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High-Gain Feedback Stability of a nonlinear Drivetrain System

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Abstract— **The development of modern automotive drivetrain systems has seen a constant increase in complexity over the past years. It is mostly driven by the increasing effort to reduce fuel consumption of passenger cars by optimizing the drivetrain to reduce losses and achieve energy-optimal set points for engine operation. Thus, electronic control units for drivetrain systems, such as automatic transmissions, have become much more complex. This increases the need for more complex control systems that traditionally have to be calibrated in a time-consuming process before production. We propose a new method of designing controllers for the shift procedure in automatic transmissions that have the same capabilities as the traditional control systems, but require less calibration. To this end, we introduce adaptive λ-tracking controllers based on the high-gain feedback principle to replace the current control scheme. In order to ensure high-gain stability, we provide a comprehensive system analysis based on a nonlinear system model and investigate the relative degree and the minimum-phase property in detail.**

I. Introduction

Automotive drivetrain systems have become much more complex in recent years. In order to reduce the fuel consumption of passenger cars, one prominent approach is to achieve a more energy-efficient set point for the internal combustion engine by expanding transmissions to more gear ratios. Also, the addition of electric drives to automotive transmissions generates hybrid drives which again allow the internal combustion engine to operate closer to its energy-optimal set point. In this context, automatic transmissions are the most common type. The existence of more gear ratios leads to the need for more shifts, which can be conducted by an automatic transmission much more often and more comfortable for the driver.

Gear shift procedures in automatic transmissions require extensive calibration. Traditionally, most of the control software is designed in the form of open-loop controllers, where control inputs are obtained from a field of parameters that have been tuned in advance to ensure constant shifting quality and comfort over a wide range of driving scenarios. Thus, the transmission control software has grown more complex. The calibration process has to be repeated for each new combination of vehicle, engine and transmission and is therefore a continuing effort. Hence, any methods that are able to shorten the calibration process are appreciated.

In this paper, we aim to replace parts of the control software responsible for conducting the gear shifts that require extensive tuning by implementing control systems that have no need for calibration: adaptive high-gain feedback λ-tracking controllers. In order to obtain the control parameters, i.e. feedback gains, an adaption law is implemented that continuously computes these parameters during operation of the controller. Thus, calibration is no longer needed. Since the system has to be high-gain-stabilizable, an extensive system analysis is required to determine whether an adaptive λ-tracking controller can be implemented.

II. Gear Shift Procedure

A common up-shift procedure in an automatic transmission is carried out as follows.

Consider two hydraulically actuated friction clutches, where one is responsible for engaging the current gear and the second is going to engage the following gear. The pressure for each clutch is controlled via individual hydraulic control valves that can be actuated by controlling their respective electric currents. By increasing the clutch control pressure, the friction disks are pushed together which reduces the differential speed of the clutch disks and allows for more torque to be transferred. If the pressure is reduced or even brought down to zero, the clutch is opened and the torque capacity is reduced as well. The transmission control scheme is designed in such a way that even during the process of shifting gears, a continuous drive torque is transmitted from the engine to the drive shaft of the vehicle. This creates both a comfortable and performance-oriented gear shift, since the vehicle does not stop accelerating. This is achieved by overlapping the hydraulic pressures of the two clutches and thus generating a continuous torque transfer between them. During this process, a number of influences and disturbances are present that have to be accounted for in the control software. These include a wide range of temperatures, terrain conditions, unknown viscosity of the hydraulic oil as well as actual clutch pressures and torque capacities. Added to this are wearing effects over the vehicles life cycle. The traditional way of dealing with these disturbances is to tune the control software for each unique set of conditions. This process is very time-consuming and therefore costly. If feedback controllers are used to replace standard open-loop controllers, the need for calibration is already somewhat reduced. However, classic feedback controllers also have to be tuned in advance. To overcome this, adaptive feedback controllers can be used (see [10]).

III. ADAPTIVE CONTROLLERS AND λ-TRACKING

Consider a standard output feedback controller, where the proportional feedback gain $k(\cdot)$ is no longer fixed, but a function of time $k(t)$:

$$e(t) = y_{\text{ref}}(t) - y(t) \tag{1}$$
$$u(t) = k(t)\, e(t) \tag{2}$$
$$k(t) = \text{Adaption law}$$

Thus, an adaption law is implemented that computes the feedback gain $k(t)$ during the runtime of the controller. It uses information from the system output to generate the control parameter.

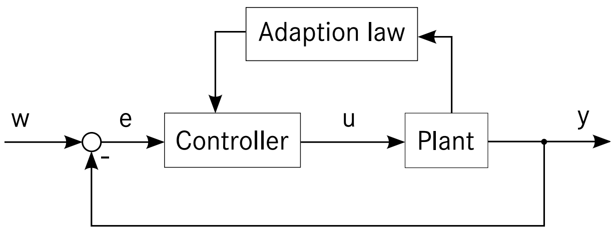

Fig. 1. Principle of adaptive control

This control scheme allows the controller to adapt to the system that is to be controlled. Therefore, no information is needed beforehand to design the control parameters and the controller is able to react to changing system parameters. Thus, it is ideal to control complex system with unknown or rapidly changing parameters (see [14], and [3] for a survey). For the purpose of this paper, we focus on λ-tracking control. Thus, the adaption law shown in Fig. 1 is designed in such a way that the system output is supposed to evolve inside a pre-specified error neighborhood λ around the set point trajectory.

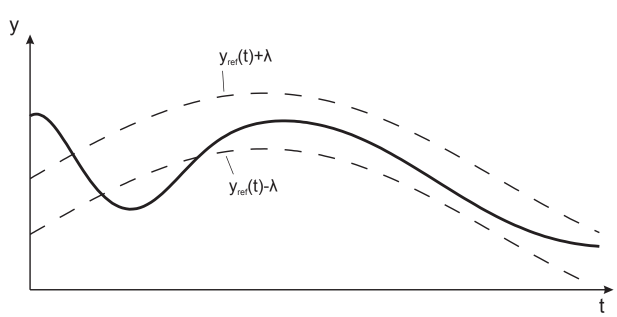

Fig. 2. Principle of λ-tracking

In order to achieve λ-tracking, the adaption law is designed to increase the gain factor $k(t)$ as long as the system output is outside the λ-neighborhood. Thus, the feedback gain is increased over time until the control goal is achieved, i.e. $e(t) < \lambda$. From then on, the gain factor is kept constant. If – due to disturbances or changing system parameters – the system output leaves the λ-neighborhood again, the adaption law reacts and increases $k(t)$ further.

A classic adaptive controller that achieves this behavior can be found in [4]:

$$e(t) = y_{\text{ref}}(t) - y(t) \tag{3}$$
$$u(t) = k(t)\, e(t) \tag{4}$$
$$\dot{k}(t) = \max\{0, |e(t)| - \lambda\}^2 \tag{5}$$
$$k(0) = 0 \tag{6}$$

with $\lambda > 0$.
Naturally, the adaption law is not limited to differential equations like the one shown above. A number of different approaches is feasible, such as algebraic equations in funnel controllers (see [5], [12]), more complex switching differential equations or fuzzy inference systems (see [1]).

Note that in the exemplary adaption law (5), the gain parameter can only be increased. It is however desirable that the parameter may not simply increase towards infinity, but that it can also be decreased. For example, this can be achieved with the help of an adaption law of the following kind (taken from [2]):

$$\dot{k}(t) = \begin{cases} \gamma\,(|e(t)| - \varepsilon\lambda)^2 & \text{, if [1]} \\ \gamma\,(|e(t)| - \varepsilon\lambda)^{\frac{1}{2}} & \text{, if [2]} \\ 0 & \text{, if [3]} \\ -\sigma\, k(t) & \text{, if [4]} \end{cases} \tag{7}$$

[1] $\varepsilon\lambda + 1 \le |e(t)|$
[2] $\varepsilon\lambda \le |e(t)| < \varepsilon\lambda + 1$
[3] $|e(t)| < \varepsilon\lambda \quad \wedge \quad t - t_e < t_d$
[4] $|e(t)| < \varepsilon\lambda \quad \wedge \quad t - t_e \ge t_d$,

with t_{e} being the point in time where the system output enters the λ-neighborhood and t_{d} being a pre-specified parameter.
Thus, after a certain time span t_{d} has passed while the error value was constantly smaller than λ, there is an exponential decrease of the gain $k(t)$. This helps reducing the energy that the controller applies to the system and thus keep the gain parameter as low as possible and as high as necessary. However, this strategy is not required for closed-loop stability. The controller only has to increase the gain parameter if necessary, so that the unknown level k^* after which the system is stabilized, will be passed. Since the magnitude of k^* is unknown due to unknown system parameters, it cannot be set beforehand, however high that initial setting may be.

In order to apply high-gain feedback controllers, the system has to be high-gain-stabilizable. Thus, no specific system data has to be known, other than certain structural properties required for a stable control loop.
According to [13] and [11], three conditions have to be met:

- The relative degree of the system δ has to equal one,
- the system has to be minimum phase and
- the sign of the high-frequency gain has to be known.

If these properties can be proven for a given system, it is high-gain-stabilizable and can be controlled with an adaptive

λ-tracking controller.

Note that the relative degree may also be greater than one. If a derivative feedback gain is inserted into the controller structure, an additional zero is created within the closed-loop system, which reduces the relative degree by 1. Thus, by implementing not only a proportional, but a proportional-derivative (PD-)feedback controller, systems of relative degree $\delta = 2$ can be controlled by high-gain feedback controllers (see [7]).

IV. System Modeling and Analysis

In order to investigate the high-gain stability of the given drivetrain system, a system model has to be obtained. In [9], a full nonlinear system model was presented that consists of the system dynamics of a electro-hydraulically actuated friction clutch in an automatic transmission. It can be somewhat simplified by using the Heaviside function instead of signum functions:

$$\underline{\dot{x}} = \underline{f}(\underline{x}) + \underline{g}(\underline{x}) \cdot u \tag{8}$$

$$y = h(\underline{x}) \tag{9}$$

$$\underline{f}(\underline{x}) = \begin{pmatrix} x_2 \\ -\frac{A_2 - A_1}{m_{\text{CV}}} x_3 - \frac{c_{\text{CV}}}{m_{\text{CV}}} x_1 - \frac{d_{\text{CV}}}{m_{\text{CV}}} x_2 \\ \frac{E\alpha C_{\text{hyd}}\sqrt{2}}{V\sqrt{\rho}} \Big(\Theta(x_1) x_1 \sqrt{p_{\text{SP}} - x_3} + \\ \Theta(-x_1) x_1 \sqrt{x_3 - p_{\text{OS}}} \Big) - \frac{E A_{\text{pist}}}{V} x_5 \\ x_5 \\ \frac{A_{\text{pist}}}{m_{\text{pist}}} x_3 - \frac{d_{\text{pist}}}{m_{\text{pist}}} x_5 - \frac{c_{\text{pist}}}{m_{\text{pist}}} x_4 \\ -\frac{z r_{\text{m}} \mu A_{\text{fric}}}{J_{\text{D}}} x_3 + \frac{z r_{\text{m}} \mu c_{\text{pist}}}{J_{\text{D}}} x_4 - \frac{d_{\text{c}}}{J_{\text{D}}} x_6 \\ \frac{z r_{\text{m}} \mu A_{\text{fric}}}{J_{\text{L}}} x_3 - \frac{z r_{\text{m}} \mu c_{\text{pist}}}{J_{\text{l}}} x_4 - \frac{d_{\text{c}}}{J_{\text{L}}} x_7 \end{pmatrix} \tag{10}$$

$$\underline{g}(\underline{x}) = \begin{pmatrix} 0 \\ \frac{K_{\text{i}}}{m_{\text{CV}}} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{11}$$

$$h(\underline{x}) = x_6 - x_7 \tag{12}$$

with constants

$$A_1, A_2, c_{\text{CV}}, m_{\text{CV}}, d_{\text{CV}}, E, \alpha, C_{\text{hyd}}, V, \rho, p_{\text{SP}}, p_{\text{OS}}, \\ A_{\text{pist}}, m_{\text{pist}}, c_{\text{pist}}, d_{\text{pist}}, z, r_{\text{m}}, \mu, A_{\text{fric}}, d_{\text{c}}, J_{\text{D}}, J_{\text{L}}, K_{\text{i}} > 0 \tag{13}$$

and with $\Theta(\cdot)$ being the Heaviside function:

$$\Theta(x) = \begin{cases} 0: & x < 0 \\ 1: & x \geq 0 \end{cases} \tag{14}$$

The system state variables have been chosen as follows:

- $x_1(t)$ - Location of control valve piston
- $x_2(t)$ - Velocity of control valve piston
- $x_3(t)$ - Hydraulic clutch pressure
- $x_4(t)$ - Location of clutch piston
- $x_5(t)$ - Velocity of clutch piston
- $x_6(t)$ - Rotational speed of clutch input side
- $x_7(t)$ - Rotational speed of clutch output side

With this, we can investigate the three conditions mentioned above.

A. Sign of High-Frequency Gain

The sign of the high-frequency gain is usually known for most systems. It merely represents the direction of action of the primary control actuator. In this case, a positive electric current leads to the control valve piston being accelerated in positive direction and thus opening the inlet to the clutch to be filled with hydraulic oil from the reservoir. As a result, the differential speed between the two sides of the clutch decreases. Therefore, the sign is known.

B. Relative Degree

The relative degree of a system determines which derivative order of the system output the control input directly acts on. For linear time-invariant (LTI) systems, it is also the difference of the number of poles and zeros in the transfer function (see [6]). Since the system at hand is nonlinear, we have to use the corresponding definition of the relative degree (see [6]):

$$L_g L_f^k h(\underline{x}) = 0, \text{ for all } \underline{x} \text{ in the vicinity of } \underline{x}_0 \\ \text{and for all } k < \delta - 1 \tag{15}$$

$$L_g L_f^{\delta-1} h(\underline{x_0}) \neq 0 \tag{16}$$

where

$$L_g h(\underline{x}) := \frac{\partial h}{\partial \underline{x}} \underline{g}(\underline{x}) \tag{17}$$

$$L_f h(\underline{x}) := \frac{\partial h}{\partial \underline{x}} \underline{f}(\underline{x}) \tag{18}$$

$$L_g L_f h(\underline{x}) := \frac{\partial \left(L_f h(\underline{x}) \right)}{\partial \underline{x}} \underline{g}(\underline{x}) \tag{19}$$

For the clutch system, the computation gives:

$$L_g L_f^0 h(\underline{x}) = L_g h(\underline{x}) = 0 \tag{20}$$

$$L_g L_f^1 h(\underline{x}) = 0 \tag{21}$$

$$L_g L_f^2 h(\underline{x}) = 0 \tag{22}$$

$$L_g L_f^3 h(\underline{x}) = -\frac{C_{\text{hyd}} E A_{\text{fric}} \mu K_{\text{i}} \alpha r_{\text{m}} z (J_{\text{D}} + J_{\text{L}})}{\sqrt{\rho} J_{\text{D}} J_{\text{L}} V m_{\text{CV}}} \cdot \\ \Big(\sqrt{x_3 - p_{\text{OS}}} + \Theta(x_1) \sqrt{p_{\text{SP}} - x_3} - \\ \Theta(x_1) \sqrt{x_3 - p_{\text{OS}}} \Big) \tag{23}$$

Since all constants are greater than zero, only the term in parentheses has to be considered:

$$0 \neq \sqrt{x_3 - p_{\text{OS}}} + \Theta(x_1) \sqrt{p_{\text{SP}} - x_3} \\ - \Theta(x_1) \sqrt{x_3 - p_{\text{OS}}} \tag{24}$$

In order to check if the term is $\neq 0$ for all $\underline{x}$, we consider two cases.

1) $\mathbf{x_1 < 0}$

If $x_1 < 0$, the control valve opens the outlet to the oil sump and the clutch is emptied. The term can now be simplified to:

$$\sqrt{x_3 - p_{\text{OS}}} \neq 0$$
$$x_3 \neq p_{\text{OS}}$$

Thus, the clutch pressure x_3 may not be equal or less than the pressure of the oil sump p_{OS}. If this were the case, the clutch could not be emptied further, although the control valve piston is moved in the respective position. This describes a borderline case, which will not be reached during operation of the system. If the clutch pressure is at its minimum, we do not aim to reduce it further.

2) $\mathbf{x_1 > 0}$

If $x_1 > 0$, the control valve opens the clutch inlet to the supply pressure and therefore fills the clutch volume with hydraulic oil. The term is now simplified to:

$$\sqrt{p_{\text{SP}} - x_3} \neq 0$$
$$x_3 \neq p_{\text{SP}}$$

Thus, the clutch pressure x_3 may not be equal or greater than the system supply pressure p_{SP}, since this would entail the clutch pressure to be increased beyond the system pressure, which is not possible, even though the control valve piston is moved in the respective position. Again, this is a borderline case that will not be reached during operation of the system. We do not require the clutch pressure to increase beyond the supply pressure.

To summarize, the term $L_g L_f^3 h(\underline{x})$ is $\neq 0$ for all $\underline{x}$, except for the borderline cases discussed above. Since those will not be reached during operation, we can conclude that the relative degree of the system is $\delta = 4$.

However, there are certain aspects of the system that have not been modeled here, especially end stops. Considering the hydraulic subsystem, the control valve piston is limited in its system dynamics in several ways:

- The piston position cannot exceed its end stops:

$$s_{\text{min}} \leq x_1(t) \geq s_{\text{max}} \tag{25}$$

- The input signal, i.e. the electric current, is bounded:

$$0 \leq u(t) \geq i_{\text{max}} \tag{26}$$

- Since the piston acceleration and position are bounded, and there is viscous damping present, the piston velocity is bounded as well:

$$-v_{\text{max}} \leq x_2(t) \geq v_{\text{max}} \tag{27}$$

These limitations are not represented in the system model (8)-(12). However, they affect the system dynamics, especially the relative degree, which can be thought of as the least number of integrators that the signal has to pass from system input to output. Since velocity and position of the control valve piston cannot exceed certain bounds, their respective integrators can be viewed as having a bounded output with small values for their limits. Thus, when these limits are reached, the dynamic behavior of the respective state variable is no longer part of the system dynamics, it rather acts as a constant. Hence, the respective integrators are not active. This reduces the systems dynamics by two states, which in turn means that when the limits are reached, the relative degree is reduced by the number of inactive integrators, i.e. 2. Since the limit values of the integrators, caused by the end stops and bounded input signal, are reached very quickly and frequently, the system dynamics are repeatedly reduced by two. Thus, they do not cause any instability. If the systems states in question diverge towards infinity, i.e. towards an unstable operating point, the limits cause them to stop. Thus, they no longer affect the macroscopic system dynamics. Effectively, the system can be viewed as having a relative degree of $\delta_{\text{eff}} = \delta - 2 = 2$ on a macroscopic scale.

C. Minimum-Phase Property

The minimum-phase property of a system defines the stability of its zero dynamics. First, consider a LTI system, where the zero dynamics are equivalent to the zeros of the system's transfer function. If all of the zeros are located in the left-half complex plane, the system is minimum-phase.

For nonlinear systems, a different definition is needed. First, the zero dynamics have to be identified for a given system, then their stability has to be investigated. One way to do so is to transform the system into the Byrnes-Isidori form and thus look at its zero dynamics (see [6]). This is rather difficult to do for system (8)-(12), since the Heaviside- and square-root-functions result in very complex terms for the transformation matrix. However, it is possible to split the system into a nonlinear subsystem containing the hydraulics, and a LTI subsystem containing the mechanical part of the clutch system, since the two are independent.

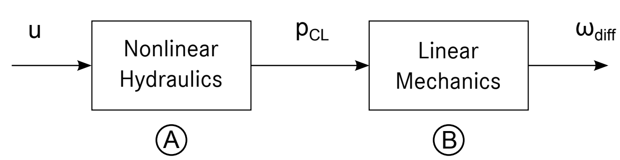

Fig. 3. Splitting the system into a nonlinear and a linear subsystem with u being the input signal, i.e. the electric current, p_{CL} being the actual clutch pressure and ω_{diff} being the differential speed of the opposing sides of the clutch, i.e. the system's output signal.

First, consider subsystem A, incorporating the hydraulics, where the input signal is the electric current and the output signal is the hydraulic clutch pressure:

$$\dot{\underline{x}}_A = \underline{f}_A(\underline{x}_A) + \underline{g}_A(\underline{x}_A) \cdot u_A + \underline{d}_A(\underline{x}_A) \tag{28}$$

$$y_A = h_A(\underline{x}_A) \tag{29}$$

$$\underline{f}_A(\underline{x}_A) = \begin{pmatrix} x_2 \\ -\frac{A_2 - A_1}{m_{\text{CV}}} x_3 - \frac{c_{\text{CV}}}{m_{\text{CV}}} x_1 - \frac{d_{\text{CV}}}{m_{\text{CV}}} x_2 \\ \frac{E\alpha C_{\text{hyd}}\sqrt{2}}{V\sqrt{\rho}} \Big(\Theta(x_1) x_1 \sqrt{p_{\text{SP}} - x_3} + \\ \Theta(-x_1) x_1 \sqrt{x_3 - p_{\text{OS}}} \Big) \end{pmatrix} \tag{30}$$

$$\underline{g}_A(\underline{x}_A) = \begin{pmatrix} 0 \\ \frac{K_{\text{i}}}{m_{\text{CV}}} \\ 0 \end{pmatrix} \tag{31}$$

$$h_A(\underline{x}_A) = x_3 \tag{32}$$

$$\underline{d}_A(\underline{x}_A) = \begin{pmatrix} 0 \\ 0 \\ -\frac{E}{V} Q_{\text{out}} \end{pmatrix} \tag{33}$$

Thus, the additional hydraulic flow $Q_{\text{out}}(t)$, caused by the movement of the clutch piston, is modeled as a disturbance signal and is therefore not part of the subsystems dynamics. Again, we can determine the relative degree of the first subsystem:

$$L_g L_f^0 h_A(\underline{x}_A) = L_g h_A(\underline{x}_A) = 0 \tag{34}$$

$$L_g L_f^1 h_A(\underline{x}_A) = 0 \tag{35}$$

$$L_g L_f^2 h_A(\underline{x}_A) = -\frac{C_{\text{hyd}} E A_{\text{fric}} \mu K_{\text{i}} \alpha r_{\text{m}} z (J_{\text{D}} + J_{\text{L}})}{\sqrt{\rho} J_{\text{D}} J_{\text{L}} V m_{\text{CV}}} \cdot \Big(\sqrt{x_3 - p_{\text{OS}}} + \Theta(x_1)\sqrt{p_{\text{SP}} - x_3} - \Theta(x_1)\sqrt{x_3 - p_{\text{OS}}} \Big) \tag{36}$$

The last term, which may be $\neq 0$, is the same as for the complete system before. Hence, the same remarks apply and we can conclude that with this term, the computation stops at $L_g L_f^2 h_A(\underline{x})$. Thus, the relative degree of subsystem A is $\delta_A = 3$.
Since we are aim to investigate the zero dynamics of the system, we look at the system order, which is $n_A = 3$. Therefore, the relative degree equals the system order, i.e. $\delta_A = n_A$. This means that all system states are part of the output dynamics of the system and therefore, subsystem A does not have any zero dynamics. Hence, we can conclude that the nonlinear subsystem A is minimum-phase, since there exist no zero dynamics that might be unstable.
Now, consider the linear subsystem B, where the input is the hydraulic clutch pressure and the output is the differential speed of the clutch disks:

$$\dot{\underline{x}}_B = \underline{A}_B \, \underline{x}_B + \underline{b}_B \, u_B \tag{37}$$

$$y_B = \underline{c}_B \, \underline{x}_B \tag{38}$$

$$\underline{A}_B = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -\frac{c_{\text{pist}}}{m_{\text{pist}}} & -\frac{d_{\text{pist}}}{m_{\text{pist}}} & 0 & 0 \\ \frac{z r_{\text{m}} \mu c_{\text{pist}}}{J_{\text{D}}} & 0 & -\frac{-d_{\text{c}}}{J_{\text{D}}} & 0 \\ -\frac{z r_{\text{m}} \mu c_{\text{pist}}}{J_{\text{L}}} & 0 & 0 & -\frac{-d_{\text{c}}}{J_{\text{L}}} \end{pmatrix} \tag{39}$$

$$\underline{b}_B = \begin{pmatrix} 0 \\ \frac{A_{\text{pist}}}{m_{\text{pist}}} \\ -\frac{z r_{\text{m}} \mu A_{\text{fric}}}{J_{\text{D}}} \\ \frac{z r_{\text{m}} \mu A_{\text{fric}}}{J_{\text{L}}} \end{pmatrix} \tag{40}$$

$$\underline{c}_B = \begin{pmatrix} 0 & 0 & 1 & -1 \end{pmatrix} \tag{41}$$

In order to investigate if the system is minimum-phase, i.e. if its zeros are located in the left half complex plane, we compute the system's zeros by use of the Rosenbrock matrix:

$$\det \begin{bmatrix} \underline{A}_B - z_k \underline{I} & \underline{b}_B \\ \underline{c}_B & 0 \end{bmatrix} = 0 \tag{42}$$

$$z_1 = -\frac{2\, d_{\text{c}}}{J_{\text{D}} + J_{\text{L}}} \tag{43}$$

$$z_{2,3} = -\frac{d_{\text{pist}}}{2\, m_{\text{pist}}} \pm \sqrt{\frac{d_{\text{pist}}^2}{4\, m_{\text{pist}}^2} - \frac{c_{\text{pist}}}{m_{\text{pist}}} + \frac{A_{\text{pist}}\, c_{\text{pist}}}{A_{\text{fric}}\, m_{\text{pist}}}} \tag{44}$$

In order to ensure that all zeros are stable, we compute:

$$0 > -\frac{d_{\text{pist}}}{2\, m_{\text{pist}}} \pm \sqrt{\frac{d_{\text{pist}}^2}{4\, m_{\text{pist}}^2} - \frac{c_{\text{pist}}}{m_{\text{pist}}} + \frac{A_{\text{pist}}\, c_{\text{pist}}}{A_{\text{fric}}\, m_{\text{pist}}}} \tag{45}$$

$$A_{\text{fric}} > A_{\text{pist}} \tag{46}$$

If the condition holds that $A_{\text{fric}} > A_{\text{pist}}$, subsystem B is minimum-phase, since $z_{1,2,3} < 0$. It is of order $n_B = 4$ and has 3 zeros. Therefore, its relative degree is $\delta_B = 1$.
Now, we merge the results found in investigating the minimum-phase condition of subsystems A and B:

- The relative degree of the complete system is $\delta_A + \delta_B = \delta = 4$, which coincides with the findings in (23).
- Subsystem A does not have any zero dynamics and is therefore minimum-phase.
- Subsystem B has three zeros, all of which are stable if $A_{\text{fric}} > A_{\text{pist}}$, which is the case here. Thus, subsystem B is minimum-phase as well.
- Hence, the complete system (8)-(12) is minimum-phase.

Note that the effective relative degree of $\delta_{\text{eff}} = 2$ on a macroscopic scale as discussed above does not contradict the findings in this section. If the zeros of the system are determined, the relative degree to be considered is still $\delta = 4$. The fact that the limitations are reached frequently and the respective system states stay constant at those times, does not the affect the investigations regarding the zero dynamics.

V. Exemplary Simulation

A first preliminary simulation of system (8)-(12) in conjunction with a λ-tracking controller has been discussed in [9]. Further detailed simulations and actual measurement data obtained from an adaptively controlled vehicle drivetrain were shown in [8]. At this point, we show system (8)-(12) once more in simulation with the λ-tracking controller (3)-(6). For our purposes, it has been extended by a derivative feedback term to account for the relative degree of 2:

$$e(t) = y_{\text{ref}}(t) - y(t) \tag{47}$$

$$u(t) = k(t)\, e(t) + k(t)\, \dot{e}(t) \tag{48}$$

$$\dot{k}(t) = \max\{0, |e(t)| - \lambda\}^2 \tag{49}$$

$$k(0) = 0 \tag{50}$$

with $\lambda > 0$.

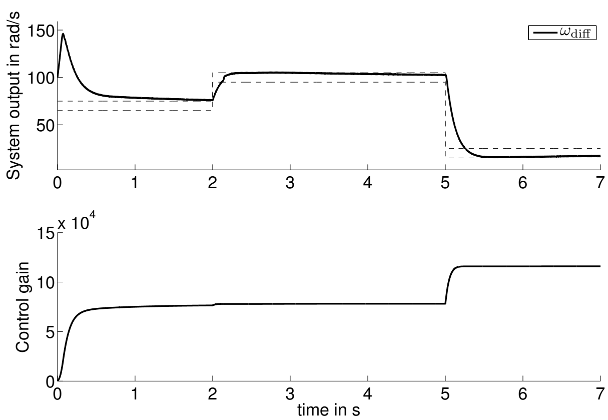

Fig. 4. Simulation of clutch system with adaptive controller (47)-(50)

The set point trajectory of the simulation shown in Fig. 4 consists of three step functions. The clutch is supposed to operate at different differential speeds: 70 rad/s, 100 rad/s and 20 rad/s. The controller forces the system output, i.e. the differential speed, into the λ-neighborhood around the piecewise constant set point trajectory. During this process, the feedback gain $k(t)$ is continuously computed by the adaption law, which causes the gain to increase whenever the system is outside the λ-neighborhood.

VI. Conclusion and Future Work

We have shown that the system (8)-(12) is high-gain-stabilizable according to the conditions stated above:

- The sign of the high-frequency gain is known.
- The effective relative degree of the system is $\delta_{\text{eff}} = 2$.
- The system is minimum-phase.

Therefore, adaptive λ-tracking controllers can be implemented and the system will be stable. This paper completes the stability analysis of the nonlinear drivetrain system and gives proof of the high-gain stability, also shown in an exemplary simulation.

Future work on this subject will focus on:

- Investigation of more complex adaption laws that are better suited for the requirements of automotive drive-train systems.
- Analysis of the potential of the approach to reduce the calibration efforts. This will be done by determining how many calibration parameters can be omitted if the traditional control scheme is replaced by a λ-tracking controller.

VII. Acknowledgment

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References

[1] C. Behn and P. Loepelmann. Adaptive versus Fuzzy Control of uncertain mechanical Systems. *International Journal of Applied Mechanics*, 4(2), 2012.

[2] C. Behn and J. Steigenberger. Experiments in adaptive control of uncertain mechanical systems. *International Review of Mechanical Engineering (IREME)*, 4(7):886 – 898, 2010.

[3] A. Ilchmann. Non-identifier-based adaptive control of dynamical systems: a survey. *IMA Journal of Mathematical Control and Information*, 8:321–366, 1991.

[4] A. Ilchmann and E.P. Ryan. Universal λ-tracking for nonlinearly-perturbed systems in the presence of noise. *Automatica*, 30(2):337–346, 1994.

[5] A. Ilchmann and H. Schuster. PI-funnel control for two mass systems. *IEEE Transactions on Automatic Control*, 2007.

[6] A. Isidori. *Nonlinear Control Systems*. Springer, London, 1995.

[7] P. Loepelmann. Design and Analysis of Control Strategies for biologically inspired Sensor Systems. Master's thesis, TU Ilmenau, Ilmenau, 2011.

[8] P. Loepelmann and B. Bäker. A novel adaptive λ-tracking approach for automotive transmission control systems. In O. Predelli W. Nietschke, editor, *Automotive Powertrain Control Systems*, volume 1, pages 1–16, 2012.

[9] P. Loepelmann and B. Bäker. Towards adaptive Control of nonlinear Drivetrain Systems. In *SDPS Conference 2012*. Society for Design and Process Science, 2012.

[10] K.J. Åström and B. Wittenmark. *Adaptive Control*. Addison-Wesley, Reading, 1989.

[11] D. Schröder. *Intelligente Verfahren - Identifikation und Regelung nichtlinearer Systeme*. Springer, Heidelberg, 2010.

[12] H. Schuster. *Hochverstärkungsbasierte Regelung nichtlinearer Antriebssysteme*. PhD thesis, TU München, München, 2009.

[13] E.D. Sontag. *Mathematical Control Theory - Deterministic Finite Dimensional Systems*. Springer, New York, 1998.

[14] J.C. Willems and C.I. Byrnes. Global adaptive stabilization in the absence of information on the sign of the high frequency gain. Part I: analysis and optimization of systems. In *6th International Conference on Analysis and Optimization*, pages 49–57. Springer, 1984.