

# Design of Multi-objective Control Systems with Optimal Failure Tolerance

Ali Mesbah and Richard D. Braatz

**Abstract**—The selection of the control structure, which is a specification of the interconnection of measurements, exogenous inputs, and manipulated variables, is a critical step in the design of a control system. This paper presents a general internal model control structure with multiple degrees-of-freedom in which each controller can be independently designed for stable systems. The control system is shown to remain optimal when controllers are taken off-line due to component failures, without requiring re-design of any of the on-line controllers. The optimality of the proposed approach is demonstrated for the control of a simulated thin-film process for a variety of component failures.

## I. INTRODUCTION

The design of the control structure, which is the specification of the interconnection of measurements, exogenous inputs, and manipulated variables, greatly influences the performance achievable by a control system. In practice, control systems are usually required to fulfill multiple objectives that are poorly described using a single performance measure as commonly proposed in the literature. This realization has led to the development of numerous control structures that have multiple degrees-of-freedom (e.g., see [1], [2], [3], [4], [5], [6], [7], [8], and the citations therein), with each controller degree-of-freedom tasked with addressing some subset of the control objectives. As failures in system components inevitably occur in practice, an important practical consideration is to design control structures and their associated controllers to have graceful performance degradation during component failures (e.g., see [9], [10], [11]).

Internal model control (IMC) is a control design method developed in the 1970s–1980s with several useful features, including that it provides a convenient theoretical framework for the design of two degrees-of-freedom control systems [12]. The basic idea is to combine an optimal controller obtained from the nominal process model with a low-pass filter to tradeoff closed-loop performance with robustness to model uncertainties. Another feature of the IMC structure is that it simplifies the task of controller design by employing the Youla parameterization to write the nominal closed-loop transfer functions as an affine function of the to-be-designed controller(s). The IMC structure can also be implemented in a manner that ensures internal nominal stability of the closed-loop system in the presence of actuator constraints [12], [13].

This paper presents a systematic procedure for the design of multiple degrees-of-freedom controllers based on an extension of the internal model control design method. The cornerstone of the design procedure is a general control structure that provides a convenient framework for multi-objective controllers design through decoupling of the performance measures defined for the different control objectives. This control structure circumvents most of the tradeoffs that are inherent in a classical feedback control structure. The approach is shown to enable the design of optimal failure-tolerant controllers without compromising on the best achievable performance and without requiring redesign of the on-line controllers. The control structure is used to design a multi-objective control system for a simulated thin-film process for a variety of disturbances and component failures.

## II. NOTATION AND PRELIMINARIES

Throughout the paper, a finite-dimensional multi-input multi-output process is denoted by  $P(s) \in \mathcal{RH}_\infty$  where  $s$  is the Laplace variable and  $\mathcal{RH}_\infty$  denotes the real rational subspace of  $\mathcal{H}_\infty$  consisting of all proper and rational stable transfer matrices. The exogenous inputs  $r(t)$ ,  $l_m(t)$ ,  $l_u(t)$ ,  $d_m(t)$ , and  $d_u(t)$  are bounded signals (i.e.,  $r(t), l_m(t), l_u(t), d_m(t), d_u(t) \in \mathcal{L}_p[0, \infty)$ , where  $\mathcal{L}_p[0, \infty)$  encompasses all signal sequences on  $[0, \infty)$  which have finite  $p$ -norm). The real-valued function  $\|\cdot\|$  denotes any norm defined over the linear vector space of the signals. The induced system norm  $|\cdot|$  is defined as the supremum of the output signal norm over a norm-bounded set of input signals [14].

*Definition 1 (Internal Stability [12]):* A continuous-time linear time-invariant (LTI) closed-loop system is *internally stable* if the transfer functions between any two points of the closed-loop system are stable (i.e., have all poles in the open left-half plane).

*Definition 2 (Robust Stability [12]):* A closed-loop system is *robustly stable* if the controller  $C$  ensures the internal stability of the closed-loop system for all  $P \in \mathcal{P}$ , where  $\mathcal{P}$  is the set of uncertain processes.

## III. SYSTEMATIC DESIGN PROCEDURE

The fundamental questions central to the design of a control strategy can be summarized as:

- 1) Control structure: Does the control structure limit the achievable performance?
- 2) Controller design: Do the closed-loop performance measures reflect the control objectives?

\*Financial support is acknowledged from Novartis Pharma AG.  
The authors are with the Massachusetts Institute of Technology, 77  
Massachusetts Avenue, Cambridge, MA 02139, USA  
amesbah@mit.edu  
braatz@mit.edu

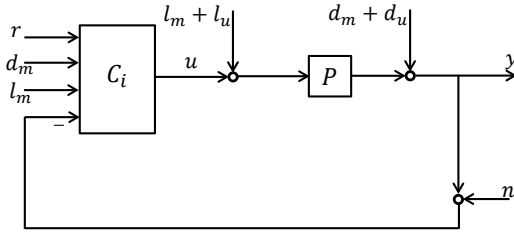


Fig. 1. General classical feedback control system where  $C_i = [C_r, C_{dm}, C_{lm}, C_y]$ .

- 3) Controller implementation: Is it feasible to realize all the control objectives given the available degrees of freedom?

This section addresses each of these questions in order.

#### A. Control Structure

The most general control structure for a process  $P$  with manipulated variable  $u$ , reference  $r$ , measured load disturbance  $l_m$ , unmeasured load disturbance  $l_u$ , measured output disturbance  $d_m$ , unmeasured output disturbance  $d_u$ , and measurement noise  $n$  is shown in Fig. 1.<sup>1</sup> All variables that can be measured are fed directly into the controller  $C_i$  that is to be designed to ensure (i) internal stability of the closed-loop system, (ii) the output  $y$  closely tracks the reference  $r$  (i.e., small error  $e = y - r$ ), and (iii) the effects of the measurement noise and measured and unmeasured disturbances on the closed-loop error  $e$  are suppressed. The mapping between all of the inputs to the closed-loop system and the process output and manipulated variable is described by

$$\begin{bmatrix} y \\ u \end{bmatrix} = H(P, C_i) \begin{bmatrix} r \\ l_m \\ l_u \\ d_m \\ d_u \\ n \end{bmatrix} \quad (1)$$

where the transfer matrix  $H(P, C_i)$  is

$$\begin{bmatrix} PC_r(I + PC_y)^{-1} & PC_{lm}(I + PC_y)^{-1} + (I + PC_y)^{-1}P \\ C_r(I + PC_y)^{-1} & C_{lm}(I + PC_y)^{-1} - C_y(I + PC_y)^{-1}P \\ (I + PC_y)^{-1}P & PC_{dm}(I + PC_y)^{-1} + (I + PC_y)^{-1} \\ -C_y(I + PC_y)^{-1}P & C_{dm}(I + PC_y)^{-1} - C_y(I + PC_y)^{-1} \\ (I + PC_y)^{-1} & -PC_y(I + PC_y)^{-1} \\ -C_y(I + PC_y)^{-1} & -C_y(I + PC_y)^{-1} \end{bmatrix}. \quad (2)$$

A standard approach in optimal control is to formulate a single performance measure in terms of an overall norm such as a weighted  $H_2$  or  $H_\infty$ -norm on  $H(P, C_i)$  [14]. However,

<sup>1</sup>Due to space limitations and to simplify the analytical expressions, explicit transfer functions for the various disturbances are not shown; the generalization of the results of this paper to include such transfer functions is straightforward.

a drawback of this approach is that a typical control problem has multiple objectives that are independently defined in terms of relationships between specific inputs and specific outputs. Several of the closed-loop transfer functions in (2) that relate the system inputs to the output  $y$  and manipulated variable  $u$  are functions of multiple controller transfer functions, so that the designs of these controller transfer functions to satisfy multiple independently defined control objectives are not independent. Next, an alternative control structure is presented that is provably general while having each term in the relationship between an input and output being a function of only one controller transfer function.

Consider the internal model control structure in Fig. 2 for the formulation of the control design problem where  $Q_i$  is the IMC controller [12]. For the class of stable linear time-invariant systems, Thm. 1 states that Fig. 2 provides a non-restrictive control structure for the design of multi-objective controllers.

**Theorem 1:** Consider a stable LTI system  $P$  with measured output  $y$ , manipulated input  $u$ , measurement noise  $n$ , and disturbances  $l_m, l_u, d_m$ , and  $d_u$ . The IMC structure in Fig. 2 is the most general LTI control structure for the design of multiple degrees-of-freedom control systems that internal stabilize the closed-loop system.

*Proof:* Let  $P = NM^{-1} = \tilde{M}^{-1}\tilde{N}$  where  $\{M, N\}$  and  $\{\tilde{M}, \tilde{N}\}$  are right and left coprime factorizations of  $P$  over  $\mathcal{RH}_\infty$ , respectively. Define  $C_{y0} = UV^{-1} = \tilde{V}^{-1}\tilde{U}$  as a stabilizing controller such that  $\tilde{V}$  and  $\tilde{U}$  satisfy the Bezout identity  $\tilde{V}M + \tilde{U}N = I$ . It is well-known that all stabilizing LTI feedback controllers are parameterized by

$$C_y = (\tilde{V} - Q_y\tilde{N})^{-1}(\tilde{U} + Q_y\tilde{M}) \quad (3)$$

where  $Q_y$  is any stable LTI transfer function (see [14]).

For a stable LTI process (i.e.,  $P \in \mathcal{RH}_\infty$ ), choose  $\tilde{N} = P$ ,  $\tilde{M} = I$ ,  $\tilde{U} = 0$ , and  $\tilde{V} = I$  [12]. This results in the Youla parameterization of all stabilizing feedback controllers for a stable LTI process  $P$  [15]:

$$C_y = (I - Q_yP)^{-1}Q_y = Q_y(I - PQ_y)^{-1}. \quad (4)$$

Insertion of this equation into (2) defines the set of all possible LTI closed-loop transfer functions that are internally stable. Hence,  $H(P, C_i)$  takes the form

$$\begin{bmatrix} PC_r(I + PC_y)^{-1} & PC_{lm}(I + PC_y)^{-1} + (I + PC_y)^{-1}P \\ C_r(I + PC_y)^{-1} & C_{lm}(I + PC_y)^{-1} - C_y(I + PC_y)^{-1}P \end{bmatrix}$$

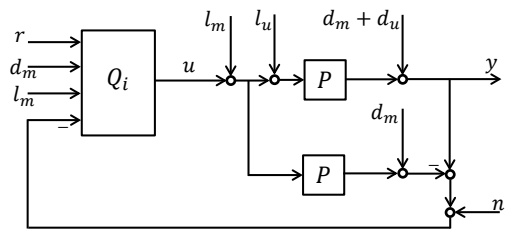


Fig. 2. General control structure for a stable LTI system where  $Q_i = [Q_r, Q_{dm}, Q_{lm}, Q_y]$ .

$$\begin{aligned}
(I - PQ_y)P & \quad PC_{dm}(I + PC_y)^{-1} + (I + PC_y)^{-1} \\
-Q_yP & \quad C_{dm}(I + PC_y)^{-1} - C_y(I + PC_y)^{-1} \\
& \quad \begin{bmatrix} I - PQ_y & -PQ_y \\ -Q_y & -Q_y \end{bmatrix}. \tag{5}
\end{aligned}$$

The expression in (5) implies that internal stability of the overall closed-loop system requires that the transfer functions

$$\begin{aligned}
C_r(I + PC_y)^{-1} \\
C_{lm}(I + PC_y)^{-1} \\
C_{dm}(I + PC_y)^{-1}
\end{aligned} \tag{6}$$

are stable.<sup>2</sup>

Define

$$\begin{aligned}
Q_r &= C_r(I + PC_y)^{-1} \\
Q_{lm} &= C_{lm}(I + PC_y)^{-1} - C_y(I + PC_y)^{-1}P \\
Q_{dm} &= C_{dm}(I + PC_y)^{-1} - C_y(I + PC_y)^{-1}.
\end{aligned} \tag{7}$$

It follows from the definition of  $C_y$  (i.e., (4)) that  $C_r$ ,  $C_{lm}$ , and  $C_{dm}$  in (7) encompass the set of all stabilizing feedback controllers for any stable  $Q_r$ ,  $Q_{lm}$ , and  $Q_{dm}$ , respectively. This implies that (5) is internally stable.

Insertion of (7) into (5) simplifies the transfer matrix  $H(P, C_i)$  to

$$\begin{bmatrix} PQ_r & P(I + Q_{lm}) & (I - PQ_y)P & I + PQ_{dm} \\ Q_r & Q_{lm} & -Q_yP & Q_{dm} \\ & I - PQ_y & -PQ_y & \\ & -Q_y & -Q_y & \end{bmatrix}. \tag{8}$$

The block diagram in Fig. 2 has the same closed-loop transfer matrix as in (8) and, therefore, is equivalent to the general classical feedback control system in Fig. 1. The control structure in Fig. 2 is non-restrictive for stable LTI systems since it entails the set of all stabilizing controllers  $C_i$  for any stable  $Q_i$ . ■

The control structure in Fig. 2 can be used for IMC implementation [12] by replacing the lower  $P$  in Fig. 2 with a process model  $\tilde{P}$ . This structure is an extension of the IMC structure to systems with four degrees of freedom. Thm. 1 indicates that the proposed IMC control structure does not restrict the set of closed-loop transfer functions that ensure internal stability of the closed-loop system. A consequence of this result is that the use of the IMC control structure does not limit the achievable closed-loop performance, regardless of the closed-loop performance measure(s) used to encode the control objectives. These characteristics are in contrast to most of the control structures that have been proposed for control systems with multiple degrees-of-freedom (e.g., see papers cited in [16]). Next, we discuss how multiple control objectives can be met by independent controllers design.

<sup>2</sup>Aside: the fact that  $(I + PC_y)^{-1}$  is stable for all stable  $Q_y$  indicates that the only unstable poles allowed in  $C_r$ ,  $C_{lm}$ , and  $C_{dm}$  for internal stability must also be unstable zeros of  $(I + PC_y)^{-1}$ .

## B. Controllers design and implementation

The control structure in Fig. 2 provides a convenient framework for the design of multi-objective controllers because the closed-loop transfer matrix (8) depends on  $Q_i$  in an affine manner, and all columns depend on only one  $Q_i$ . Hence, the optimal controllers  $Q_r$ ,  $Q_{lm}$ ,  $Q_{dm}$ , and  $Q_y$  for multiple objectives between the system inputs and outputs can be designed independently.

Based on the closed-loop mapping obtained from (8) between  $[e, u]^T$  and each of the system inputs, the following multiple control objectives can be defined:

- Reference tracking:

$$\inf_{Q_r} \left\| \begin{bmatrix} PQ_r - I \\ Q_r \end{bmatrix} r \right\|. \tag{9}$$

- Measured load disturbance rejection:

$$\inf_{Q_{lm}} \left\| \begin{bmatrix} P(I + Q_{lm}) \\ Q_{lm} \end{bmatrix} l_m \right\|. \tag{10}$$

- Unmeasured load disturbance rejection:

$$\inf_{Q_y} \left\| \begin{bmatrix} (I - PQ_y)P \\ -Q_yP \end{bmatrix} l_u \right\|. \tag{11}$$

- Measured output disturbance rejection:

$$\inf_{Q_{dm}} \left\| \begin{bmatrix} I + PQ_{dm} \\ Q_{dm} \end{bmatrix} d_m \right\|. \tag{12}$$

- Unmeasured output disturbance rejection:

$$\inf_{Q_y} \left\| \begin{bmatrix} I - PQ_y \\ -Q_y \end{bmatrix} d_u \right\|. \tag{13}$$

- Measurement noise suppression:

$$\inf_{Q_y} \left\| \begin{bmatrix} -PQ_y \\ -Q_y \end{bmatrix} n \right\|. \tag{14}$$

In practice, a single signal  $v$

$$v = Pl_u + d_u \tag{15}$$

is typically used to represent the combined effect of unmeasured load and unmeasured output disturbances. This implies that the two performance measures (11) and (13) can be replaced with a single expression

$$\inf_{Q_y} \left\| \begin{bmatrix} I - PQ_y \\ -Q_y \end{bmatrix} v \right\|. \tag{16}$$

As such, the above norm expressions are applicable to both continuous-time and discrete-time transfer functions and for different norms used as performance measures. For each control objective, an alternative performance measure is to replace the signal norm with an induced system norm. For example, the measurement noise suppression objective is often expressed in terms of its induced system norm as

$$\inf_{Q_y} \left\| \begin{bmatrix} -PQ_y \\ -Q_y \end{bmatrix} \right\|. \tag{17}$$

The performance measures in (9)-(14) indicate that  $Q_r$ ,  $Q_{lm}$ , and  $Q_{dm}$  only influence one control objective. Hence,

the latter controller transfer functions can be designed independently of each other and independently of  $Q_y$ . The only tradeoff in each of the designs of  $Q_r$ ,  $Q_{lm}$ , and  $Q_{dm}$  is that fast speed of response (i.e., the effect on  $y$ ) will be associated with faster and larger changes in the manipulated variable  $u$ . This tradeoff can be implemented by placing a weight on one or both of the closed-loop error  $e$  and manipulated variable  $u$  signals. In addition, the design of  $Q_y$  requires prioritizing the multiple control objectives in view of their importance, as  $Q_y$  is the only controller to suppress the effects of unmeasured disturbances and measurement noise (see expressions (14) and (16)). Next, we discuss how the control structure in Fig. 2 enables the controllers to be implemented such that they lead to optimal achievable performance in the presence of system failures.

### C. Failure-tolerant Controller Design

A common approach to failure-tolerant control is to design a single control system using robust control techniques to deal with all potential actuator and/or sensor failures (e.g., see the discussion in [5]). Since this approach designs the control system for the worst-case performance, it may lead to very conservative performance when no actuator and/or sensor failures occur.

The special feature of the proposed control structure that the controllers  $Q_r$ ,  $Q_{lm}$ ,  $Q_{dm}$ , and  $Q_y$  are designed independently of each other is particularly significant for failure-tolerant control when a system component (actuator or sensor) needs to be taken out of service due to a failure. The design of the controllers in Section III-B can be posed as the multi-objective optimization

$$\inf_{\mathbf{Q}_i} \sum_{k=1}^4 w_k F_k(Q_i) \quad (18)$$

where  $w_k \geq 0$  and the objective functions  $F_k(Q_i)$  are the signal or induced system norms of the columns of the closed-loop mapping between  $[e, u]^T$  and the system inputs (see (9)-(17)). In (18), the vector  $\mathbf{Q}_i$  consists of the to-be-designed controllers  $Q_r$ ,  $Q_{lm}$ ,  $Q_{dm}$ , and  $Q_y$ .

*Theorem 2:* Consider the multi-objective optimization (18). The solution to the optimization remains globally optimal as the controller(s)  $Q_i$  and the respective objective function(s)  $F_k(Q_i)$  are eliminated from the multiple-objective function in (18).

*Proof:* Let the optimal solution to the convex optimization (18) be

$$\mathbf{Q}_i^* = \arg \min_{\mathbf{Q}_i} \sum_{k=1}^4 w_k F_k(Q_i). \quad (19)$$

Since each objective function depends on only one decision variable  $Q_i$ , the optimal solution to every objective function  $F_k(Q_i)$  is independent of the other objective functions, implying that

$$Q_i^* = \arg \min_{Q_i} F_k(Q_i). \quad (20)$$

Hence, when  $F_{k'}(Q_i)$  is eliminated from the multiple-objective function, the optimal solution

$$\mathbf{Q}_i^* = \arg \min_{\mathbf{Q}_i} \sum_{k=1, k \neq k'}^4 w_k F_k(Q_i) \quad (21)$$

remains the same as that in (19) for the rest of the  $Q_i$ . ■

Thm. 2 indicates that when optimal control is used to design each  $Q_i$  independently, the rest of the  $Q_i$  remain optimal if one or more of the other controllers are taken out of service (i.e., set to 0) due to an actuator or sensor failure. In other words, the overall control system remains optimal for the multiple achievable objectives when any  $Q_i$  is taken out of service. This implies that the remaining controllers need not be redesigned to realize optimal failure-tolerant control. Such an approach to optimal failure tolerance leads to vastly superior performance under most conditions than designing a controller to optimize the worst-case performance for all possible failure conditions.

The proposed control structure also possesses a distinct feature for fault-tolerant control when abnormal system operation results from a change in process dynamics and/or disturbance characteristics. When a change in process dynamics can be characterized by model uncertainty, the lower  $P$  in Fig. 2 is replaced by a process model  $\tilde{P} \in \mathcal{P}$ . In this case, the internal stability of the closed-loop system in Fig. 2 for any  $\tilde{P} \in \mathcal{P}$  depends on only  $Q_y$ .

*Theorem 3:* For stable  $Q_r$ ,  $Q_{lm}$ , and  $Q_{dm}$ , robust stability of the closed-loop system in Fig. 2 depends on only  $Q_y$ ,  $\tilde{P}$ , and  $\mathcal{P}$ , where  $\mathcal{P}$  is the set of uncertain processes. In particular, robust stability does not depend on  $Q_r$ ,  $Q_{lm}$ , or  $Q_{dm}$ .

*Proof:* Suppose that the process dynamics are described by the model  $\tilde{P}$  that belongs to the uncertainty set  $\mathcal{P}$ . Replace the lower  $P$  in the general control structure of Fig. 2 with  $\tilde{P}$ . The closed-loop transfer matrix  $H(\tilde{P}, Q_i)$  for the mapping between  $[y, u]^T$  and the system inputs (see (1)) takes the form

$$\begin{bmatrix} PQ_r S & PQ_{lm} S + (I - \tilde{P}Q_y)SP & (I - \tilde{P}Q_y)SP \\ Q_r S & Q_{lm} S - Q_y SP & -Q_y SP \\ PQ_{dm} S + (I - \tilde{P}Q_y)S & (I - \tilde{P}Q_y)S & -PQ_y S \\ Q_{dm} S - Q_y S & -Q_y S & -Q_y S \end{bmatrix} \quad (22)$$

where  $S$  is

$$S = (I + (P - \tilde{P})Q_y)^{-1}. \quad (23)$$

Robust stability of the control structure in Fig. 2 requires that all transfer functions in (22) are stable for any  $\tilde{P} \in \mathcal{P}$ . For stable  $Q_r$ ,  $Q_{lm}$ , and  $Q_{dm}$ , (22) indicates that the robust stability of the closed-loop system is determined by the stability of  $S$ . Hence, the robust stability of the system in Fig. 2 depends on only  $Q_y$ ,  $\tilde{P}$ , and  $P \in \mathcal{P}$ . ■

Thm. 3 implies that any stable  $Q_r$ ,  $Q_{lm}$ , and  $Q_{dm}$  do not influence the internal stability of the closed-loop system in the presence of model uncertainties. This observation motivates an approach similar to [5] in which  $Q_y$  can be

designed to be robust to some modest fault conditions as well as to model uncertainties, while the rest of the controllers are designed independently of these fault conditions. The control structure therefore alleviates the need for redesigning  $Q_r$ ,  $Q_{lm}$ , and  $Q_{dm}$  when faults occur in the closed-loop system. What distinguishes the proposed control structure from that presented in [5] is its ability to realize optimal failure-tolerant control for multi-objective controllers.

#### IV. MULTI-OBJECTIVE CONTROLLER DESIGN FOR A THIN-FILM DRYER

Consider the thickness control problem in a thin-film dryer. In this process, the feed solution is cast on a moving metal slab that is heated to remove the volatile components of the solution through evaporation and to form a thin film with desired chemical and mechanical properties [17]. A key process variable is the film thickness as it largely affects the mechanical characteristics of the manufactured thin films. The film thickness is assumed to be controlled by varying the gap size between the casting knife and the metal slab.

In the thin-film dryer investigated here, the film thickness dynamics are described by the first-order-plus-dead-time model

$$p(s) = \frac{0.2e^{-5s}}{0.5s + 1}. \quad (24)$$

The system is affected by different measured and unmeasured disturbances

$$y(s) = p(s)(u(s) + l_m(s)) + P_{dm}(s)d_m(s) + d_u(s) \quad (25)$$

where  $P_{dm} = \frac{s+0.01}{s+1}$  is the measured output disturbance transfer function. The output measurements are corrupted by stochastic sensor noise having a zero mean Gaussian distribution with  $\sigma^2 = 10^{-4}$ .

The control problem concerns maintaining the film thickness at a desired value  $r$  during drying while the process is perturbed by a measured load disturbance  $l_m$ , measured output disturbance  $d_m$ , and unmeasured output disturbance  $d_u$ . The control structure in Section III was used to cast the thickness control problem as a multi-objective controller design problem. Four independent control objectives were formulated to realize adequate reference tracking while rejecting the measured and unmeasured disturbances (see expressions (9)-(16)). The optimal IMC controllers  $Q_r$ ,  $Q_{lm}$ ,  $Q_{dm}$ , and  $Q_y$  were obtained using Thm 4.1-1 in [12] for a step change in  $r$ ,  $l_m$ ,  $d_m$ , and  $d_u$ , respectively. The IMC controllers were made proper so as to be physically realizable by augmenting with a first-order low-pass filter with  $\lambda_f = 1.5$ .

The optimal controllers are listed in Table I. Fig. 3 shows the film thickness profile when a step change is applied to the inputs  $r$ ,  $l_m$ ,  $d_m$ , and  $d_u$  of the closed-loop system. As can be seen,  $Q_r$  enables very good reference tracking and the controllers  $Q_{lm}$ ,  $Q_{dm}$ , and  $Q_y$  adequately reject the measured and unmeasured disturbances. The suppression of the measured load disturbance  $l_m$  is perfect for these simulations for the nominal process model. The closed-loop response for the measured and unmeasured output disturbance are limited

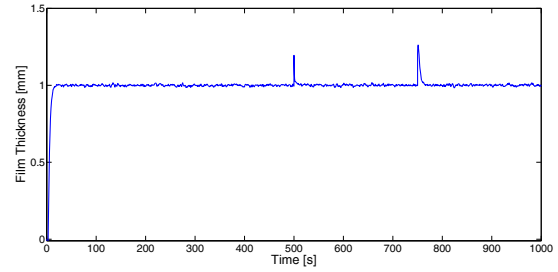


Fig. 3. Dynamic behavior of the thin-film dryer for a step change in the reference  $r$ , the measured load disturbance  $l_m$ , the measured output disturbance  $d_m$ , and the unmeasured output disturbance  $d_u$  at  $t = 0, 250, 500$ , and  $750$  s, respectively.

by the same nonminimum phase behavior of the process. The closed-loop speed of response for the measured output disturbance and reference tracking is the same because the two inputs act through the same controller transfer function  $Q_y$ .

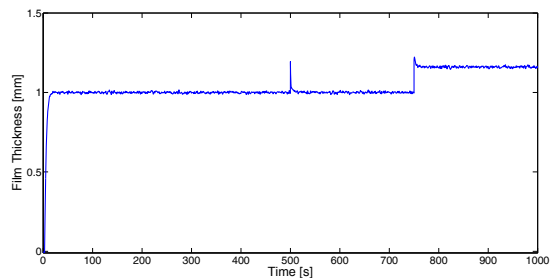
Next, we investigate the performance of the multi-objective control system in response to failures in the sensors of the measurable system. It is assumed that the sensor failures can be detected using fault detection and diagnosis methods (e.g., as described in [18], [19], and citations therein). When a sensor fails, its measurements can no longer be used for control and the respective controller(s) is switched off. Fig. 4 shows the system responses in the event of sensor failures for the measurable variables  $r$ ,  $l_m$ ,  $d_m$ , and  $y$ . The observed film thickness profiles show that the closed-loop responses with respect to the measurable variables with working sensors are unaffected by removal of the failed sensors, as indicated by the analysis in the previous section. The output responses to the inputs in  $r$ ,  $l_m$ , and  $d_m$  are completely unaffected by a failure in the output sensor  $y$  (see Fig. 4a), as their associated feedforward controllers remain intact by the loss in the feedback of  $y$ . Under these conditions, the output response to the unmeasured disturbance is affected by the loss of  $y$ , as the measurement of the output is the only way in which the control system can detect the presence of  $d_u$ .

Fig. 4b shows that losing the controller  $Q_{lm}$  influences the output response at  $t = 250$  s while having no effects on the output responses to the reference  $r$  and unmeasured output disturbance  $d_u$ . The integrating action of  $Q_y$  forces the output response to follow the reference signal  $r$  after the load disturbance perturbs the system. Fig. 4b also indicates that the loss of the measured output disturbance  $d_m$  and, consequently, the controller  $Q_{dm}$  does not influence the output responses of the working sensors  $Q_r$  and  $Q_y$ .

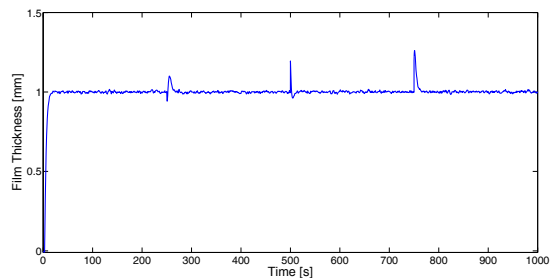
TABLE I

MULTI-OBJECTIVE CONTROLLERS FOR THE THIN-FILM DRYER

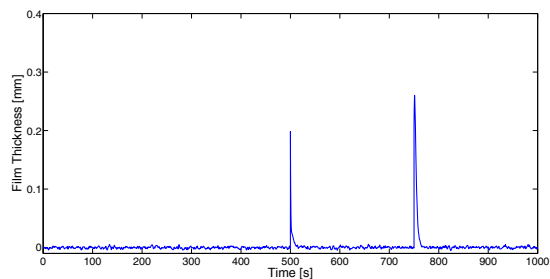
$Q_r = \frac{2.5s+5}{1.5s+1}$	$Q_{dm} = \frac{1.025s^2+2.025s-0.05}{1.5s^2+1.015s+0.01}$
$Q_{lm} = -1$	$Q_y = \frac{2.5s+5}{1.5s+1}$



(a) Sensor failure for  $y$ ; with  $Q_y$  taken out of service.



(b) Sensor failures for  $l_m$  and  $d_m$ ; with  $Q_{l_m}$  and  $Q_{d_m}$  taken out of service.



(c) Loss of the reference signal  $r$ ; with  $Q_r$  taken out of service

Fig. 4. Dynamic behavior of the simulated thin-film dryer during various failures in the sensors of the measurable variables for a step change in the reference  $r$ , the measured load disturbance  $l_m$ , the measured output disturbance  $d_m$ , and the unmeasured output disturbance  $d_u$  at  $t = 0, 250, 500$ , and  $750$  s, respectively.

Fig. 4c shows the output response for a loss in the reference signal, which could occur due to loss in a communication line between an upper level supervisory control loop and a lower level regulatory control system. The output responses to the disturbances  $l_m$ ,  $d_m$ , and  $d_u$  are completely unaffected by the loss of reference signal; they are just shifted to a different baseline (compare Figs. 3 and 4c). This numerical example demonstrates the optimal failure tolerance of the proposed control structure and design method. It shows that the control structure alleviates the need to redesign controllers for optimal failure-tolerant control.

## V. CONCLUSIONS AND FUTURE DIRECTIONS

A general control structure was presented for the design of multiple degrees-of-freedom controllers for MIMO systems. The control structure is shown to be non-restrictive in terms of achievable performance for stable linear time-invariant systems. A design procedure was proposed in which the

four degrees-of-freedom in the control system are designed independently, and that these independent designs are optimal when the control objectives are defined separately for each input. This approach enables the design of optimal failure-tolerant controllers without compromising the best achievable performance.

The proposed design procedure provides a unified framework to investigate and to compare the performance of different multiple degrees-of-freedom controllers in a systematic manner. More theoretical work would be needed to extend the design procedure to unstable and nonlinear systems.

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