 Hz and 90 Hz.

IV. CONCLUSION

An method of adaptive disturbance attenuation is proposed and applied to the active vibration control benchmark. The key idea is decomposing of the disturbance as a linear combination of outputs of filters. The parameters of the decomposition are estimated and used for compensation. This approach allows to deal with the systems with unstable zeros. Such a decomposition exists even if the exact number of sinusoidal signals in the disturbance is unknown. This allows designing one controller that is suitable for any number of sinusoidal signals below the known upper bound.

Real-time results demonstrate good attenuation performance; the transient processes are fast and allow to deal with the step-wise or linearly varying frequencies. However, a high amplification can be observed at the frequencies other then the disturbance.

The performance of the controller depends mainly on the design of filters and the estimation routine. One solution is to design filters using the base filter and time-shifted signals. The base filter is designed according to an a priori available information about the disturbance. The choice of the estimation routine is not analyzed. It is possible that usage of more complicated and robust methods will improve the performance.

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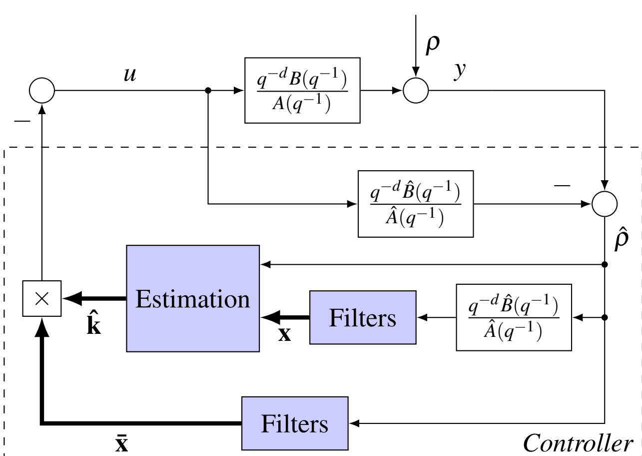

Fig. 3. Control scheme with another computational cost.

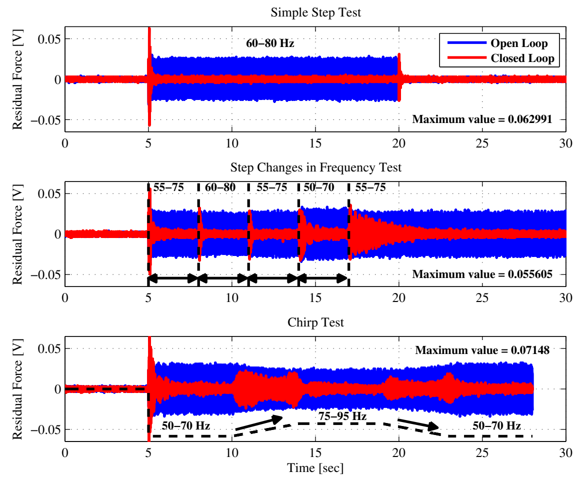

Fig. 4. The results of the real-time experiments for different protocols, Level 2.

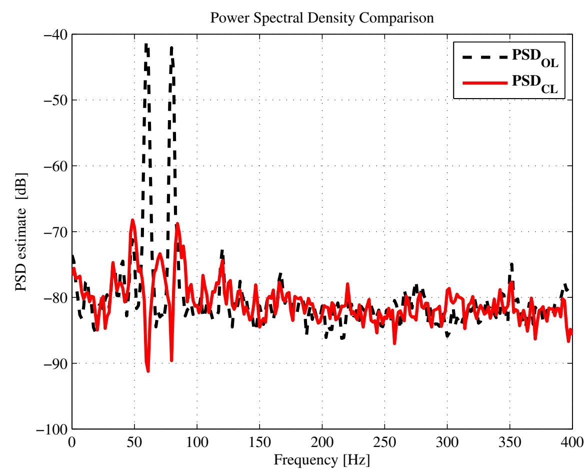

Fig. 5. The PSD estimation for real-time experiment, Level 2, Simple Step protocol.

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TABLE II

TABLE OF REAL-TIME RESULTS FOR THE STEP CHANGES PROTOCOL, LEVELS 1-3.

Level	Frequency (Hz)	Norm^2 Trans. ($\times 10^{-3}$)	Max. Val. ($\times 10^{-3}$)
Level 1	**SEQUENCE - 1**		
	60→70	17.9	24.1
	70→60	18.9	24.1
	60→50	66.7	24.1
	50→60	125.9	25.3
	SEQUENCE - 2		
	60→70	14.7	17.9
	70→60	15.5	23.7
	60→50	16.8	23.7
	50→60	17.5	22.5
	SEQUENCE - 3		
	60→70	18.8	16.7
	70→60	30.6	21.6
	60→50	16.0	23.7
	50→60	14.9	19.2
Level 2	**SEQUENCE - 1**		
	[55-75] → **[60-80]**	27.9	32.6
	[60-80] → **[55-75]**	30.6	31.4
	[55-75] → **[50-70]**	93.0	32.6
	[50-70] → **[55-75]**	197.1	36.0
	SEQUENCE - 2		
	[70-90] → **[75-95]**	26.0	25.3
	[75-95] → **[70-90]**	44.7	26.2
	[70-90] → **[65-85]**	21.3	28.6
	[65-85] → **[70-90]**	19.2	25.3
Level 3	**SEQUENCE - 1**		
	[55-70-85] → **[60-75-90]**	48.2	55.5
	[60-75-90] → **[55-70-85]**	50.1	56.0
	[55-70-85] → **[50-65-80]**	126.7	58.0
	[50-65-80] → **[55-70-85]**	147.6	57.2
	SEQUENCE - 2		
	[60-75-90] → **[65-80-95]**	46.7	45.8
	[65-80-95] → **[60-75-90]**	59.1	53.5
	[60-75-90] → **[55-70-85]**	52.6	59.3
	[55-70-85] → **[60-75-90]**	48.8	56.8

TABLE III

TABLE OF REAL-TIME RESULTS FOR THE CHIRP CHANGES PROTOCOL, LEVELS 1-3.

	Maximum $\times 10^{-3}$		**Mean Square** $\times 10^{-6}$	
	↗	↘	↗	↘
Chirp-Level 1	17.6	20.4	42.8	18.6
Chirp-Level 2	18.8	24.1	67.9	22.2
Chirp-Level 3	14.3	24.9	50.8	15.4

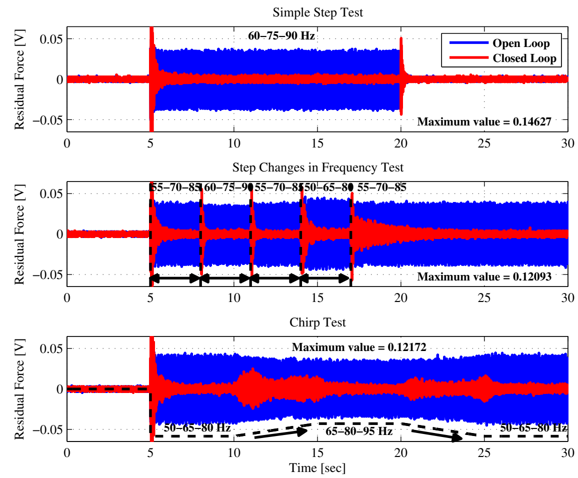

Fig. 6. The results of the real-time experiments for different protocols, Level 3.

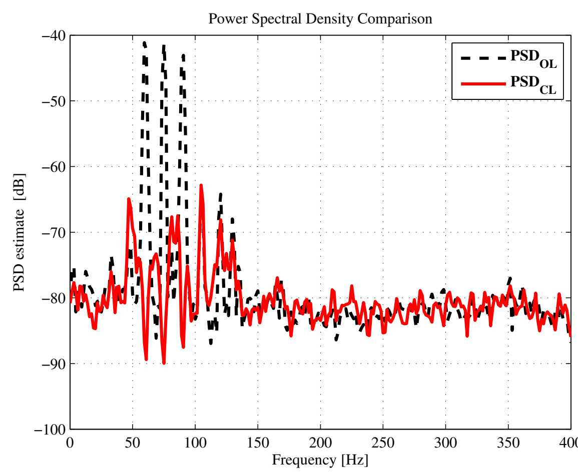

Fig. 7. The PSD estimation for real-time experiment, Level 3, Simple Step protocol.

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