

Mixed-integer NMPC for predictive cruise control of heavy-duty trucks

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Abstract—We present a novel numerical method for nonlinear model-predictive control of heavy-duty trucks. This method realizes a predictive online cruise controller, and includes the opportunity for multiple predictive gear choices. The combination of nonlinear dynamics, constraints, and objective with the hybrid nature of the gear choice makes for a challenging combinatorial prediction problem. The numerical algorithm presented to attack this problem is based on a direct and simultaneous method for the efficient solution of optimal control problems in ordinary differential equations and differential-algebraic equations. Partial outer convexification is applied to treat the mixed-integer aspect of the problem, maintaining certificates of ε -optimality. A vanishing constraint formulation for engine speed constraints ensures feasibility of predictive gear choices. Suitable exploitation of direct multiple shooting structures allows to achieve encouraging real-time capable mixed-integer control feedback rates. Numerical results for a heavy-duty truck cruise controller including predictive gear shifts are presented for several artificial and real-world scenarios.

I. INTRODUCTION

Today's heavy duty trucks include a powertrain that contains, among other units, a gearbox with 8 to 24 gears. In many cases, an experienced truck driver chooses to accelerate, brake, or shift gears based on his ability to predict load changes of the powertrain. He chooses the truck's operation point in a fashion suitable for an oncoming period of time rather than for the current observable system state only. Obvious examples include, for example, timely downshifts before entering a steep slope, or upshifts accompanying acceleration. Especially with the advent of hybrid engines using multiple energy sources, optimal operation of vehicle systems has become increasingly nonstraightforward. Mixed-integer predictive control may hence reveal a significant and previously inaccessible potential for energy savings. To this end, one aims at the design and implementation of intelligent cruise controllers, supporting the human driver in his or her decisions. It is, however, evident that automatic cruise controllers operating solely on the knowledge of the truck's current system state inevitably will make control decisions inferior to those of an experienced driver, cf. [6], [20]. This article describes the algorithmic design of a fully automatic

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predictive cruise control system for heavy-duty trucks including multiple predictive gear shifts. This system anticipates the truck's behavior over a long prediction horizon, reacts to traffic and road conditions, and automatically chooses optimal discrete and continuous truck control inputs under real-time conditions. To achieve this, we develop and implement a novel mathematical optimization algorithm for fast nonlinear model predictive control (NMPC) of nonlinear constrained switched systems in ordinary differential equations (ODE).

Structure of the Article

In §II, we present a switched nonlinear ODE model of a heavy-duty truck. We formulate the associated nonlinear constrained mixed-integer optimal control problem to be solved in each of the successive steps of the moving horizon optimal control algorithm. In §III we present several efficient formulations and methods for the treatment of integer controls and discuss them with respect to feasibility, tightness, and real-time tractability. In §IV we propose an efficient moving horizon algorithm for mixed-integer NMPC, based on the direct multiple shooting method, on partial outer convexification, and on a vanishing constraint formulation. In §V numerical results obtained for the heavy-duty truck model and selected road scenarios are presented. We conclude with a summary and an outlook on future research directions in §VI.

II. THE HEAVY-DUTY TRUCK CONTROL PROBLEM

A. Dynamics

We describe the truck model introduced in [20] used for all computations. The truck's dynamics are modeled as a switched ODE system. A summary of model parameters, states, and controls can be found in tables I-III. Location $s \in [s_0, s_f]$ (in meters) is chosen as the independent variable. This limits the model's applicability to the domain of strictly positive velocities. Time $t(s)$ depending on position s and velocity $v(s)$ is found from

$$v(s)\dot{t}(s) = 1, \quad t(s_0) = 0. \quad (1)$$

The truck's acceleration is computed from the summation of accelerating torques M_{ac} , braking torques M_{br} , and resisting torques M_{ai} , M_{rd} ,

$$mv(s)\dot{v}(s) = \frac{i_A}{r_{stat}} (M_{ac}(s) - M_{br}(s)) - M_{ai}(s) - M_{rd}(s). \quad (2)$$

The parameter m denotes the truck's mass; the rear axle's transmission ratio is denoted by i_A ; and the static rear tire radius is r_{stat} . For the rate-limited indicated engine

torque M_{ind} and engine brake torque M_{EB} , we control the corresponding rates of change R_{ind} , R_{EB} , and have

$$v(s)\dot{M}_{\text{ind}}(s) = R_{\text{ind}}(s), \quad v(s)\dot{M}_{\text{EB}}(s) = R_{\text{EB}}(s). \quad (3)$$

With this, the accelerating torque M_{ac} depends on the transmission ratio $i_{\text{T}}(y)$ and the degree of efficiency $\eta_{\text{T}}(y)$ of the selected gear y , and is computed from

$$M_{\text{ac}}(s) := i_{\text{T}}(y(s)) \eta_{\text{T}}(y(s)) M_{\text{ind}}(s). \quad (4)$$

The sum of braking torques M_{br} is computed from the controlled engine brakes torque M_{EB} , increased by resisting torques due to friction M_{fric} in the engine. The value n_{eng} denotes the engine's speed in revolutions per minute.

$$M_{\text{br}}(s) := M_{\text{EB}}(s) + i_{\text{T}}(y(s)) M_{\text{fric}}(n_{\text{eng}}(s)). \quad (5)$$

Additional braking torques, independent of the selected gear, due to turbulent friction M_{ai} and road conditions M_{rd} are taken into account. The parameter A denotes the effective flow surface, c_w the shape coefficient; and ρ_{ai} the air density,

$$M_{\text{ai}}(s) := \frac{1}{2} c_w A \rho_{\text{ai}} v^2(s). \quad (6)$$

The term M_{rd} accounts for rolling friction with coefficient f_r and downhill force depending on the slope $\gamma(s)$,

$$M_{\text{rd}}(s) := m g (\sin \gamma(s) + f_r \cos \gamma(s)) \quad (7)$$

The parameter g is the gravity constant. Finally, the engine's speed in revolutions per minute, depending on the selected gear y , is obtained from the truck's current velocity,

$$n_{\text{eng}}(s) := i_{\text{A}} i_{\text{T}}(y(s)) v(s) \cdot 60[\text{s}] / (2\pi r_{\text{stat}}). \quad (8)$$

B. Objective

We present objective functions combined into a performance criterion. The integral cost to be minimized is composed of a weighted sum of three different objectives.

TABLE I: Parameters of the truck model.

Name	Description	Unit
A	Front facing area	m^2
c_w	Aerodynamic shape coefficient	-
f_r	Coefficient of rolling friction	-
γ	Road's slope	rad
g	Gravity constant	m/s^2
i_{A}	Rear axle transmission ratio	-
$i_{\text{T}}(y)$	Gearbox transmission ratio	-
κ	Road's curvature	-
m	Vehicle mass	kg
$\eta_{\text{T}}(y)$	Gearbox degree of efficiency	-
ρ_{ai}	Air density	kg/m^3
r_{stat}	Static rear tire radius	m

TABLE II: Controls of the truck model.

Name	Description	Unit
R_{ind}	Indicated engine torque rate	Nm/s
R_{EB}	Engine brake torque rate	Nm/s
y	Selected gear number	-

1) *Deviation from a desired velocity*: The deviation of the truck's actual velocity from the desired one is penalized in a least-squares sense over the prediction horizon $[s_0, s_f]$ starting at the truck's current position s_0 on the road.

$$\Phi_{\text{dev}}[v(\cdot)] := \int_{s_0}^{s_f} (v(s) - v_{\text{des}}(s))^2 ds. \quad (9)$$

2) *Fuel consumption*: The fuel consumption is identified from a fuel consumption map $Q_f(n_{\text{eng}}, M_{\text{ind}})$:

$$\Phi_{\text{fuel}}[n_{\text{eng}}(\cdot), M_{\text{ind}}(\cdot)] := \int_{s_0}^{s_f} \frac{1}{v(s)} Q_f(n_{\text{eng}}(s), M_{\text{ind}}(s)) ds. \quad (10)$$

3) *Driving comfort*: Rapid changes of the indicated engine torque degrade the driving comfort as experienced by the truck driver:

$$\Phi_{\text{comf}}[R_{\text{ind}}(\cdot), R_{\text{EB}}(\cdot)] := \int_{s_0}^{s_f} \frac{1}{v(s)} (R_{\text{ind}}(s) + R_{\text{EB}}(s))^2 ds. \quad (11)$$

Weighting and summing up, we obtain the combined objective function

$$\Phi := \lambda_1 \Phi_{\text{dev}} + \lambda_2 \Phi_{\text{fuel}} + \lambda_3 \Phi_{\text{comf}}. \quad (12)$$

The choice of λ_1 and λ_2 allows for a compromise between matching the desired velocity as set by the truck driver at the cost of increased fuel consumption ($\lambda_1 \gg \lambda_2$) and following an economic operating mode of the truck at the cost of longer travel times ($\lambda_2 \gg \lambda_1$). In our computations, λ_3 is chosen comparatively small.

C. Bounds and Constraints

In this section, we discuss state and control bounds as well as nonlinear path constraints. Besides simple bounds on the truck controls for $s \in [s_0, s_f]$,

$$0 \leq R_{\text{ind}}(s) \leq R_{\text{ind,max}}, \quad 0 \leq R_{\text{EB}}(s) \leq R_{\text{EB,max}}, \quad (13)$$

and the integer restrictions in the gear choice,

$$y(s) \in \{1, \dots, y_{\text{max}}\}, \quad s \in [s_0, s_f], \quad (14)$$

where y_{max} is the constant number of available gears and may range from 8 to 24. The truck's driving strategy is subject to several constraints, a significant one being velocity limits imposed by law. From 3D map data, curvature $\kappa(s)$ of the road at position s is known and translated to an additional velocity limit $v_{\text{curve}}(\kappa(s))$. For $s \in [s_0, s_f]$, we have

$$0 < v(s) \leq v_{\text{max}} := \min\{v_{\text{law}}(s), v_{\text{curve}}(\kappa(s))\}. \quad (15)$$

The indicated and brake torques must respect upper limits as specified by the engine characteristics

$$0 \leq M_{\text{ind}}(s) \leq M_{\text{ind,max}}(n_{\text{eng}}(s)), \quad s \in [s_0, s_f], \quad (16a)$$

$$0 \leq M_{\text{EB}}(s) \leq M_{\text{EB,max}}(n_{\text{eng}}(s)). \quad (16b)$$

TABLE III: Differential states of the truck model.

Name	Description	Unit
t	Time	s
v	Velocity	m/s
M_{ind}	Indicated engine torque	Nm
M_{EB}	Engine brake torque	Nm

Finally, the engine speed n_{eng} itself must stay within prescribed limits according to the engine's specification,

$$n_{\text{eng,min}} \leq n_{\text{eng}}(s) \leq n_{\text{eng,max}}, \quad s \in [s_0, s_f]. \quad (17)$$

D. Control Problem Formulation

The mixed-integer optimal control problem to be solved in the loop by a predictive cruise control algorithm can now be formulated for a given state estimate $(\hat{v}, \hat{M}_{\text{ind}}, \hat{M}_{\text{EB}})$:

$$\begin{aligned} & \text{minimize} && \text{(Objective (12))} \\ & \text{subject to} && \text{(ODE system (1 – 3))} \\ & && \text{(Constraints (13 – 17))} \\ & && (t, v, M_{\text{ind}}, M_{\text{EB}})(s_0) = (0, \hat{v}, \hat{M}_{\text{ind}}, \hat{M}_{\text{EB}}) \end{aligned}$$

Numerical values for all parameters of Tab. I and for all constraint bounds, as well as polynomial approximations to the engine specific characteristics $M_{\text{ind,max}}$, $M_{\text{EB,max}}$, M_{fric} , and Q_f , can be found in [8]. This problem falls into the following more general class of mixed-integer optimal control problems (MIOCP) on $s \in [s_0, s_f]$:

$$\begin{aligned} \min_{x(\cdot), u(\cdot), \omega(\cdot)} & \int_{s_0}^{s_f} l(s, x(s), u(s), \omega(s)) \, ds + e(x(s_f)) & (18a) \\ \text{s.t.} & \dot{x}(s) = f(s, x(s), u(s), \omega(s)), & (18b) \\ & x(s_0) = \hat{x}_0, & (18c) \\ & 0 \leq c(s, x(s), u(s), \omega(s)), & (18d) \\ & 0 \leq d(s, x(s), u(s)), & (18e) \\ & \omega(s) \in \Omega := \{\omega^1, \dots, \omega^{n_\Omega}\} \subset \mathbb{R}^{n_\omega}, & (18f) \end{aligned}$$

wherein we denoted the state vector by $x = (t, v, M_{\text{ind}}, M_{\text{EB}})$ and the continuous control vector by $u = (R_{\text{ind}}, R_{\text{EB}})$. The integer control $\omega(\cdot)$ is chosen from a discrete set Ω comprising n_Ω choices $\omega^1, \dots, \omega^{n_\Omega}$ representing the different modes the system can switch between. For the truck model, $\omega = (i_T, \eta_T)^T$ and hence $\omega^j := (i_T(j), \eta_T(j))^T$ with $n_\Omega = y_{\text{max}}$. Constraints (18d) and (18e) comprise the set (16a, 16b, 17) of constraints that depend on ω , and the set (13, 15) of constraints independent of ω , respectively.

III. MIOCP FORMULATIONS AND METHODS

Problem (18) is inherently combinatorial, nonlinear, possibly nonconvex, and generally challenging to solve especially under real-time conditions and when targeting automotive embedded systems with limited computational resources. In this section, we briefly review a number of possible approaches. We propose a formulation and a numerical method that makes problem (18) tractable.

A. Approaches to MIOCP

In [4] a branch-and-bound algorithm solving continuous optimal control problems (OCP) in the tree nodes was described for an automotive control problem, and demonstrates the combinatorial effort required for its solution by enumeration. A comparison to our approach proposed in §III-C was carried out in [11].

Direct discretization in time of problem (18), e.g. using direct collocation [2] or direct multiple shooting methods [3], [12], yields a possibly nonconvex mixed-integer nonlinear

program (MINLP). Although sophisticated methods have been developed for MINLP, real-time feasible computation times have not yet been achieved.

Naïve relaxation of the integer constraint (14) on the other hand poses the challenge of defining ratios i_T and efficiencies η_T for fractional gear choices $y \in [1, y_{\text{max}}]$. Feasibility or optimality guarantees for solutions obtained by rounding a relaxed optimal solution can in general not be given.

B. Partial Outer Convexification and Relaxation

To address this issue, it was proposed in [14] to introduce a new binary control function $\omega_k(s) \in \{0, 1\}$ on $[s_0, s_f]$ for each choice $\omega^k \in \Omega$ available for $\omega(s)$, using the bijection

$$\omega(s) = \omega^k \iff \omega_k(s) = 1, \quad \sum_{j=1}^{n_\Omega} \omega_j(s) = 1. \quad (19)$$

Model functions l (18a) and f (18b) depending on $\omega(\cdot)$ are replaced by their outer convexifications $\sum_{j=1}^{n_\Omega} \omega_j(\cdot) f(\cdot, \omega^j)$. A single continuous OCP has to be solved after substituting $\omega(s) \in \{0, 1\}^{n_\Omega}$ with its relaxed counterpart $\alpha(s) \in [0, 1]^{n_\Omega}$. The relaxed optimal solution to this convexified OCP can be shown to constitute a lower bound to the MIOCP solution [16]. Moreover, given a sufficiently fine time discretization, the optimality gap can be made arbitrarily small [15] when applying, where necessary, a special sum-up rounding scheme [14] to the relaxed solution.

C. Complementarity and Vanishing Constraints

Constraints (16a, 16b, 17) depend on the integer gear choice $y(s)$. Special attention must be paid to these constraints when rounding optimal solutions computed from reformulations that relax the integrality requirement (14). We propose two different constraint formulations: Applying partial outer convexification also to the residual, we obtain

$$0 \leq \sum_{j=1}^{n_\Omega} \alpha_j(s) \cdot c(s, x(s), u(s), \omega^j, p). \quad (20)$$

This formulation is numerically well-behaved, but suffers from compensatory effects: A fractional, convex combination of two or more infeasible residuals may turn out to be feasible, but any integer choice $\alpha_j(s) = 1$ may violate $0 \leq c(\cdot, \omega^j)$. As an alternative, we may ask for feasibility of each choice,

$$0 \leq \alpha_j(s) \cdot c(s, x(s), u(s), \omega^j, p), \quad 1 \leq j \leq n_\Omega. \quad (21)$$

This second formulation guarantees feasibility after rounding of any $\alpha_j(s) \neq 0$, but violates constraint qualifications if $\alpha_j(s) = 0$, see e.g. [1], [17], and is known as a Mathematical Program with Vanishing Constraints (MPVC). It is structurally combinatorial and nonconvex. With few exceptions, nonlinear solvers struggle with problems of this structure.

IV. FAST MIXED-INTEGER NMPC ALGORITHM

In view of the real-time demands of mixed-integer NMPC, we follow the partial outer convexification and relaxation approach. To guarantee feasibility, we adopt the vanishing constraint formulation (21) for engine torque and speed constraints despite its inherent nonconvexity. A new and tailored

sequential nonconvex quadratic programming method [10] is used to solve the arising MPVCs. Block structures induced by the time discretization are exploited in the factorizations [19] and during updates carried out in this active-set approach [9].

A. Time Discretization and MPVC Formulation

The partially convexified relaxation of problem (18) is discretized in time to obtain a finite-dimensional nonlinear program (NLP) using, for example, direct collocation [2], or, in this article, the direct multiple shooting method for optimal control. For details we refer the reader to [3], [12] and only mention here that this approach introduces a time grid $s_0 < s_1 < \dots < s_N = s_f$ on the horizon $[s_0, s_f]$, together with state variables $x_i \in \mathbb{R}^{n_x}$ for the grid points indexed by $0 \leq i \leq N$ and control variables $q_i \in \mathbb{R}^{n_u}$, $\alpha_i \in [0, 1]^{n_\Omega}$ on the grid intervals indexed by $0 \leq i < N$. This results in a large but structured MPVC of the form

$$\begin{aligned} \min_{x, q, \alpha} \quad & \sum_{i \in \mathcal{I}} L_i(x_i, q_i, \alpha_i) + E(x_N) & (22a) \\ \text{s.t.} \quad & 0 = X_i(x_i, q_i, \alpha_i) - x_{i+1}, \quad i \in \mathcal{I} \setminus \{N\}, & (22b) \\ & 0 = \hat{x}_0 - x_0, & (22c) \\ & 0 \leq D_i(x_i, q_i), \quad i \in \mathcal{I}, & (22d) \\ & 0 \leq \alpha_{ij} \cdot C_{ij}(x_i, q_i, \omega^j), \quad i \in \mathcal{I}, j \in \mathcal{J}, & (22e) \\ & 0 \leq \alpha_{ij}, \sum_{j \in \mathcal{J}} \alpha_{ij} = 1, \quad i \in \mathcal{I}, j \in \mathcal{J}, & (22f) \end{aligned}$$

wherein for brevity $\mathcal{I} := \{1, \dots, N\}$ indexes the discretization points and $\mathcal{J} := \{1, \dots, n_\Omega\}$ the switching mode numbers. Constraint (22b) represents the multiple shooting parameterized ODE system, with embedded state estimate or measurement \hat{x}_0 (22c). Eq. (22d) subsumes all constraints independent of the integer control $\omega(\cdot)$, and (22e) gives the vanishing constraint formulation of the gear choice dependent constraints. Eq. (22f), is the SOS1-relaxation (20) of the convexified integer control.

B. Mixed-Integer Moving Horizon NMPC Algorithm

Summarizing, after modeling the predictive switched optimal control problem using the partial outer convexification approach and adopting the vanishing constraint formulation, a single iteration of the mixed-integer NMPC algorithm reads as follows:

- 1) Evaluate functions, derivatives, and Hessian of problem (22) in iterate (x^k, q^k, α^k) . This requires solving the IVPs and computing sensitivities w.r.t. (x^k, q^k, α^k) . Set up a local linear-quadratic subproblem, a Quadratic Problem with Vanishing Constraints (QPVC), as far as possible without knowledge of \hat{x}_0 .
- 2) Obtain system state measurement or estimate \hat{x}_0 . Solve the QPVC, see [10], to find the relaxed feedback control vector (q_0^k, α_0^k) . Round α_0^k according to the sum-up rounding scheme [14]. Feed rounded mixed-integer feedback controls to truck.
- 3) Continue solution of QPVC for $x_1^k, \dots, x_N^k, q_1^k, \dots, q_{N-1}^k$ and $\alpha_1^k, \dots, \alpha_{N-1}^k$ to find new MPVC iterate. Loop with $k \leftarrow k + 1$.

C. Solution of QPVCs and Structure Exploitation

For a general background on sequential nonconvex programming, we refer the reader to [17] who discusses the case of complementarity programs. In step 2), we need to solve a QPVC of the general structure ($0 \leq i \leq N, 1 \leq j \leq n_\Omega$)

$$\min_{\Delta w^k} \Delta w^k{}^T B^k \Delta w^k + b^k{}^T \Delta w^k \quad (23a)$$

$$\text{s.t.} \quad 0 = G^k \Delta w^k + g^k, \quad (23b)$$

$$0 = \Delta x_0^k + (x_0^k - \hat{x}_0^k), \quad (23c)$$

$$0 \leq D^k \Delta w^k + d^k, \quad (23d)$$

$$0 \leq (\Delta \alpha_{ij}^k + \alpha^k) \cdot (C_{ij}^k \Delta w^k + c_{ij}^k), \quad (23e)$$

$$0 \leq \Delta \alpha^k + \alpha^k, \sum_{j \in \mathcal{J}} \Delta \alpha_{ij}^k = 0 \quad (23f)$$

with column vector $\Delta w^k = (\Delta x^k, \Delta q^k, \Delta \alpha^k)$. In (23), B^k is (an approximation of) the Hessian of the MPVC-Lagrangian, see [1]; b^k is the gradient of the objective, G^k , C^k , D^k are Jacobians of the constraints (22b), (22d), and (22e); and g^k , c^k , d^k are the respective constraint residuals. All are evaluated in the current MPVC iterate $w^k = (x^k, q^k, \alpha^k)$. Note that C_{ij}^k denotes the row of C^k associated with the multiplier α_{ij} of choice v^j at discretization point i ; similarly c_{ij}^k is a scalar entry of c^k .

For this QPVC, a tailored active set method is described in [8], [10]. Extensive exploitation of direct multiple shooting structures present in this QPVC is crucial for linear algebra efficiency. Block structured factorizations suitable for a direct multiple shooting discretization are described in [19]. In active set methods, one wants to compute an initial factorization only, and update this after every active set exchange. For direct multiple shooting, a block structured approach with $O(N)$ and $O(n_\Omega)$ runtime complexity is described in [8], [9].

V. NUMERICAL RESULTS

In this section we present numerical results for a simulated mixed-integer NMPC environment using the truck model and the algorithmic framework proposed in the previous sections. All numerical results have been computed on an Intel Pentium 4 machine with 2.67 GHz and 1 GB of RAM, running SuSE Linux 10.1.

A. Two Artificial Scenarios

For the following scenarios, we choose $y_{\max} = 16$ gears and a prediction horizon length of 2km, discretized into 40 intervals of 50m length each. Hence, 40 possible gear shifts on the prediction horizon are considered for optimization in each feedback step. Unless indicated otherwise, feedback is given every 50m. Objective function weights are chosen as $\lambda_1 = 10$, $\lambda_2 = 10^{-2}$, $\lambda_3 = 10^{-4}$ to realize a speed-oriented controller.

Figure 1 presents the computational results for a steep slope scenario, showing optimal trajectories for the engine torque M_{ind} and the gear y , as well as the resulting velocity. The truck starts at a position of 0m, and aims at keeping a velocity of $v_{\text{des}} = 80\text{km/h}$, not exceeding $v_{\text{max}} = 85\text{km/h}$. Fig. 1 Given the prediction horizon length of 1km, the

controller recognizes the oncoming slope when passing the 3.0km mark. It accelerates shortly before the 4.5km mark, early enough for the truck to reach the maximum allowed velocity of 85km/h when the sloped section starts at 5.0km. By gaining initial momentum and by running at maximum engine torque during the slope, a terminal velocity at the slope's exit around the 5.5km mark of about 45km/h is achieved. The truck requires another 0.5 km to return to the desired velocity v_{des} .

Fig. 2 presents the computational results for a sharp curve scenario. The curve comes into view of the predictive controller at the 3.0km mark. The controller initiates a braking maneuver approximately 0.4km later to account for the decrease in the desired velocity starting at 4.5km. At the same time, the engine torque is reduced to zero. Since usage of brakes is not penalized, this maneuver is initiated as late and as strong as possible. It is further assisted by gradually shifting to lower gears, thus running the engine at higher speeds to support engine braking. The curve is traversed in a gear optimal for the maximum velocity of approximately 25km/h allowed due to lateral forces. Once the truck enters the straight part of the road again, the desired velocity returns to 80km/h and is attained by accelerating and shifting gears up again to maintain feasible engine speeds. Spikes in the optimal indicated engine torque trajectories in Fig. 2b during acceleration are caused by these gear shifts. As the transmission ratio and thus the engine's speed n_{eng} changes abruptly, a jump in the engine torque M_{ind} is optimal with respect to the combined objective function.

B. Scenarios using Real-World Data

Fig. 3 shows mixed-integer NMPC results for a section

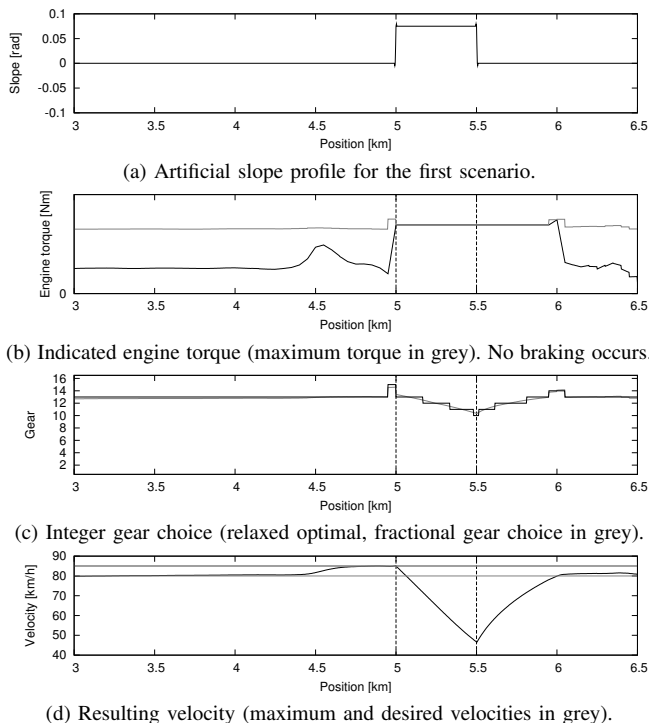


Fig. 1: Results for first scenario, tackling a steep slope.

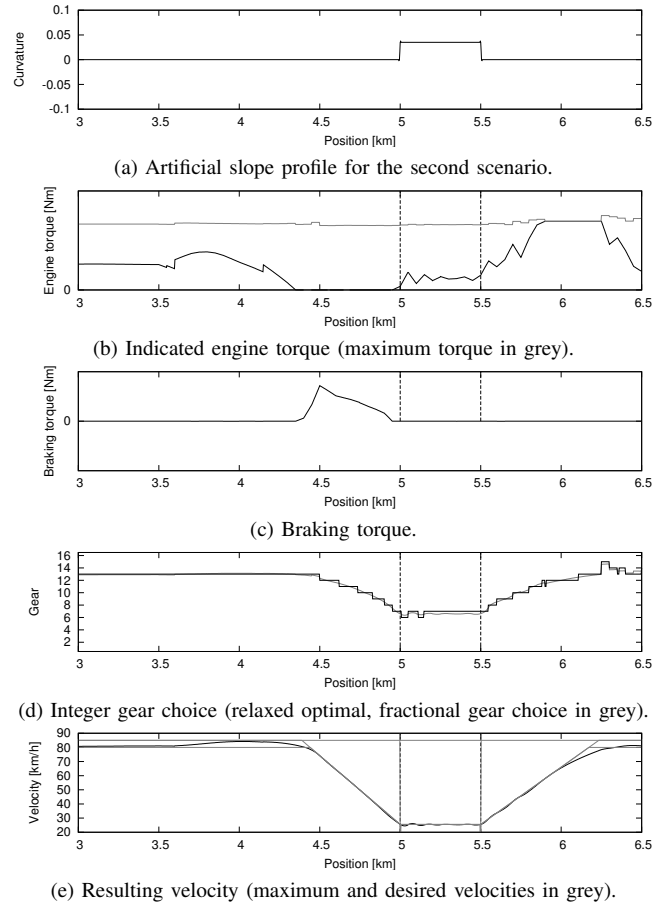


Fig. 2: Results for second scenario, tackling a sharp curve.

of the German Autobahn A8, tracking $v_{des} = 80$ km/h. The highway profile is characterized by several steep slopes, see Fig. 3a. These lead to significant deviations from the desired velocity, see Fig. 3c, while the truck's engine is providing maximum torque, see Fig. 3b. In order to maintain a feasible engine speed, these velocity losses and gains are accompanied by down- and upshift sequences, see Fig. 3d.

In this scenario, feedback is given every 10m to cope with the strongly nonlinear slope profile. The total travel time for 150km using the predictive control profiles of Figure 3 is 2 h 08min. The average computation time was 20 ms for step 1) and 3ms for step 2) in §IV-B. Step 3) is negligible for the problem at hand. The delay is safely below 10m : 80 km/h = 450ms. Hence, the described approach would be real-time feasible even on an industry-type embedded platform with up to 20 times less computing power.

We also evaluated the influence of the objective weights λ_1 , λ_2 on the behavior of the algorithm. Feedback was again given every 50m only. In Table IV we summarize the performance of the algorithm on this highway section for different choices of the velocity tracking and fuel consumption weights λ_1 and λ_2 , while λ_3 remained fixed at 10^{-4} . As can be seen, compared to a reference choice taken to constitute 100%, fuel savings as large as 20%, and increases as large as 10%, can be realized at a cost, or gain, in travel time.

TABLE IV: Performance on a 150km highway section.

λ_1	λ_2	fuel cons.	travel time	total/per-step CPU time
0.1	1.0	81%	3h 24min 32s	1min 28s / 30ms
1.0	0.1	83%	3h 01min 53s	1min 18s / 26ms
1.0	0.01	100%	2h 33min 08s	1min 21s / 27ms
10.0	0.01	109%	2h 09min 21s	1min 22s / 28ms
20.0	0.01	112%	2h 02min 29s	1min 27s / 29ms

VI. CONCLUSION AND OUTLOOK

We have presented a novel numerical algorithm for mixed-integer optimal control of nonlinearly constrained dynamic systems, and investigated the special case of model-predictive control of heavy-duty trucks. Information about road sections ahead of the truck is employed to predict acceleration and braking actions as well as gear choices which are optimal with regard to a compromise between fuel consumption and deviation from a desired velocity. An ODE model used for simulation of the heavy-duty truck's behavior, as well as physical and engineering bounds and constraints to be respected have been presented. We have formulated a predictive mixed-integer optimal control problem, and have described the numerical methods used for its efficient solution. We have discussed several approaches on how to treat the discrete choice of gears within an NLP framework. Compared to previous work in [20], our method is capable of computing up to 40 optimal gear shifts on the

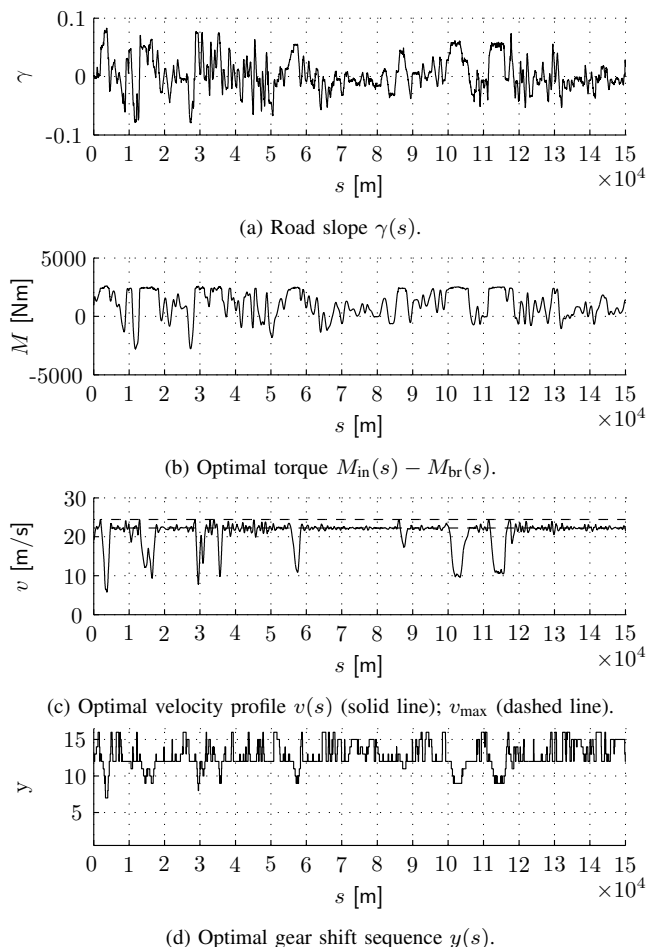


Fig. 3: Results for a 150km section of Autobahn A8.

prediction horizon, instead of only one. Numerical results for two case studies on exemplary road sections have been presented and discussed. We have reported travel times and fuel consumptions achieved by application of the presented algorithm to a real-world highway section. These results are encouraging in terms of both performance and computational effort required to obtain them.

Further research on the presented topic of mixed-integer nonlinear model predictive control will have to focus on hot-startable numerical methods for mathematical programs with complementarity and vanishing constraints. In [7] we also investigated the connection of generalized disjunctive programming, e.g. [5], to tight convex relaxations of problem (18). In addition, relaxation-linearization techniques, e.g. [18] may in the future help to avoid the peculiar nonconvex structure induced by the vanishing constraint formulation. Finally, mixed-integer pareto front techniques, such as those investigated in [13], will be used to deepen our insight into the compromising objective.

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