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Synchronization in a heterogeneous network of discrete-time introspective right-invertible agents

Xu Wang[1], Ali Saberi[1], and Tao Yang[2]

***Abstract*— This paper studies output synchronization problem for a heterogenous network of discrete-time introspective right-invertible agents. We propose a decentralized control scheme to solve the output synchronization problem for a set of communication topologies. Moreover, if the synchronization trajectories are assumed to be bounded, a universal controller can be constructed for all communication topologies which contain a directed spanning tree.**

I. INTRODUCTION

The synchronization problem in a network has received substantial attention in recent years, e.g., [1]–[4].

The research can be generally divided into two categories: one studies *homogeneous* networks, i.e., networks consisting of identical agents–and the other studies *heterogeneous* networks in which non-identical agents are interconnected.

In homogeneous networks, each agent may measure its own state or output relative to that of neighbors. The study on state synchronization in homogeneous network has been quite fruitful. Depending on what information the agents collect from the network, synchronization in homogeneous networks can be classified into two categories. In some networks, each agent measures its own state relative to that of neighbors, which is referred to as *full-state coupling* [5]–[10]; In other networks, the agents may collect information of its output relative to that of its neighboring agents, which we refer to as *partial-state coupling* [11]–[14]. A key idea in the work of [13], is the development of a distributed observer-based synchronization controller. These controllers make additional use of the network by allowing the agents to exchange about their own internal estimates. It is shown in [14] that for agents that are at most critically unstable, such communication among the controllers is not necessary and a decentralized controller can be designed via a low-gain approach.

Compared with the study on homogeneous networks, relatively limited results have been obtained for *heterogeneous networks*. For heterogenous networks, the notion of state synchronization may no longer make sense as each agent possesses a set of state information which may be inherently different from others. In this case, it is more natural to study an alternative problem of *output synchronization*, that is, all the agents should agree on a set of pre-selected outputs, e.g., [15]–[20]. As demonstrated in [18], [19], it is necessary to further classify the heterogeneous network as having *introspective* agents or *non-introspective* agents. While they both collect information of their output relative to that of its neighbors from the network, an introspective agent also acquires absolute full or partial measurement of its own states. For introspective agents, a recent paper [18] studies continuous-time agents and solves the synchronization problem using a two-step control strategy. By utilizing local measurement and system structural properties of the agents, a local pre-compensator can be *properly* designed to convert the non-identical agents to "asymptotically identical" agents with additional desired properties. Then high level decentralized synchronization controller can be constructed on top of the homogenized network using existing techniques. This homogenization has the freedom in choosing the homogeneous model which facilitate the design of high level controller so that the synchronization can be achieved in a larger range of networks. The synchronization problem is more difficult for non-introspective agents, yet effort has already been made in this direction [19], [21].

Although the research is primarily focused on continuous-time case, synchronization in homogeneous networks of discrete-time agents has been studied in e.g., [6], [22]–[24]. A *distributed* observer-based synchronization controller was developed in [22] which communicates information over the same network. In [23], the author considers a very special case of neutrally stable agent with full actuation ($B = I$). A network of first-order agents with Laplacian communication topology is studied in [6] where the topology can be switching. All the aforementioned work only considers identical agents, namely, homogeneous networks.

In this paper, we consider output synchronization problem for a heterogeneous network of non-identical discrete-time introspective right-invertible agents. The goal is to design a *decentralized* controller to achieve output synchronization in a set of unknown networks. We adopted the same strategy as in [18]. However, as later will become clear, the technical development is very different from continuous-time case. For example, the network configuration is discrete-time case and continuous-time case are different and hence the synchronization conditions. Consequently, we have taken this into account in our homogenization process. Another major difference is that in this paper we aim to design a decentralized control scheme. In the continuous-time case, once the network is casted into a homogenous one, this

[1]School of Electrical Engineering and Computer Science, Washington State University, Pullman, WA 99164-2752, USA. E-mail: Email: xw665@ieee.org, saberi@eecs.wsu.edu.

[2]ACCESS Linnaeus Centre, Royal Institute of Technology (KTH), Sweden. E-mail: taoyang@kth.se.

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is known problem. However, the decentralized controller designs are not available in discrete-time case even for homogeneous networks. We show that exchange of information among controllers is not needed and propose an explicit decentralized control design methodology, which also extends the results in [22]. In the journal version of this paper [25], we also considered two other problems, namely, formation problem and regulated synchronization problem. Due to the space limitation, we have omitted most of the proofs, which can be found in [25].

The paper is organized as follows: The network structure and preliminary assumptions and definitions are given in Section II. The output synchronization problem is solved in Section III.Our design methodology is illustrated in Section IV by numerical examples. Section V concludes the paper.

II. Network structure

Consider a heterogeneous network of N introspective agents

$$\begin{cases} x^i(k+1) = A^i x^i(k) + B^i u^i(k), \\ y^i(k) \quad = C_y^i x^i(k), \\ z^i(k) \quad = C_z^i x^i(k), \\ \zeta^i(k) \quad = \sum_{j=1}^N d_{ij}(y^i(k) - y^j(k)), \end{cases} \tag{1}$$

where $x^i \in \mathbb{R}^{n_i}$, $y^i \in \mathbb{R}^p$, $z^i \in \mathbb{R}^{q_i}$ and $u^i \in \mathbb{R}^{m_i}$. The matrix $D = \{d_{ij}\} \in \mathbb{R}^{N\times N}$ is a row-stochastic matrix that satisfies $d_{ii} > 0$, $d_{ij} \geq 0$ and $\sum_j d_{ij} = 1$. This D matrix defines a communication topology that can be captured by a directed graph $G = (\mathscr{V}, \mathscr{E})$. The set $\mathscr{N}$ contains all the node and $\mathscr{E}$ is the edge set such that an arc $(j,i) \in \mathscr{E}$ if $d_{ij} > 0$.

Assumption 1: Assume that the communication topology G contains a directed spanning tree, and the matrix $D = \{d_{ij}\}_{N\times N}$ is a row stochastic matrix with $d_{ii} > 0$ for all $i = 1,\dots,N$.

Since Assumption 1 is satisfied, it then follows from [8, Corollary 3.5] that D has a simple eigenvalue at 1 with the corresponding right eigenvector **1** and all other eigenvalues are strictly within the unit disk. Let $\lambda_1,\dots,\lambda_N$ denote the eigenvalues of D such that $\lambda_1 = 1$ and $|\lambda_i| < 1$, $i = 2,\dots,N$. We can define a set of communication topologies as follows:

Definition 1: For $\delta \in (0,1]$, let $\mathscr{G}_\delta$ denote the set of communication topologies for which $|\lambda_i| < \delta$, $i = 2,...,N$.

Remark 1: For $\delta = 1$, $\mathscr{G}_1$ is the set of all communication topologies that satisfies Assumption 1. In this case, we shall drop the subscription 1 and simply denote it as $\mathscr{G}$ but it implies $\delta = 1$.

In the network (1), each agent collects two measurements:

1) a network measurement $\zeta^i \in \mathbb{R}^p$ which is a combination of its own output relative to that of neighboring agents;
2) a local measurement $z^i \in \mathbb{R}^{q_i}$ of its internal dynamics.

For each agent, we make the following standard assumption.

Assumption 2: Assume that for each agent $i = 1,\dots,N$,

1) (A^i, B^i) is stabilizable;
2) (A^i, C_z^i) is detectable;
3) (A^i, C_y^i) is detectable; and
4) (A^i, B^i, C_y^i) is right-invertible.

Remark 2: The definition of right-invertibility of a linear system can be found in [26]. Right-invertibility of a triple (A^i, B^i, C_y^i) means that, given a reference output $y_r(k)$, there exist an initial condition $x^i(0)$ and an input $u^i(k)$ such that $y^i(k) = y_r(k)$ for all the non-negative integers k. For example, every single-input single-output system is right-invertible, unless its transfer function is identically zero.

III. Output Synchronization

The first problem studied in this paper is the output synchronization problem. The output synchronization in a heterogeneous network of the form (1) is defined as follows:

Definition 2: Consider a heterogeneous network of the form (1). *The agents in the network achieve output synchronization if*

$$\lim_{k\to\infty}(y^i(k) - y^j(k)) = 0, \quad \forall i,j \in \{1,\dots,N\}.$$

The output synchronization problem is formulated below:

Problem 1: Consider a heterogenous network of the form (1). For $\delta \in (0,1]$ and a given set $\mathscr{G}_\delta$, the output synchronization problem with a set of communication topologies $\mathscr{G}_\delta$ is to design a local linear dynamical controller

$$\begin{cases} \hat{x}^i(k+1) = A_c^i \hat{x}^i(k) + B_c^i \zeta^i(k) + E_c^i z^i(k), \\ u^i(k) \quad = C_c^i \hat{x}^i(k) + D_c^i \zeta^i(k) + M_c^i z^i(k), \end{cases} \tag{2}$$

such that the output synchronization can be achieved in the network with any communication topology belonging to $\mathscr{G}_\delta$.

Remark 3: Since (A^i, C_z^i) is detectable, one can always design a local stabilizing measurement feedback controller so that the network achieves output synchronization in the sense that $y^i(k) \to 0$ as $k \to \infty$. Such a case is not of interest in this paper. We are aiming to reach synchronization with a non-trivial and possibly desirable synchronization trajectory.

The synchronization trajectories considered in most applications are either bounded or polynomially increasing. We shall also present the main results respectively for these two cases. The first theorem is concerned with bounded synchronization trajectories.

Theorem 1: For the set $\mathscr{G}$, Problem 1 with bounded synchronization trajectories is always solvable via decentralized dynamic controllers (2).

Remark 4: Theorem 1 indicates that in the case of bounded synchronization trajectories, a universal synchronization controller can be constructed which solve Problem 1 for any communication topology satisfying Assumption 1.

If unbounded synchronization trajectories are demanded, the admissible set of communication topologies has to be more restricted. This is stated in the next theorem

Theorem 2: For $\delta \in (0,1)$ and a given set $\mathscr{G}_\delta$, Problem 1 with unbounded increasing synchronization trajectories is solvable via decentralized dynamic consensus controllers (2).

We shall prove Theorem 1 and Theorem 2 by explicitly constructing the synchronization controllers. The design and analysis is done in the next four subsections. First, by exploiting the self-measurement of each agent, in Section III-A, we design a local pre-compensator such that the agent model can be re-shaped as asymptotically identical. Next, in the resulting (asymptotically) homogeneous network, we show that solvability of the output synchronization problem can be connected to that of a robust stabilization problem in Section III-B. Finally, the last step is to solve this robust stabilization problem by designing a compensator using a low-gain approach. In this stage, depending on different types of synchronization trajectories, two controllers are proposed in Section III-C and Section III-D respectively.

A. Homogenization of the network

For introspective agents, their self-reflection of internal dynamics provides us with additional freedom to manipulate the agent models so as to disguise them as being almost identical to the rest of the network viewed from their output. This is shown in the next lemma.

Lemma 1: Consider a heterogeneous network of the form (1). Let n_d denote the maximum order of infinite zeros of (A^i, B^i, C^i). Suppose a triple (A,B,C) is given such that

1) $\operatorname{rank}(C) = p$.
2) (A,B,C) is invertible, of uniform rank $n_q \geq n_d$ and has no invariant zeros.

There exists a compensator

$$\begin{cases} \xi^i(k+1) = A_h^i \xi^i(k) + B_h^i z^i(k) + E_h^i v^i(k), \\ u^i(k) \quad = C_h^i \xi^i(k) + D_h^i v^i(k), \end{cases} \tag{3}$$

such that the closed-loop system of (1) and (3) can be written in the following form:

$$\begin{cases} \bar{x}^i(k+1) = A\bar{x}^i(k) + B\left(v^i(k) + d^i(k)\right), \\ y^i(k) \quad = C\bar{x}^i(k), \\ \zeta^i(k) \quad = \sum_{j=1}^N d_{ij}(y^i(k) - y^j(k)), \end{cases} \tag{4}$$

where d^i are generated by

$$\begin{cases} e^i(k+1) = A_s^i e^i(k), \quad i = 1, ..., N, \\ d^i(k) \quad = C_s^i e^i(k), \end{cases} \tag{5}$$

and A_s^i are Schur stable.

Proof: The homogenization is philosophically similar to that in continuous-time case [18], which was built upon earlier work in [27] and [28]. We shall design a squaring-down pre-compensator, a rank-equalizing precompensator, and an observed-based pre-feedback within each agent to yield a network of asymptotically identical agents. Due to the space limitation, we have omitted the proof. ■

Remark 5: We would like to make several observations:

1) The property that the triple (C,A,B) is invertible and has no invariant zero implies that (A,B) is controllable and (C,A) is observable.
2) The triple (C,A,B) is arbitrarily assignable as long as the conditions are satisfied. They play a role as design parameters. We shall use this freedom in various places in order to obtain our results in a more broad range of scenario. For example, from the existing literature, e.g. [22], it is unclear how to achieve synchronization in a set of networks with unknown communication topologies satisfying Assumption 1 but *without* explicit knowledge of δ, that is the upper bound on the eigenvalues of associated row stochastic matrix. However, using the homogenization technique, we can reshape the dynamic in such a special way that the synchronization can still be attainable in the homogenized network. This is done in Section III-C.
3) The triple (A,B,C) always exists and, without loss of generality, takes the following form:

$$A = A_0 + BF, \quad A_0 = \begin{bmatrix} 0 & I_{(n_q-1)p} \\ 0 & 0 \end{bmatrix},$$
$$B = \begin{bmatrix} 0 \\ I_p \end{bmatrix}, \quad C = \begin{bmatrix} I_p & 0 \end{bmatrix}, \tag{6}$$

and F is such that $A_0 + BF$ has desired eigenvalues.

B. Connection to robust stabilization problem

In this subsection, we shall show that the output synchronization in an (asymptotically) homogeneous network (4) and (5) can be solved by equivalently solving a robust stabilization problem.

Suppose the synchronization problem for network (4) and (5) with any communication topology in $\mathscr{G}_\delta$ can be solved by a compensator

$$\begin{cases} \chi^i(k+1) = A_c\chi^i(k) + B_c\zeta^i(k), \\ v^i(k) \quad = C_c\chi^i(k). \end{cases} \tag{7}$$

Let $\tilde{x}^i = [\bar{x}^i; \chi^i]$. Then the closed-loop of each agent can be written as

$$\begin{cases} \tilde{x}^i(k+1) = \begin{bmatrix} A & BC_c \\ 0 & A_c \end{bmatrix} \tilde{x}^i(k) + \begin{bmatrix} 0 \\ B_c \end{bmatrix} \zeta^i(k) + \begin{bmatrix} B \\ 0 \end{bmatrix} d^i(k), \\ y^i(k) \quad = \begin{bmatrix} C & 0 \end{bmatrix} \tilde{x}^i(k), \\ \zeta^i(k) \quad = y^i(k) - \sum_{j=1}^N d_{ij} y^j(k). \end{cases} \tag{8}$$

Define $\tilde{x} = [\tilde{x}^1; \cdots; \tilde{x}^N]$,

$$\bar{A} = \begin{bmatrix} A & BC_c \\ 0 & A_c \end{bmatrix}, \quad \bar{B} = \begin{bmatrix} 0 \\ B_c \end{bmatrix}, \quad \bar{C} = \begin{bmatrix} C & 0 \end{bmatrix} \text{ and } \bar{E} = \begin{bmatrix} B \\ 0 \end{bmatrix}.$$

The overall dynamics of the N agents can be written as

$$\tilde{x}(k+1) = \left[I_N \otimes \bar{A} + (I_N - D) \otimes \bar{B}\bar{C}\right] \tilde{x}(k) + (I_N \otimes \bar{E}) d(k).$$

Define $\eta = [\eta^1; \cdots; \eta^N] = (T \otimes I_n)\tilde{x}$ where $\eta^i \in \mathbb{C}^n$ and T is such that $J_L = T(I_N - D)T^{-1}$ is in the Jordan canonical form and $J_L(1,1) = 0$ where $J_L(1,1)$ denote the $(1;1)-th$ element of J_L. In the new coordinates, the dynamics of η can be written as

$$\eta(k+1) = \left[I_N \otimes \bar{A} + J_L \otimes \bar{B}\bar{C}\right] \eta(k) + (T \otimes \bar{E}) d(k).$$

Lemma 2: The network of the form (8) achieves output synchronization if $\eta^i(k) \to 0$ as $k \to \infty$ for $i = 2, ..., N$.

Remark 6: It also becomes clear from Lemma 2 that the synchronization trajectory is given by $\eta^1(k)$ which is governed by

$$\eta^1(k+1) = A\eta^1(k) + (w \otimes \bar{E})d(k), \quad \eta^1(0) = (w \otimes I_n)\tilde{x}(0),$$

where w is the first row of T, i.e. the left eigenvector associated with eigenvalue 1. Note that $d(k) \to 0$ as $k \to \infty$. This shows that the modes of the synchronization trajectory are determined by the eigenvalues of A and the complete dynamics depends on both A and a weighted average of the agents' initial conditions.

Define $\bar{\eta} = [\eta^2; \cdots; \eta^N]$. Taking the dynamics of d into account, we can write

$$\begin{bmatrix} \bar{\eta}(k+1) \\ e(k+1) \end{bmatrix} = \begin{bmatrix} I_{N-1} \otimes \bar{A} + \bar{J}_L \otimes \bar{B}\bar{C} & (\bar{I}T \otimes \bar{E})\bar{C}_s \\ 0 & \bar{A}_s \end{bmatrix} \begin{bmatrix} \bar{\eta}(k) \\ e(k) \end{bmatrix}, \tag{9}$$

where $e = [e^1; \ldots; e^N]$,

$$\bar{C}_s = \text{blkdiag}\{C_s^i\}_{i=1}^N, \quad \bar{I} = [0, I_{N-1}], \quad \bar{A}_s = \text{blkdiag}\{A_s^i\}_{i=1}^N,$$

and $\bar{J}_L$ is such that $J_L = \begin{bmatrix} 0 & \\ & \bar{J}_L \end{bmatrix}$. Clearly $\bar{\eta} \to 0$ for any initial condition if the system (9) is globally asymptotically stable. Since $\bar{A}_s$ is Schur stable, the next lemma is straightforward:

Lemma 3: The network of the form (8) achieves output synchronization if the system

$$\tilde{\eta}(k+1) = (I_{N-1} \otimes \bar{A} + \bar{J}_L \otimes \bar{B}\bar{C})\tilde{\eta}(k) \tag{10}$$

is globally asymptotically stable.

Due to upper-triangular structure of $I_{N-1} \otimes \bar{A}$ and $(\bar{J}_L \otimes \bar{B}\bar{C})$, the system (10) is essentially a family of $N-1$ subsystems:

$$\tilde{\eta}^i(k+1) = (\bar{A} + (1-\lambda_i)\bar{B}\bar{C})\tilde{\eta}^i(k), \quad i = 2, \ldots, N, \tag{11}$$

where λ_i, $i = 2, \ldots, N$ are those eigenvalues of D that are not equal to 1.

Lemma 4: The network (8) achieves output synchronization if (11) is globally asymptotically stable for λ_i, $i = 2, \ldots, N$.

Note that (11) can be viewed as the closed-loop of

$$\begin{cases} x(k+1) = Ax(k) + Bu(k), \\ z(k) \quad = (1-\lambda)Cx(k), \end{cases} \tag{12}$$

and a compensator

$$\begin{cases} \chi(k+1) = A_c\chi(k) + B_c z(k), \\ u(k) \quad = C_c\chi(k) \end{cases} \tag{13}$$

with unknown λ satisfying $|\lambda| < \delta$. It is easy to see that owing to linearity, (13) stabilizes (12) if it stabilizes

$$\begin{cases} x(k+1) = Ax(k) + (1-\lambda)Bu(k), \\ z(k) \quad = Cx(k). \end{cases} \tag{14}$$

Therefore, we arrive at the following conclusion by the end of this subsection.

Lemma 5: Problem 1 is solved via a composite controller of (3) and (7) if the closed-loop of (14) and (13) is globally asymptotically stable for all $|\lambda_i| < \delta$.

Proof: By establishing Lemma 2-4, we have shown that if the closed-loop of (14) and (13) is globally asymptotically stable for all $|\lambda| < \delta$, then the interconnections of the closed-loop of compensator (7) and (4), which is the network (8), will reach synchronization. This implies that the composite controller of (3) and (7) solves Problem 1. ■

So far, we have converted the output synchronization problem to a simultaneous stabilization problem. Next, depending on different types of synchronization trajectories, the design bifurcates into two approaches.

C. Bounded synchronization trajectories

It has been shown that the eigenvalues of A dictate the modes in the synchronization trajectories. If the trajectories are required to be bounded, we can choose A matrix in Lemma 1 to have only semi-simple eigenvalues on the unit circle. This can be done by choosing proper F matrix in (6). Note that in this case, we can always assume without loss of generality that $A'A = I$. The controller designed based on this type of A matrix can be easily modified by a state transformation so as to be applicable to the agents with a more general form.

Based on the analysis in the preceding subsection, to prove Theorem 1, we need to design data (A_c, B_c, C_c) in the compensator (7) for which the closed-loop of (14) and (13) is globally asymptotically stable with any $|\lambda| < 1$.

We can construct (7) in the following form:

$$\begin{cases} \chi^i(k+1) = (A+KC)\chi^i(k) - K\zeta^i(k), \\ v^i(k) \quad = -\varepsilon B'A\chi^i(k), \end{cases} \tag{15}$$

where K is such that $A + KC$ is Schur stable and $\varepsilon > 0$ is a design parameter to be chosen later. In other words, we choose $A_c = A + KC$, $B_c = -K$ and $C_c = -\varepsilon B'A$. With this set of data, the closed-loop (14) and (13) can be written as

$$\begin{cases} x(k+1) = Ax(k) - (1-\lambda)\varepsilon BB'A\chi(k), \\ \chi(k+1) = (A+KC)\chi(k) - KCx(k). \end{cases} \tag{16}$$

Lemma 6: There exists an $\varepsilon^* > 0$ such that for $\varepsilon \in (0, \varepsilon^*]$, (16) is globally asymptotically stable for $|\lambda| \in (0, 1)$.

D. Unbounded synchronization trajectories

We proceed to consider synchronization trajectories that are possibly unbounded. In most applications, those unbounded synchronization trajectories are normally polynomially increasing. This can be achieved by choosing an A matrix that has all the eigenvalues on the unit circle, some of which may be degenerate. It should be pointed out we can not only assign the eigenvalues of A to arbitrary locations, but we are also able to assign the multiplicity structures of the eigenvalues as long as they are compatible, that is, the summation of all algebraic multiplicities equals to the dimension of A.

Our design is built upon the solution of the following discrete algebraic Riccati equation (DARE) which is also used in [22]:

$$P_\varepsilon = A'P_\varepsilon A + \varepsilon I - (1-\delta^2)A'P_\varepsilon B(B'P_\varepsilon B + I)^{-1}B'P_\varepsilon A. \tag{17}$$

The next lemma can be proved following the work in [29], [30] (see also [22]).

Lemma 7: For any $\varepsilon > 0$ and $\delta \in (0, 1)$, the DARE (17) has a unique positive definite solution P_ε and moreover $A - (1-\lambda)B(B'P_\varepsilon B + I)^{-1}B^*P_\varepsilon A$ is Schur stable for $|\lambda| < \delta$.

The compensator (7) can be designed as follows

$$\begin{cases} \chi^i(k+1) = (A+KC)\chi^i(k) - K\zeta^i(k), \\ v^i(k) \quad = F_\varepsilon\chi^i(k). \end{cases} \tag{18}$$

where K is such that $A + KC$ is Schur stable and

$$F_\varepsilon = -(B'P_\varepsilon B + I)^{-1}B'P_\varepsilon A. \tag{19}$$

In this case, $A_c = A + KC$, $B_c = -K$ and $C_c = F_\varepsilon$. We shall prove that with this set of data, the closed-loop of (14) and (13) is globally asymptotically stable for $|\lambda| < \delta$. The closed-loop system can be written as:

$$\begin{cases} x(k+1) = Ax(k) + (1-\lambda)BF_\varepsilon \chi(k), \\ \chi(k+1) = (A+KC)\chi(k) - KCx(k). \end{cases} \quad (20)$$

Lemma 8: Let $\delta \in (0,1)$ be given. There exists an ε^* such that for $\varepsilon \in (0, \varepsilon^*]$, the system (20) is globally asymptotically stable for $|\lambda| < \delta$.

Remark 7: Notice that the proposed compensator (15) for bounded synchronization trajectories and the compensator (18) for unbounded synchronization trajectories is different from the compensator used in [22] in that we do not require the communication among the compensators.

IV. Illustrative Example

A. Output synchronization

We illustrate our design procedure on a network of four agents. The agents dynamics are of form (1) with

$$A^1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & -1 \\ 0 & -1 & 0 & 0 \end{bmatrix}, \quad B^1 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix},$$

$$C_y^1 = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}, \quad C_z^1 = \begin{bmatrix} 0 & -1 & 0 & 1 \end{bmatrix},$$

$$A^2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \quad B^2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix},$$

$$C_y^2 = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}, \quad C_z^2 = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix},$$

$$A^3 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, B^3 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C_y^3 = \begin{bmatrix} 1 & 0 \end{bmatrix}, C_z^3 = \begin{bmatrix} 1 & 1 \end{bmatrix},$$

$$A^4 = \begin{bmatrix} 0 & 1 \\ -2 & -2 \end{bmatrix}, B^4 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C_y^4 = \begin{bmatrix} 1 & 0 \end{bmatrix}, C_z^4 = \begin{bmatrix} 1 & 1 \end{bmatrix}.$$

The network topology is given by Figure 1. In order to

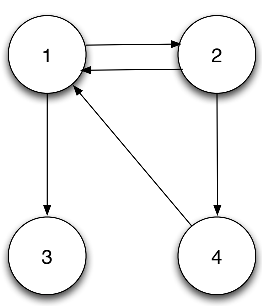


Fig. 1. Network Topologies

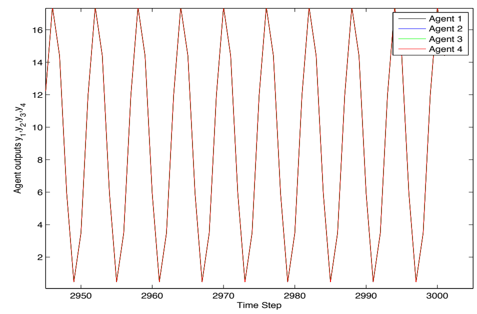
illustrate that our design scheme work for a set of network topologies, we choose two different matrices D as

$$D = \begin{bmatrix} 0.5 & 0.3 & 0 & 0.2 \\ 0.2 & 0.8 & 0 & 0 \\ 0.4 & 0 & 0.6 & 0 \\ 0 & 0.3 & 0 & 0.7 \end{bmatrix}, D = \begin{bmatrix} 0.6 & 0.1 & 0 & 0.3 \\ 0.4 & 0.6 & 0 & 0 \\ 0.5 & 0 & 0.5 & 0 \\ 0 & 0.3 & 0 & 0.7 \end{bmatrix}. \quad (21)$$

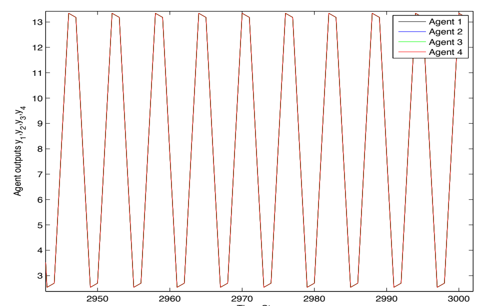
Choose $n_q \geq n_d = 3$. If bounded synchronization trajectories are demanded, we choose matrices A, B, C as below

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -2 & 2 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}.$$

It is easy to see that the above matrices A, B, C satisfy the conditions of Lemma 1. We then $\varepsilon = 0.01$ and $K = \begin{bmatrix} 1.0773 \\ -0.0774 \\ -0.3333 \end{bmatrix}$ for the dynamic low-gain controllers (15). Figure 3(b) and Figure 2(b). shows that the output synchronization is achieved for two networks with two different D given in (21), respectively. If the unbounded synchronization trajectories



(a) Network 1



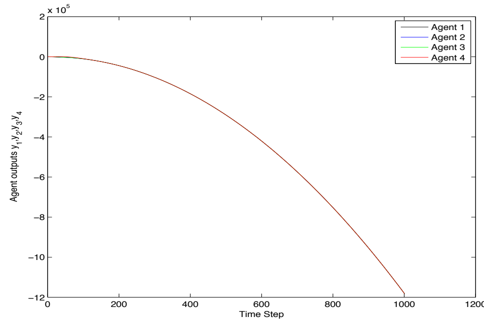
(b) Network 2

Fig. 2. Output Synchronization with a bounded trajectory

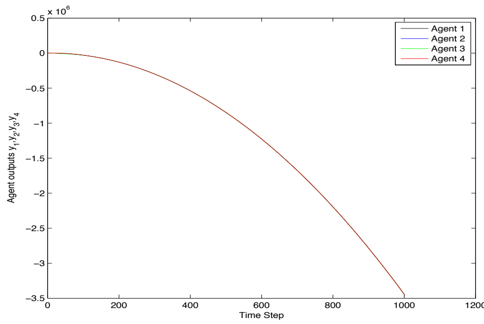
are demanded, we choose matrices A, B, C as below

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -3 & 3 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}.$$

It is easy to see that the above matrices A, B, C satisfy the conditions of Lemma 1. Given $\gamma = 0.7$, we have the resulting set $\mathscr{G}_{0.7}$. It can be easily see that the networks with D given in (21) belong to this set. We then choose $\varepsilon = 10^{-8}$ and $K = \begin{bmatrix} -2.1667 \\ -1.5 \\ -0.3333 \end{bmatrix}$ for the dynamic low-gain controller (18). Figure 3 shows that the output synchronization is achieved for two networks with given matrices D given in (21), respectively.



(a) Network 1



(b) Network 2

Fig. 3. Output Synchronization with an unbounded trajectory

V. CONCLUSION

In this paper, a decentralized control scheme is developed to solve the output synchronization problem in a heterogeneous network of discrete-time introspective right-invertible agents. The essence of the proposed design is two-folds: first, by exploiting the introspection and right-invertibility property of each agent, we design a local shaping pre-compensator to manipulate the agent's internal dynamics as being asymptotically identical to a new model in which we enjoy a substantial freedom in assigning its eigenstructures. Then, different synchronization controllers depending on the two types of synchronization trajectories can be constructed on top of the new model so that the output synchronization can be achieved for a set of communication topologies.

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