

# LMI-based design of robust fault isolation filters for linear systems

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**Abstract**—In this paper we employ general linear dynamic filters to robustly isolate faults in linear systems. The concept can be regarded as a generalization of observer based approaches and offers more degrees of freedom and less structural constraints than fault isolation observers (FIOs). We propose an LMI-based design approach for fault isolation filters (FIFs), where the existence of a solution for exactly known system matrices is guaranteed by the constructive design of an attainable reference model. The approach can readily be applied to systems with disturbances and/or parametric uncertainties, which is a main advantage over observer-based approaches. Its applicability is demonstrated by means of both simulations and lab experiments with a gantry crane system.

## I. INTRODUCTION

Over the past decades, model-based fault diagnosis has been receiving constant attention in both the scientific community and applications, which is due to increasing demands on reliability and rising system complexity (s. e.g. [3], [5] for an overview). Two fundamental tasks are *fault detection* and *fault isolation*. While fault detection is devoted to determine whether any fault(s) is (are) active in a system, fault isolation aims at localizing faults. Thus isolation enables to distinguish between different faults and to determine, which fault(s) is (are) active.

The most common approach to achieve fault isolation is to split the task into several robust fault detection problems by using banks of observers [3], [7], [9]. In these approaches, several observers run in parallel and a decision logic is used to determine which fault is active.

Recently, designs have been proposed that achieve isolation by only employing a single, specifically parameterized observer. The idea was introduced in [13] for a limited class of linear systems. In [2], [11], the concept of fault isolation observers (FIOs) is further elaborated on using methods similar to the classical approach to non-interacting controller design [6], while in [22] a parametric eigenstructure assignment approach dual to [15] is used.

However, all real-world systems suffer from exogenous disturbances. While robustness issues have been extensively studied for fault detection problems (s. e.g. [12], [14], [23], [24]), there are less results considering robustness of fault isolation observers. In [2], [10], [11], [22] the rejection of disturbances is studied. Apart from that, real-world systems are also subject to parametric uncertainties in the system matrices. These might result from inexact knowledge of

some parameters, component aging, or deviation from the operating point if the system results from the linearization of a nonlinear model, to name a few. While these uncertainties can be recast as exogenous disturbances, this approach introduces conservatism as elaborated on in [5]. For the observer-based approaches, directly incorporating parametric uncertainties is very difficult. While it is possible to use model-matching techniques as presented for fault detection in [25], conservatism has to be introduced again to obtain a convex optimization problem with LMI constraints.

In [17], fault isolation filters (FIFs) are introduced for the purpose of fault detection. These filters have a more general structure compared to observers. It is shown that robustness with respect to uncertain parameters can comparatively easily be incorporated into the design, which reduces to an LMI-problem. While the approach is successfully applied for fault detection, it has not been systematically exploited for fault isolation as well. This is due to the fact that finding an attainable reference model for the fault to residual transfer matrix is crucial for the fault isolation filter (FIF) approach.

Apart from presenting some extensions for the design approach given in [17], the main contribution of this paper is to give sufficient conditions for the existence of an attainable reference model for fault isolation. Furthermore, we provide a simple yet constructive design for attainable reference models for fault isolation based on recent results on FIO design [22]. Therewith, we are able to obtain an LMI-based design for fault isolation filters (FIFs). Exogenous disturbances are considered in the design as well as parametric uncertainties. Furthermore, additional measurement information, which results in additional degrees of freedom, can be exploited by the proposed approach to improve the fault isolation performance without any changes in the design procedure.

The paper is structured as follows. After giving some notational aspects and a proper problem description in Section II, we briefly recall some results on fault isolation observers (FIOs) in Section III. These results guarantee the existence of an attainable reference model for the FIF design presented in the main part of this paper in Section IV. To verify the applicability of the proposed approach, we apply it to isolate faults in a gantry crane system both in simulations as well as in experiments with real measurement data.

## II. PRELIMINARIES

### A. Notation

With  $I_n$  we denote an identity matrix of dimension  $n$  and  $0$  is written for a zero matrix of appropriate dimensions. For a square matrix  $P \in \mathbb{R}^{n \times n}$ , the symbol  $\succ$  ( $\prec$ ) denotes positive

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$$\Omega = \left\{ \begin{bmatrix} A_\xi & B_\xi & E_\xi & B_{d,\xi} \\ C_\xi & D_\xi & 0 & D_{d,\xi} \end{bmatrix} \mid \begin{bmatrix} A_\xi & B_\xi & E_\xi & B_{d,\xi} \\ C_\xi & D_\xi & 0 & D_{d,\xi} \end{bmatrix} = \sum_{i=1}^N \xi_i \begin{bmatrix} A_i & B_i & E_i & B_{d,i} \\ C_i & D_i & 0 & D_{d,i} \end{bmatrix}, \xi_i \geq 0, \sum_{i=1}^N \xi_i = 1 \right\} \quad (2)$$

(negative) definiteness of the matrix and we write  $\text{He}(P) = P + P^\top$  as an abbreviation. In symmetric matrices,  $\star$  stands for elements, which are readily inferred by symmetry. For (block-)diagonal matrices with diagonal elements  $q_i$ , we employ the notation  $Q = \text{diag}(q_1, \dots, q_n)$ . Furthermore, a linear dynamical system  $\dot{x} = Ax + Bu$ ,  $y = Cx + Du$  with transfer matrix  $G_{yu}(s) = C(sI - A)^{-1}B + D$  is written as  $(A, B, C, D)$  or  $(A, B, C)$  if  $D = 0$ . The  $\mathcal{H}_\infty$ -norm of a (stable) system is defined as

$$\|G_{yu}(s)\|_\infty = \sup_{\omega \in [0, \infty)} \bar{\sigma}(G_{yu}(j\omega)),$$

where  $\bar{\sigma}(G_{yu}(j\omega))$  is the maximum singular value of  $G_{yu}(j\omega)$ .

### B. Problem description

We are considering linear, time-invariant systems described by

$$\dot{x} = A_\xi x + B_\xi u + E_\xi f + B_{d,\xi} d, \quad (1a)$$

$$y = C_\xi x + D_\xi u + D_{d,\xi} d, \quad (1b)$$

where  $x \in \mathbb{R}^n$  are the system states,  $u \in \mathbb{R}^{n_u}$  are the control inputs,  $f \in \mathbb{R}^{n_f}$  are the faults,  $d \in \mathbb{R}^{n_d}$  are the disturbances,  $y \in \mathbb{R}^{n_y}$  are the measurable outputs, and all matrices are of appropriate dimensions. The matrices are uncertain but known to belong to a polyhedral region as described by (2), which is given at the top of this page. Furthermore, we assume a nominal model

$$\dot{x} = A_0 x + B_0 u + E_0 f + B_{b,0} d, \quad (3a)$$

$$y = C_0 x + D_0 u + D_{d,0} d, \quad (3b)$$

with

$$\begin{bmatrix} A_0 & B_0 & E_0 & B_{b,0} \\ C_0 & D_0 & 0 & D_{d,0} \end{bmatrix} \in \Omega. \quad (4)$$

Note that we do not explicitly consider sensor faults in this setup. However, this does not impose a loss of generality, as they can be recast as pseudo-actuator faults as shown in [19]. Furthermore, the method presented in this paper can readily be extended to directly handle sensor faults as well, which is omitted for the sake of notational brevity.

The objective is to find a general linear dynamic filter

$$\dot{x}_f = A_f x_f + B_{f,u} u + B_{f,y} y, \quad (5a)$$

$$r = C_f x_f + D_{f,u} u + D_{f,y} y \quad (5b)$$

with  $x_f \in \mathbb{R}^{n_{\text{fit}}}$  that generates residuals  $r \in \mathbb{R}^{n_f}$ . These residuals can be used to isolate faults for the nominal model if the transfer matrix  $G_{rf,0}(s)$  relating them to faults is rendered diagonal. In this case, the  $i$ -th residual is only influenced by the  $i$ -th fault and thus by inspecting a residual, the associated fault can not only be detected but isolated from

all other faults even if they occur simultaneously. Hence, the objective is to achieve

$$G_{rf,0}(s) = C' (sI_{n+n_{\text{fit}}} - A')^{-1} E' \quad (6a)$$

$$= \text{diag}(g_{1,1}(s), \dots, g_{n_f, n_f}(s)). \quad (6b)$$

Therein, the matrices  $A'$ ,  $C'$ ,  $E'$  are defined in

$$\begin{aligned} \begin{bmatrix} \dot{x} \\ \dot{x}_f \end{bmatrix} &= \underbrace{\begin{bmatrix} A_0 & 0 \\ B_{f,y} C_0 & A_f \end{bmatrix}}_{A'} \begin{bmatrix} x \\ x_f \end{bmatrix} + \underbrace{\begin{bmatrix} B_0 \\ B_{f,u} + B_{f,y} D_0 \end{bmatrix}}_{B'} u \\ &+ \underbrace{\begin{bmatrix} E_0 \\ 0 \end{bmatrix}}_{E'} f + \underbrace{\begin{bmatrix} B_{b,0} \\ B_{f,y} D_{d,0} \end{bmatrix}}_{B'_d} d, \end{aligned} \quad (7a)$$

$$\begin{aligned} r &= \underbrace{\begin{bmatrix} D_{f,y} C_0 & C_f \end{bmatrix}}_{C'} \begin{bmatrix} x \\ x_f \end{bmatrix} + \underbrace{\begin{bmatrix} D_{f,u} + D_{f,y} D_0 \end{bmatrix}}_{D'} u \\ &+ \underbrace{\begin{bmatrix} D_{f,y} D_{d,0} \end{bmatrix}}_{D'_d} d. \end{aligned} \quad (7b)$$

Similar to [13], we define the *fault detectability indices*  $\delta_i$  with  $i = 1, \dots, n_f$  as

$$\delta_i = \min_k \{k \geq 1 : C_0 A_0^{k-1} e_{0i} \neq 0\}, \quad (8)$$

where  $e_{0i}$  is the  $i$ -th column of  $E_0$ . Therewith, the *fault detectability matrix*  $D_0^* \in \mathbb{R}^{n_y \times n_f}$  is defined as

$$D_0^* = \begin{bmatrix} C_0 A_0^{\delta_1 - 1} e_{01} & \dots & C_0 A_0^{\delta_{n_f} - 1} e_{0n_f} \end{bmatrix}. \quad (9)$$

With these definitions, we pose the following assumptions for the remainder of the paper.

*Assumption 1:* The pair  $(A_0, C_0)$  is observable.

*Assumption 2:* The matrices  $E_0$  and  $C_0$  as well as  $E_i$  and  $C_i$  with  $i = 1, \dots, N$  are full rank.

*Assumption 3:* The matrix  $D_0^*$  satisfies  $\text{rank}(D_0^*) = n_f$ .

*Assumption 4:* The system  $(A_0, E_0, C_0)$  is minimum phase.

While Assumption 1 ensures that all observer eigenvalues can be arbitrarily placed, Assumptions 2 and 3 guarantee the existence of an FIO for the nominal system. Assumption 4 ensures that the nominal system does not have any invariant zeros in the closed right half plane and thus guarantees stability of the designed FIO as we show in Section III.

### III. FIO DESIGN

In this section, we briefly recall some results on FIO design. For details, we refer to [22].

The idea of the FIO approach is to use structures like

$$\dot{\hat{x}} = A_0 \hat{x} + B_0 u + L(y - C_0 \hat{x} - D_0 u), \quad (10a)$$

$$r = V(y - C_0 \hat{x} - D_0 u) \quad (10b)$$

to generate the residuals. As shown in e.g. [10], [11], [22], this allows to diagonalize  $\mathbf{G}_{rf,0}(s)$  for the nominal system, which is summarized in the following theorem.

*Theorem 1:* Given a system (3) fulfilling Assumptions 1 – 4. Then an FIO of the form (10) diagonalizes  $\mathbf{G}_{rf,0}(s)$  for the nominal model with diagonal elements

$$g_{i,i}(s) = \frac{z_{i,0}}{s^{\delta_i} + q_{i,\delta_i-1}s^{\delta_i-1} + \dots + q_{i,1}s + q_{i,0}}$$

if  $\mathbf{L}$  and  $\mathbf{V}$  are chosen according to  $\mathbf{L} = \mathbf{M}_0\mathbf{D}_0^{*-1}$ ,  $\mathbf{V} = \mathbf{N}_0\mathbf{D}_0^{*-1}$  with

$$\mathbf{M}_0 = \begin{bmatrix} \left( \mathbf{A}_0^{\delta_1} \mathbf{e}_{0_1} + \sum_{k=0}^{\delta_1-1} q_{1,k} \mathbf{A}_0^k \mathbf{e}_{0_1} \right)^\top \\ \vdots \\ \left( \mathbf{A}_0^{\delta_{n_f}} \mathbf{e}_{0_{n_f}} + \sum_{k=0}^{\delta_{n_f}-1} q_{n_f,k} \mathbf{A}_0^k \mathbf{e}_{0_{n_f}} \right)^\top \end{bmatrix}^\top,$$

$$\mathbf{N}_0 = \text{diag}(z_{1,0}, \dots, z_{n_f,0}).$$

*Proof:* The proof relies on the duality between FIO design and non-interacting controller design dating back to [6]. Its details can be found in [22] and are omitted here due to space restrictions. ■

It should be noticed that an FIO (10) is a special case of an FIF with  $\mathbf{A}_f = \mathbf{A}_0 - \mathbf{L}\mathbf{C}_0$ ,  $\mathbf{B}_{f,u} = \mathbf{B}_0 - \mathbf{L}\mathbf{D}_0$ ,  $\mathbf{B}_{f,y} = \mathbf{L}$ ,  $\mathbf{C}_f = -\mathbf{V}\mathbf{C}_0$ ,  $\mathbf{D}_{f,u} = -\mathbf{V}\mathbf{D}_0$ , and  $\mathbf{D}_{f,y} = \mathbf{V}$ . While all observer eigenvalues can be arbitrarily placed due to Assumption 1, the structure of  $\mathbf{A}_f$  is fixed to  $\mathbf{A}_f = \mathbf{A}_0 - \mathbf{L}\mathbf{C}_0$ . This structural constraint, which is inherent to the observer-based approach, is making it difficult to optimize robustness of FIO by means of convex optimization with LMI constraints. Furthermore, Theorem 1 only considers square systems. While it is possible to extend it to non-square systems with  $n_y > n_f$ , this extension increases the design effort [10], [11], [22]. It should be noticed, that for non-square systems, the interpretation of  $\delta_i$  being the minimum relative degree of the diagonal element  $g_{i,i}(s)$  remains the same as pointed out in [22].

#### IV. FIF DESIGN

For FIFs, all matrices in (5) are arbitrary and no structural constraints are imposed, which results in more degrees of freedom in the design compared to FIOs. The drawback is that it is very difficult to give a closed solution for  $(\mathbf{A}_f, \mathbf{B}_{f,u}, \mathbf{B}_{f,y}, \mathbf{C}_f, \mathbf{D}_{f,u}, \mathbf{D}_{f,y})$  achieving (6). However, since FIOs can be considered as a special case of FIFs, we know that an FIF diagonalizing  $\mathbf{G}_{rf,0}(s)$  exists if the conditions of Theorem 1 hold. As a by-product of Theorem 1, we also know the resulting transfer matrix  $\mathbf{G}_{rf,0}(s)$  with its diagonal elements  $g_{i,i}(s)$ . Furthermore, for perfect fault isolation neither the control inputs  $\mathbf{u}$  nor the disturbances  $\mathbf{d}$  should have any effect onto the generated residuals.

The basic idea of the model-matching based FIF design approach [17] is to select a reference model, which meets the design specifications regarding control inputs, faults, and disturbances mentioned above. The system consisting of the actual plant and FIF (marked by the dashed box in Fig. 1) is then to generate residuals  $\mathbf{r}$ , which match the residuals

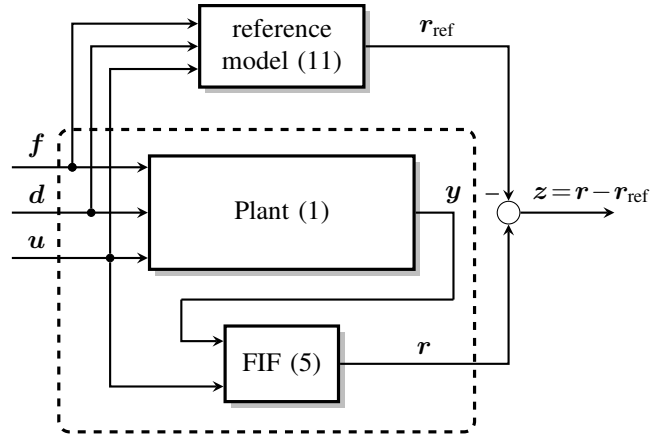


Fig. 1. Virtual reference model for FIF design

$\mathbf{r}_{\text{ref}}$  generated by the virtual reference model as closely as possible.

#### A. Design of an attainable reference model for fault isolation

As pointed out in [8], the selection of a reference model

$$\dot{\mathbf{x}}_{\text{ref}} = \mathbf{A}_{\text{ref}}\mathbf{x}_{\text{ref}} + \mathbf{B}_{\text{ref},u}\mathbf{u} + \mathbf{B}_{\text{ref},f}\mathbf{f} + \mathbf{B}_{\text{ref},d}\mathbf{d}, \quad (11a)$$

$$\mathbf{r}_{\text{ref}} = \mathbf{C}_{\text{ref}}\mathbf{x}_{\text{ref}} + \mathbf{D}_{\text{ref},u}\mathbf{u} + \mathbf{D}_{\text{ref},f}\mathbf{f} + \mathbf{D}_{\text{ref},d}\mathbf{d} \quad (11b)$$

with  $\mathbf{x}_{\text{ref}} \in \mathbb{R}^{n_{\text{ref}}}$  is crucial for the successful application of the model-matching approach. The selection of a reference model that is unsuitable for the system can lead to very poor robustness results [8]. In [17], the model-matching approach to design fault detection filters is introduced and its possible application to fault isolation is also briefly mentioned. However, no results on the design of a suitable reference model for fault isolation are given, hindering the wide use of general linear filters for fault isolation using the model-matching based FIF design approach.

To be more precise, we use a definition for suitable or *attainable* reference models similar to [8].

*Definition 1:* For a nominal system (3), a reference model

$$\mathbf{G}_{\text{ref},rf}(s) = \mathbf{C}_{\text{ref}}(s\mathbf{I}_{n_{\text{ref}}} - \mathbf{A}_{\text{ref}})^{-1}\mathbf{B}_{\text{ref},f}$$

is called *attainable* if there exists a filter (5), such that for  $\mathbf{G}_{rf,0}(s)$  defined in (6a),

$$\|\mathbf{G}_{rf,0}(s) - \mathbf{G}_{\text{ref},rf}(s)\|_{\infty} < \varepsilon$$

holds with an arbitrarily small scalar  $\varepsilon > 0$ .

Since we know the resulting structure of  $\mathbf{G}_{rf}(s)$  when using FIOs, we can select this transfer matrix as a reference model, which is then guaranteed to be attainable. At this point it is important to emphasize that the actual design of an FIO does not have to be conducted. Only the resulting transfer matrix  $\mathbf{G}_{rf}(s)$  is needed, since it is used to design the reference model for the model-matching approach to design the FIF. Since  $\mathbf{G}_{rf}(s)$  is diagonal, the matrices  $\mathbf{A}_{\text{ref}}$ ,

$B_{\text{ref},f}$ ,  $C_{\text{ref}}$ , and  $D_{\text{ref},f}$  can be set to

$$A_{\text{ref}} = \text{diag}(A_{\text{ref},f,1}, \dots, A_{\text{ref},f,n_f}), \quad (12a)$$

$$B_{\text{ref},f} = \text{diag}(B_{\text{ref},f,1}, \dots, B_{\text{ref},f,n_f}), \quad (12b)$$

$$C_{\text{ref}} = \text{diag}(C_{\text{ref},f,1}, \dots, C_{\text{ref},f,n_f}), \quad (12c)$$

$$D_{\text{ref},f} = \mathbf{0}, \quad (12d)$$

where  $(A_{\text{ref},f,i}, B_{\text{ref},f,i}, C_{\text{ref},f,i})$  is any canonical state space realization of the SISO transfer functions  $g_{i,i}(s)$ .

Notice that for the notion of attainability, we only consider the fault to residual transfer matrix. Since for perfect fault isolation, control inputs and disturbances should have no effect onto the residuals, we choose  $B_{\text{ref},u} = \mathbf{0}$ ,  $B_{\text{ref},d} = \mathbf{0}$ ,  $D_{\text{ref},u} = \mathbf{0}$ , and  $D_{\text{ref},d} = \mathbf{0}$ .

*Remark 1:* As discussed in [25], it is also possible to include the influence of the disturbances into the reference model. Following this paradigm, we set  $B_{\text{ref},u} = \mathbf{0}$ ,  $D_{\text{ref},u} = \mathbf{0}$ ,  $B_{\text{ref},d} = B_{b,0} - LD_{d,0}$ , and  $D_{\text{ref},d} = VD_{d,0}$  having designed an FIO. This approach takes into account the fact that even without parametric uncertainties, it is generally not possible to design a residual generator which isolates faults and decouples disturbances perfectly at the same time. However, the actual FIO design has to be carried out to select  $B_{\text{ref},d}$  and  $D_{\text{ref},d}$  in contrast to the simpler choice  $B_{\text{ref},d} = \mathbf{0}$ ,  $D_{\text{ref},d} = \mathbf{0}$ .

### B. LMI-based FIF design approach

To describe the input-output behavior of the overall system depicted in Fig. 1, we introduce a generalized input vector  $\tilde{u} = [u^T \ f^T \ d^T]^T \in \mathbb{R}^{n_u+n_f+n_d}$  and a performance output  $z = r - r_{\text{ref}} \in \mathbb{R}^{n_f}$ . Using  $x_g = [x_{\text{ref}}^T \ x^T \ x_f^T]^T \in \mathbb{R}^{n_{\text{ref}}+n+n_{\text{fit}}}$  and (1), (5), and (11) with  $B_{\text{ref},u} = \mathbf{0}$  and  $D_{\text{ref},u} = \mathbf{0}$ , we can deduce the state space model of the overall system as (13), which is given at the bottom of this page. With (13), the FIF design problem can be formulated as

$$\begin{aligned} & \underset{A_f, B_{f,u}, B_{f,y}, C_f, D_{f,u}, D_{f,y}}{\text{minimize}} \quad \gamma \text{ s.t.} \quad (14a) \\ & \left\| C_{z\tilde{u},\xi} (sI - A_{z\tilde{u},\xi})^{-1} B_{z\tilde{u},\xi} + D_{z\tilde{u},\xi} \right\|_{\infty} < \gamma \quad \forall \xi. \quad (14b) \end{aligned}$$

Employing the Bounded Real Lemma (BRL), [1], the constraint (14b) can be recast as

$$\mathbb{R}^{n_{\text{ref}}+n+n_{\text{fit}}} \ni P^T = P \succ \mathbf{0}, \quad (15a)$$

$$\begin{bmatrix} \text{He}(PA_{z\tilde{u},i}) & PB_{z\tilde{u},i} & C_{z\tilde{u},i}^T \\ * & -\gamma I_{n_u+n_f+n_d} & D_{z\tilde{u},i}^T \\ * & * & -\gamma I_{n_f} \end{bmatrix} \prec \mathbf{0}, \quad (15b)$$

with  $i = 1, \dots, N$ . It is important to notice that since a common matrix  $P$  is used for all  $i$ , the finite number of matrix inequalities in (15) ensures that the model-matching level  $\gamma$  is met for all systems that belong to  $\Omega$ . However, since  $A_{z\tilde{u},i}$ ,  $B_{z\tilde{u},i}$ ,  $C_{z\tilde{u},i}$ , and  $D_{z\tilde{u},i}$  contain the parameters of the FIF, (15) are bilinear matrix inequalities (BMIs) and hence non-convex and difficult to solve. In the following we adopt the approach presented in [17] to formulate equivalent LMI conditions.

First, we partition  $P$  and  $P^{-1}$  as

$$P = \begin{bmatrix} Y_{11} & U \\ * & Y_{22} \end{bmatrix}, \quad P^{-1} = \begin{bmatrix} X_{11} & W \\ * & X_{22} \end{bmatrix}, \quad (16)$$

where  $Y_{11} = Y_{11}^T \in \mathbb{R}^{(n_{\text{ref}}+n) \times (n_{\text{ref}}+n)}$ ,  $Y_{22} \in \mathbb{R}^{n_{\text{fit}} \times n_{\text{fit}}}$ ,  $U \in \mathbb{R}^{(n_{\text{ref}}+n) \times n_{\text{fit}}}$ ,  $X_{11} = X_{11}^T \in \mathbb{R}^{(n_{\text{ref}}+n) \times (n_{\text{ref}}+n)}$ ,  $X_{22} \in \mathbb{R}^{n_{\text{fit}} \times n_{\text{fit}}}$ , and  $W \in \mathbb{R}^{(n_{\text{ref}}+n) \times n_{\text{fit}}}$ . This allows to employ a linearizing change of variables similar as proposed in [20]. Inspecting  $PP^{-1} = I$  reveals that

$$Y_{11}X_{11} + UW^T = I_{n_{\text{ref}}+n}. \quad (17)$$

Next, we define  $Z = X_{11}^{-1}$  and the positive definite matrix

$$\Pi = \begin{bmatrix} I_{n_{\text{ref}}+n} & I_{n_{\text{ref}}+n} \\ W^T Z & \mathbf{0} \end{bmatrix}. \quad (18)$$

Due to [1], the matrix inequality (15a) is equivalent to  $\Pi^T P \Pi \succ \mathbf{0}$ . This congruence transformation results in

$$\Pi^T P \Pi = \begin{bmatrix} Z & Z \\ * & Y_{11} \end{bmatrix} \succ \mathbf{0}. \quad (19)$$

Using the Schur complement lemma [1], this can be formulated as

$$Z \succ \mathbf{0}, \quad (20a)$$

$$Y_{11} - Z \succ \mathbf{0}. \quad (20b)$$

In the following, we introduce the matrices

$$\bar{A}_i = \begin{bmatrix} A_{\text{ref}} & \mathbf{0} \\ \mathbf{0} & A_i \end{bmatrix}, \quad \bar{B}_{1,i} = \begin{bmatrix} \mathbf{0} & B_{\text{ref},f} \\ B_i & E_i \end{bmatrix}, \quad \bar{B}_{2,i} = \begin{bmatrix} B_{\text{ref},d} \\ B_{d,i} \end{bmatrix}, \quad (21a)$$

$$\bar{C}_i = [\mathbf{0} \ C_i], \quad \hat{A}_f = UA_fW^T Z, \quad \hat{B}_{f,u} = UB_{f,u}, \quad (21b)$$

$$\hat{B}_{f,y} = UB_{f,y}, \quad \hat{C}_f = C_fW^T Z, \quad \hat{D}_{f,u} = D_{f,u}, \quad \hat{D}_{f,y} = D_{f,y}. \quad (21c)$$

$$\dot{x}_g = \underbrace{\begin{bmatrix} A_{\text{ref}} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & A_{\xi} & \mathbf{0} \\ \mathbf{0} & B_{f,y}C_{\xi} & A_f \end{bmatrix}}_{A_{z\tilde{u},\xi}} x_g + \underbrace{\begin{bmatrix} \mathbf{0} & B_{\text{ref},f} & B_{\text{ref},d} \\ B_{\xi} & E_{\xi} & B_{d,\xi} \\ B_{f,u} + B_{f,y}D_{\xi} & \mathbf{0} & B_{f,y}D_{d,\xi} \end{bmatrix}}_{B_{z\tilde{u},\xi}} \tilde{u}, \quad (13a)$$

$$z = \underbrace{\begin{bmatrix} -C_{\text{ref}} & D_{f,y}C_{\xi} & C_f \end{bmatrix}}_{C_{z\tilde{u},\xi}} x_g + \underbrace{\begin{bmatrix} D_{f,u} + D_{f,y}D_{\xi} & -D_{\text{ref},f} & D_{f,y}D_{d,\xi} - D_{\text{ref},d} \end{bmatrix}}_{D_{z\tilde{u},\xi}} \tilde{u} \quad (13b)$$

Performing a congruence transformation of (15b) with  $\text{diag}(\mathbf{\Pi}, \mathbf{I}, \mathbf{I})$ , we therewith obtain

$$\begin{bmatrix} \Xi_{11,i} & [\Xi_{121,i} \ \Xi_{122,i}] & \Xi_{13,i} \\ \star & -\gamma \mathbf{I}_{n_u+n_f+n_d} & \Xi_{23,i} \\ \star & \star & -\gamma \mathbf{I}_{n_f} \end{bmatrix} \prec \mathbf{0}, \quad (22a)$$

$$\Xi_{11,i} = \text{He} \left( \begin{bmatrix} \mathbf{Z}\bar{\mathbf{A}}_i & \mathbf{Z}\bar{\mathbf{A}}_i \\ \mathbf{Y}_{11}\bar{\mathbf{A}}_i + \hat{\mathbf{B}}_{f,y}\bar{\mathbf{C}}_i + \hat{\mathbf{A}}_f & \mathbf{Y}_{11}\bar{\mathbf{A}}_i + \hat{\mathbf{B}}_{f,y}\bar{\mathbf{C}}_i \end{bmatrix} \right), \quad (22b)$$

$$\Xi_{121,i} = \begin{bmatrix} \mathbf{Z}\bar{\mathbf{B}}_{1,i} \\ \mathbf{Y}_{11}\bar{\mathbf{B}}_{1,i} + \hat{\mathbf{B}}_{f,u}[\mathbf{I} \ \mathbf{0}] + \hat{\mathbf{B}}_{f,y}[\mathbf{D}_i \ \mathbf{0}] \end{bmatrix}, \quad (22c)$$

$$\Xi_{122,i} = \begin{bmatrix} \mathbf{Z}\bar{\mathbf{B}}_{2,i} \\ \mathbf{Y}_{11}\bar{\mathbf{B}}_{2,i} + \hat{\mathbf{B}}_{f,y}\mathbf{D}_{d,i} \end{bmatrix}, \quad \Xi_{13,i} = \begin{bmatrix} -\mathbf{C}_{\text{ref}}^\top \\ \mathbf{C}_i^\top \hat{\mathbf{D}}_{f,y}^\top + \hat{\mathbf{C}}_f^\top \\ -\mathbf{C}_{\text{ref}}^\top \\ \mathbf{C}_i^\top \hat{\mathbf{D}}_{f,y}^\top \end{bmatrix}, \quad (22d)$$

$$\Xi_{23,i} = \begin{bmatrix} \left( \hat{\mathbf{D}}_{f,u} + \hat{\mathbf{D}}_{f,y}\mathbf{D}_i \right)^\top \\ -\mathbf{D}_{\text{ref},f}^\top \\ \left( \hat{\mathbf{D}}_{f,y}\mathbf{D}_{d,i} - \mathbf{D}_{\text{ref},d} \right)^\top \end{bmatrix}. \quad (22e)$$

Observe that therein, all variables enter linearly and hence both (20) and (22) are LMIs.

From results on  $\mathcal{D}$ -regions [4] it is known that the eigenvalues of a matrix  $\mathbf{\Lambda}$  lie in a circular region with radius  $\varrho$  around the origin if

$$\mathbf{P} \succ \mathbf{0}, \quad \begin{bmatrix} -\varrho \mathbf{P} & \mathbf{P}\mathbf{\Lambda} \\ \star & -\varrho \mathbf{P} \end{bmatrix} \prec \mathbf{0} \quad (23)$$

holds. Evaluating (23) for the virtual system matrix  $\mathbf{A}_{z\tilde{u},i}$  as defined in (13) and performing a congruence transformation with  $\text{diag}(\mathbf{\Pi}, \mathbf{\Pi})$  yields

$$\begin{bmatrix} \mathbf{\Gamma}_{11} & \mathbf{\Gamma}_{12,i} \\ \star & \mathbf{\Gamma}_{22} \end{bmatrix} \prec \mathbf{0}, \quad (24a)$$

$$\mathbf{\Gamma}_{11} = \mathbf{\Gamma}_{22} = -\varrho \begin{bmatrix} \mathbf{Z} & \mathbf{Z} \\ \star & \mathbf{Y}_{11} \end{bmatrix}, \quad (24b)$$

$$\mathbf{\Gamma}_{12,i} = \begin{bmatrix} \mathbf{Z}\bar{\mathbf{A}}_i & \mathbf{Z}\bar{\mathbf{A}}_i \\ \mathbf{Y}_{11}\bar{\mathbf{A}}_i + \hat{\mathbf{B}}_{f,y}\bar{\mathbf{C}}_i + \hat{\mathbf{A}}_f & \mathbf{Y}_{11}\bar{\mathbf{A}}_i + \hat{\mathbf{B}}_{f,y}\bar{\mathbf{C}}_i \end{bmatrix} \quad (24c)$$

with (20). The additional LMI (24) can be used to prevent overly large filter eigenvalues, which might cause numerical and implementational problems. Since  $\mathbf{A}_{z\tilde{u},i}$  is in lower block-triangular structure, its set of eigenvalues consists of the eigenvalues of  $\mathbf{A}_{\text{ref}}$ ,  $\mathbf{A}_i$ , and  $\mathbf{A}_f$ . Due to this fact, care has to be taken that  $\varrho$  is selected such that the resulting  $\mathcal{D}$ -region contains the eigenvalues of  $\mathbf{A}_{\text{ref}}$  and  $\mathbf{A}_i$ .

Summarizing the design, the model-matching problem (14) can be reformulated as

$$\underset{\mathbf{z}, \mathbf{Y}_{11}, \hat{\mathbf{A}}_f, \hat{\mathbf{B}}_{f,u}, \hat{\mathbf{B}}_{f,y}, \hat{\mathbf{C}}_f, \hat{\mathbf{D}}_{f,u}, \hat{\mathbf{D}}_{f,y}}{\text{minimize}} \quad \gamma \text{ s.t.} \quad (25a)$$

$$(20), (22), (24), i = 1, \dots, N. \quad (25b)$$

We point out again that the LMI-based design using a reference model is similar to [17]. However, we make use of

the results on FIO design in Section III to obtain an attainable reference model for fault isolation.

*Remark 2:* Note that (25) tries to match the input-output behavior of the combined system of plant (1) and FIF (5) to the reference model with respect to control inputs  $\mathbf{u}$ , faults  $\mathbf{f}$ , and disturbances  $\mathbf{d}$  at the same time. Of course it is possible to add weighting factors into the optimization problem to trade off between the model-matching quality with respect to the three inputs  $\mathbf{u}$ ,  $\mathbf{f}$ , and  $\mathbf{d}$ . To achieve this, we have to multiply the inputs with positive scalars  $\alpha_u$ ,  $\alpha_f$ ,  $\alpha_d$ , which is equivalent to setting

$$[\mathbf{B}_\xi \ \mathbf{B}_{f,u} \ \mathbf{D}_\xi \ \mathbf{D}_{f,u}] := \alpha_u [\mathbf{B}_\xi \ \mathbf{B}_{f,u} \ \mathbf{D}_\xi \ \mathbf{D}_{f,u}], \quad (26a)$$

$$[\mathbf{B}_{\text{ref},f} \ \mathbf{E}_\xi \ \mathbf{D}_{\text{ref},f}] := \alpha_f [\mathbf{B}_{\text{ref},f} \ \mathbf{E}_\xi \ \mathbf{D}_{\text{ref},f}], \quad (26b)$$

$$[\mathbf{B}_{\text{ref},d} \ \mathbf{B}_{d,\xi} \ \mathbf{D}_{d,\xi} \ \mathbf{D}_{\text{ref},d}] := \alpha_d [\mathbf{B}_{\text{ref},d} \ \mathbf{B}_{d,\xi} \ \mathbf{D}_{d,\xi} \ \mathbf{D}_{\text{ref},d}] \quad (26c)$$

in optimization problem (25).

Once (25) has been solved, it remains to recover the filter parameters  $\mathbf{A}_f$ ,  $\mathbf{B}_{f,u}$ ,  $\mathbf{B}_{f,y}$ ,  $\mathbf{C}_f$ ,  $\mathbf{D}_{f,u}$ , and  $\mathbf{D}_{f,y}$ . In order to do that we have to compute  $\mathbf{U}$  and  $\mathbf{W}$  and from (17) we deduce

$$\mathbf{U}\mathbf{W}^\top = \mathbf{I}_{n_{\text{ref}}+n} - \mathbf{Y}_{11}\mathbf{Z}^{-1}. \quad (27)$$

If the filter order is selected as  $n_{\text{filt}} = n_{\text{ref}} + n$ ,  $\mathbf{U}$  and  $\mathbf{W}$  are square matrices and can be obtained by e.g. an LU-factorization of (27). Because  $\mathbf{U}$  and  $\mathbf{W}$  are non-singular in this case, we readily obtain

$$\mathbf{A}_f = \mathbf{U}^{-1} \hat{\mathbf{A}}_f \left( \mathbf{W}^\top \mathbf{Z} \right)^{-1}, \quad \mathbf{B}_{f,u} = \mathbf{U}^{-1} \hat{\mathbf{B}}_{f,u}, \quad (28a)$$

$$\mathbf{B}_{f,y} = \mathbf{U}^{-1} \hat{\mathbf{B}}_{f,y}, \quad \mathbf{C}_f = \hat{\mathbf{C}}_f \left( \mathbf{W}^\top \mathbf{Z} \right)^{-1}, \quad (28b)$$

$$\mathbf{D}_{f,u} = \hat{\mathbf{D}}_{f,u}, \quad \mathbf{D}_{f,y} = \hat{\mathbf{D}}_{f,y}. \quad (28c)$$

If  $n_{\text{filt}} < n_{\text{ref}} + n$  is chosen, (27) can be factorized into full-rank matrices  $\mathbf{U}$  and  $\mathbf{W}$  if the rank constraint  $\text{rank}(\mathbf{I} - \mathbf{Y}_{11}\mathbf{Z}^{-1}) = n_{\text{filt}}$  is fulfilled (s. [20]). This can be formulated as an additional constraint in optimization problem (25) as

$$\text{rank}(\mathbf{Y}_{11} - \mathbf{Z}) = n_{\text{filt}}. \quad (29)$$

This rank constraint renders the optimization problem non-convex. However, from Theorem 1 we know that a full-order observer exists, which achieves perfect fault isolation for the nominal system with  $\mathbf{d} = \mathbf{0}$ . As pointed out already in Section III, such an observer is a special case of an FIF with  $n_{\text{filt}} = n$ . Hence we know that the rank constrained optimization problem (25), (29) can be solved for arbitrarily small  $\gamma$  for the nominal system with  $\mathbf{d} = \mathbf{0}$ , if  $n_{\text{filt}} = n < n_{\text{ref}} + n$  is chosen, which justifies the application of specialized solvers such as presented in [18]. In this case, the filter parameters are obtained by

$$\mathbf{A}_f = \mathbf{U}^+ \hat{\mathbf{A}}_f \left( \mathbf{W}^\top \mathbf{Z} \right)^+, \quad \mathbf{B}_{f,u} = \mathbf{U}^+ \hat{\mathbf{B}}_{f,u}, \quad (30a)$$

$$\mathbf{B}_{f,y} = \mathbf{U}^+ \hat{\mathbf{B}}_{f,y}, \quad \mathbf{C}_f = \hat{\mathbf{C}}_f \left( \mathbf{W}^\top \mathbf{Z} \right)^+, \quad (30b)$$

$$\mathbf{D}_{f,u} = \hat{\mathbf{D}}_{f,u}, \quad \mathbf{D}_{f,y} = \hat{\mathbf{D}}_{f,y} \quad (30c)$$

using Moore-Penrose-Inverses after factorizing (27).

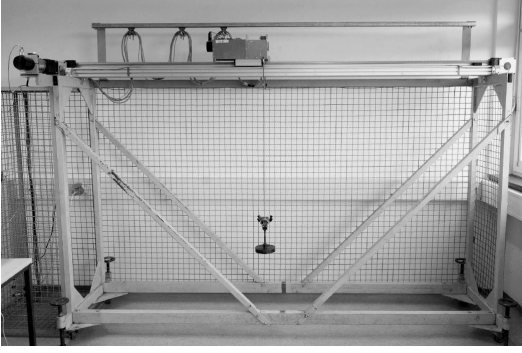


Fig. 2. Gantry crane lab experiment

## V. EXAMPLE

To demonstrate our approach, we apply it to isolate faults in the gantry crane system depicted in Fig. 2. It consists of an actuated cart with a load mass attached to a pendulum. Assuming an uncertain load mass, the nominal system matrices of our lab experiment system read

$$\mathbf{A}_0 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -70.31 & a_{2,3_0} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -70.31 & a_{4,3_0} & 0 \end{bmatrix}, \quad \mathbf{B}_0 = \begin{bmatrix} 0 \\ 5.89 \\ 0 \\ 5.89 \end{bmatrix}, \quad (31a)$$

$$\mathbf{E}_0 = \begin{bmatrix} 0 & 0 \\ 0 & 5.89 \\ 0 & 0 \\ -0.10 & 5.89 \end{bmatrix}, \quad \mathbf{C}_0 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}, \quad (31b)$$

with  $\mathbf{D}_0 = \mathbf{0}$ . Therein, we model two potential faults. The fault  $f_1$  describes a horizontal force applied to the load mass. This can e.g. be caused by the load mass hitting an obstacle. The second fault models an actuator fault, i.e. the actuator is not producing the signals the controller commanded. The states of the system are cart position and velocity and pendulum angle and angular velocity, where the first and third state can be measured. By inspecting the system dynamics, we observe that the uncertain load mass enters linearly into  $a_{2,3}$

and  $a_{4,3}$ . For a nominal load mass of  $m_{L_0} = 9.6$  kg we obtain  $a_{2,3_0} = -2.12$  and  $a_{4,3_0} = -14.04$ .

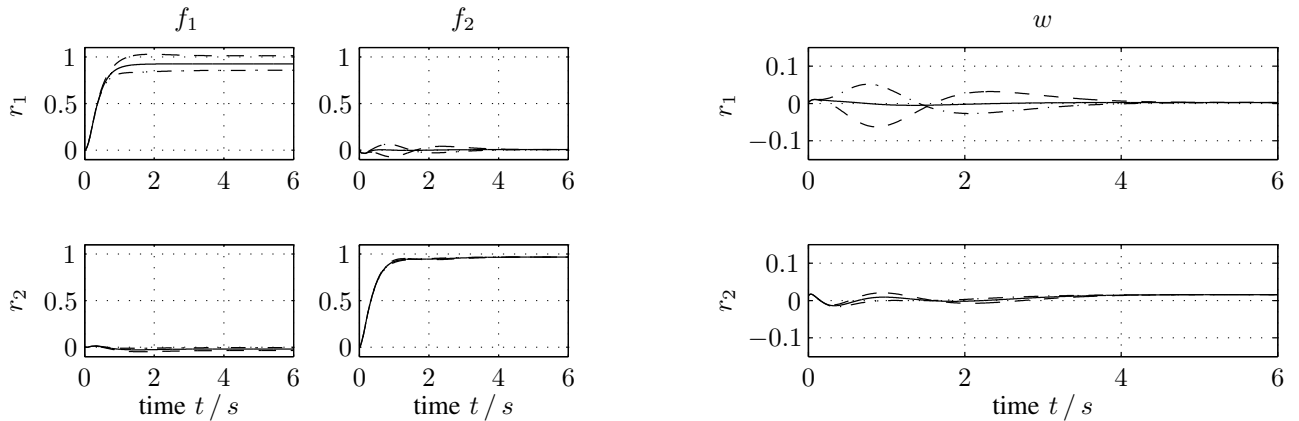
The system is operated using an observer-based output-feedback controller  $\mathbf{u} = -\mathbf{K}\hat{\mathbf{x}} + \mathbf{F}w$ , where  $w$  is the commanded cart position and  $\hat{\mathbf{x}}$  is the estimated state generated by a state observer parameterized by  $\mathbf{L}$ . Note that this observer is not an FIO but a standard Luenberger-type observer to estimate the system states. The closed-loop system with faults is thus given by

$$\begin{bmatrix} \dot{\hat{\mathbf{x}}} \\ \dot{\hat{\mathbf{x}}} \end{bmatrix} = \underbrace{\begin{bmatrix} \mathbf{A}_\xi & -\mathbf{B}_0\mathbf{K} \\ \mathbf{L}\mathbf{C}_0 & \mathbf{A}_0 - \mathbf{B}_0\mathbf{K} - \mathbf{L}\mathbf{C}_0 \end{bmatrix}}_{\hat{\mathbf{A}}_\xi} \begin{bmatrix} \mathbf{x} \\ \hat{\mathbf{x}} \end{bmatrix} + \dots \\ + \underbrace{\begin{bmatrix} \mathbf{B}_0\mathbf{F} \\ \mathbf{B}_0\mathbf{F} \end{bmatrix}}_{\hat{\mathbf{B}}} w + \underbrace{\begin{bmatrix} \mathbf{E}_0 \\ \mathbf{0} \end{bmatrix}}_{\hat{\mathbf{E}}} f, \quad (32a)$$

$$\mathbf{y} = \underbrace{\begin{bmatrix} \mathbf{C}_0 & \mathbf{0} \end{bmatrix}}_{\hat{\mathbf{C}}} \begin{bmatrix} \mathbf{x} \\ \hat{\mathbf{x}} \end{bmatrix}. \quad (32b)$$

Since the uncertain parameter only affects  $\mathbf{A}_\xi$ , we use the nominal matrices for  $\hat{\mathbf{B}}$ ,  $\hat{\mathbf{C}}$ , and  $\hat{\mathbf{E}}$ . To account for sensor noise, we set  $\mathbf{B}_d = \mathbf{0}$  and  $\mathbf{D}_d = \text{diag}(0.1, 0.1)$ .

Analyzing the closed-loop system  $(\hat{\mathbf{A}}_0, \hat{\mathbf{E}}, \hat{\mathbf{C}})$  reveals that Assumptions 1 – 4 are fulfilled and that the fault detectability indices are  $\delta_1 = \delta_2 = 2$ . Due to this, we choose a diagonal transfer matrix with poles  $\lambda_{\text{ref},1,1} = -4$  and  $\lambda_{\text{ref},1,2} = -6$  for the first channel and  $\lambda_{\text{ref},2,1} = -5$  and  $\lambda_{\text{ref},2,2} = -5.5$  for the second fault with static gains of 1 as a reference model (11), (12). To describe the set of system matrices  $\Omega$ , we assume a minimum load mass of  $m_{L_1} = 5$  kg and a maximum load mass of  $m_{L_2} = 14$  kg. The resulting matrices  $\hat{\mathbf{A}}_1$  and  $\hat{\mathbf{A}}_2$  are used to design an FIF according to (25). To emphasize on fault isolation and attenuation of the influence the commanded position onto the residuals, we set weighting factors  $\alpha_u = \alpha_f = 1$  and  $\alpha_d = 0.01$ . Furthermore, the eigenvalues are constrained by (24) with  $\varrho = 60$ . Designing a full-order FIF, we choose  $n_{\text{filt}} = 2n + n_{\text{ref}} = 12$  and



(a) Faults to residuals step response (— nominal load mass, --- minimum load mass, ··· maximum load mass)

(b) Commanded position to residuals step response (— nominal load mass, --- minimum load mass, ··· maximum load mass)

Fig. 3. Simulated step responses

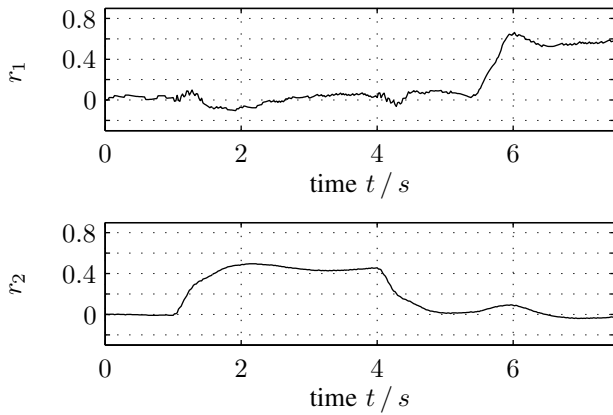


Fig. 4. Time plot of measured residuals

employing the LMI-solver SeDuMi [21] with the YALMIP [16] interface we achieve  $\gamma = 0.2621$  when solving (25) with (26). The actual filter parameters are then obtained by factorizing (27) and evaluating (28).

Fig. 3 visualizes the simulated step responses of faults (Fig. 3(a)) and the commanded cart position (Fig. 3(b)) with respect to the generated residuals for the nominal load mass as well as for  $m_{L_1}$  and  $m_{L_2}$ . From Fig. 3(a) we observe that for all configurations, fault isolation is achieved since there are hardly any cross-couplings between the faults and the generated residuals. Furthermore, the commanded signals only have a neglectable influence onto the residuals (notice the y-axis scaling in Fig. 3(b)).

Furthermore, the FIF is implemented in a dSPACE-environment and applied to the lab experiment. The generated residuals are given in Fig. 4. During  $t \in [1s, 4s]$ , the actuator fault is set to  $f_2 = 0.5$ , which is readily isolated by inspecting  $r_2$  after approx.  $0.2s$ . After approximately  $t = 5.7s$ , we manually apply a horizontal force to the load mass, which is also detected and isolated from  $r_1$  after  $0.2s$ . Notice that Fig. 4 shows real-time measurement results, where the load mass is approximately  $m_L = 8$  kg.

## VI. CONCLUSION

We showed how general linear filters can be employed to isolate faults in linear systems using an LMI-based design. The key to the solution is the determination of an attainable reference model using analysis tools from the design of fault isolation observers (FIOs). This guarantees the existence of a solution, which is then obtained by solving an LMI-problem. Therein, robustness with respect to both disturbances as well as uncertain parameters can be regarded in a straightforward manner. The design of fault isolation filters (FIFs) furthermore easily allows to integrate additional measurements. The design was successfully applied to a closed-loop gantry crane system in both simulations and experiments.

Future work includes the extension of the proposed FIF design taking into account finite frequency ranges for disturbances, control signals, and faults.

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