

Design of Performance-adaptive PID Control System Using Model Predictive Approach

Takao Sato, Toru Yamamoto, Nozomu Araki and Yasuo Konishi

Abstract—The present paper discusses a new design method for a PID control system using model predictive approach. The PID compensator is designed based on generalized predictive control (GPC). The PID parameters are adaptively updated such that the control performance is improved because the design parameters of GPC are automatically selected to attain a user-specified control performance. In the proposed scheme, estimated plant parameters are updated only when the estimation error increases. Therefore, the control system is not updated frequently. The control system is updated only when the control performance is sufficiently improved. Numerical examples demonstrate the effectiveness of the proposed method.

I. INTRODUCTION

Proportional-integral-derivative control [1], [2], [3], [4], [5], [6] is the most commonly used control method and has been widely used in various fields. However, in order to obtain advanced control performance by PID control, a number of design methods have been investigated.

The present paper discusses a design method of a PID control system based on generalized predictive control (GPC) [7], [8], [9]. In recent decades, most design methods for GPC-based PID control systems focused on how to precisely achieve GPC performance by PID control [10], [11], [12], [13]. In these methods, since GPC performance is decided based on design parameters, the design parameters must be carefully selected. The realization of GPC performance by PID control is important, but performance improvement is more important.

In the design of a generalized minimum variance control (GMVC)-based PID control system, a performance-adaptive control method has been proposed in order to achieve a user-specified control performance [14]. Furthermore, this method has been applied to a weigh feeder control system [15], [16]. In the present study, a performance-adaptive design method is extended to the design of a GPC-based PID control system. In conventional methods [14], [15], [16], one design parameter is adjusted in order to achieve the desired performance. In the design of GPC, there are two design parameters, namely, weighting factor and maximum predictive horizon. These parameters cannot be easily decided because the ranges of these parameters are large. Therefore, in the proposed method, two parameters are automatically decided in order to achieve a user-specified control performance.

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II. PLANT MODEL AND PID CONTROLLER

Most controlled plants are nonlinear, time-variant, and high-order systems. However, it is difficult to obtain precise models. Since the dynamic characteristics of a controlled plant can be locally approximated as a linear model, the present paper discusses a design method based on a linear first-order system with the dead time given as:

$$G(s) = \frac{K}{1 + Ts} e^{-Ls} \quad (1)$$

where K , T and L are the system gain, the time constant and the dead-time, respectively. In order to compensate the dead-time by PID control, the dead-time is replaced with the first-order Pade approximation, and the following second-order model is obtained.

$$G'(s) = \frac{K(1 - \frac{L}{2}s)}{(1 + Ts)(1 + \frac{L}{2}s)} \quad (2)$$

The following discrete-time model corresponding to the second-order model is obtained.

$$\begin{aligned} A(z^{-1})y(k) &= B(z^{-1})u(k-1) + \frac{\chi(k)}{\Delta} \quad (3) \\ A(z^{-1}) &= 1 + a_1z^{-1} + a_2z^{-2} \\ B(z^{-1}) &= b_0 + b_1z^{-1} \\ \Delta &= 1 - z^{-1} \end{aligned}$$

where $y(k)$, $u(k)$ and $\chi(k)$ are the plant output, the control input and the modeling error and/or integrated white noise disturbances driven by Gaussian white noise, respectively. z^{-1} denotes the backward shift operator which means $z^{-1}y(k) = y(k-1)$. (3) is the so-called CARIMA (Controlled Auto-Regressive and Integrated Moving Average) model, and has been widely used in the process industry.

Based on (3), a control system is designed using the following PID control law given as:

$$\Delta u(k) = C(1)r(k) - C(z^{-1})y(k) \quad (4)$$

$$C(z^{-1}) = k_c \left(\Delta + \frac{T_s}{T_I} + \frac{T_D}{T_s} \Delta^2 \right) \quad (5)$$

where $r(k)$ is the reference input, T_s denotes the sampling time, and k_c , T_I , and T_D are the proportional gain, integral time, and derivative time, respectively.

In the present study, the PID parameters are designed based on GPC. To this end, the plant parameters are estimated using a recursive least squares method, and the design parameters of GPC are automatically updated using

estimated plant parameters such that the user-desired performance is attained. Consequently, a performance-adaptive GPC-based PID controller is obtained.

III. GPC-BASED PID CONTROLLER

A. GPC[7]

The design objective of GPC is to minimize the following cost function:

$$J = E \left[\sum_{j=N_1}^{N_2} \{y(k+j) - r(k)\}^2 + \sum_{j=1}^{N_u} \lambda \{\Delta u(k+j-1)\}^2 \right] \quad (6)$$

where N_1 , N_2 , N_u , and λ are the minimum predictive horizon, the maximum predictive horizon, the control horizon, and the weighting factor for the difference in the control input, respectively. In the original GPC method, a future reference trajectory is used for a smooth approach. However, a future reference trajectory is not used in the present study, and the reference input is assumed to be constant in order to obtain a GPC-based PID controller.

The GPC law (7) is obtained using the following equations.

$$\begin{aligned} G(z^{-1})\Delta u(k) &= Pr(k) - F(z^{-1})y(k) \quad (7) \\ P &= p_{N_2} + p_{N_2-1} + \dots + p_{N_1} \\ [p_{N_1} \ \dots \ p_{N_2}] &= [1 \ \underbrace{0 \ \dots \ 0}_{N_u-1} (R^T R + \Lambda)^{-1} R^T \\ G(z^{-1}) &= 1 + z^{-1}S(z^{-1}) \\ S(z^{-1}) &= p_{N_1}S_{N_1}(z^{-1}) + \dots + p_{N_2}S_{N_2}(z^{-1}) \\ F(z^{-1}) &= p_{N_1}F_{N_1}(z^{-1}) + \dots + p_{N_2}F_{N_2}(z^{-1}) \end{aligned}$$

where

$$\begin{aligned} 1 &= \Delta A(z^{-1})E_j(z^{-1}) + z^{-j}F_j(z^{-1}) \quad (8) \\ E_j(z^{-1}) &= e_0 + e_1z^{-1} + \dots + e_{j-1}z^{-(j-1)} \\ F_j(z^{-1}) &= f_{j0} + f_{j1}z^{-1} + f_{j2}z^{-2} \\ E_j(z^{-1})B(z^{-1}) &= R_j(z^{-1}) + z^{-j}S_j(z^{-1}) \quad (9) \\ R_j(z^{-1}) &= r_0 + r_1z^{-1} + \dots + r_{j-1}z^{-(j-1)} \\ S_j(z^{-1}) &= s_{j0} + s_{j1}z^{-1} + \dots + s_{jm-1}z^{-(m-1)} \\ R &= \begin{bmatrix} r_{N_1-1} & \dots & r_0 & & \mathbf{0} \\ \vdots & & & \ddots & \\ r_{N_u-1} & & & & r_0 \\ \vdots & & & & \vdots \\ r_{N_2-1} & \dots & \dots & \dots & r_{N_2-N_u} \end{bmatrix} \\ \Lambda &= \begin{bmatrix} \lambda & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \lambda \end{bmatrix} \end{aligned}$$

B. Design of PID Parameters Based on GPC Law

In order to approximate the PID controller using the derived GPC law, polynomial $G(z^{-1})$ is replaced with its steady-state gain, and the GPC law is rewritten as follows:

$$\Delta u(k) = \frac{P}{\nu}r(k) - \frac{F(z^{-1})}{\nu}y(k) \quad (10)$$

$$\nu = G(1) \quad (11)$$

Since $P = F(1)$, comparison of (4) with (10) yields the following relations:

$$C(z^{-1}) = \frac{F(z^{-1})}{\nu} \quad (12)$$

From the above equation, the PID parameters are calculated as follows:

$$k_c = -\frac{f_1 + 2f_2}{\nu} \quad (13)$$

$$T_I = -\frac{f_1 + 2f_2}{f_0 + f_1 + f_2}T_s \quad (14)$$

$$T_D = -\frac{f_2}{f_1 + 2f_2}T_s \quad (15)$$

where

$$F(z^{-1}) = f_0 + f_1z^{-1} + f_2z^{-2}$$

IV. PERFORMANCE-ADAPTIVE PID CONTROLLER

A. Decision of Design Parameters

The minimum predictive horizon N_1 is set to the estimated minimum dead-time, or to 1 if the plant has no dead time. Control horizon N_u is simply set to 1. λ and N_2 are designed as follows:

- 1) The acceptable maximum value of the control error is set first.
- 2) By changing λ and N_2 , the minimum value of the variance of the difference in the control input is searched under the above condition.

The image of the trade-off curves is shown in Fig. 1. In this figure, the variances of the control error and the difference in the control input are plotted by changing λ and N_2 .

B. Update of Estimated Plant Parameters

To enhance the reliability of model identification, the plant parameters are updated based on the following methods:

- The plant parameters are estimated using a recursive least squares method using N steps of input-output data only when the absolute value of the estimation error exceeds the pre-specified value.
- As few as possible parameters are estimated.

A discrete-time model corresponding to continuous-time model (1) is considered.

$$\begin{aligned} \alpha(z^{-1})y(k) &= z^{-(d+1)}\beta(z^{-1})u(k) + \frac{\xi(k)}{\Delta} \quad (16) \\ \alpha(z^{-1}) &= 1 + \alpha_1z^{-1} \\ \beta(z^{-1}) &= \beta_0 + \beta_1z^{-1} \end{aligned}$$

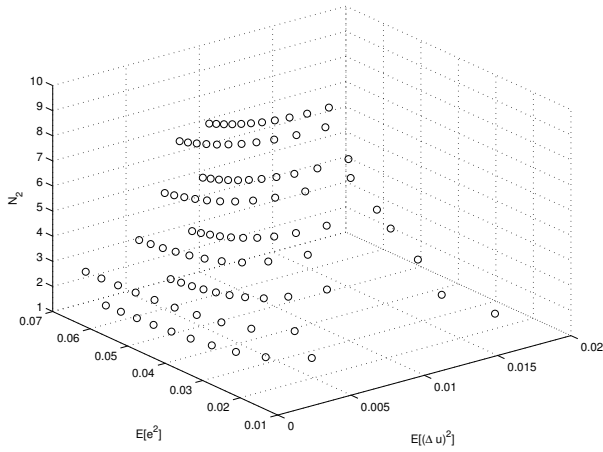


Fig. 1. Image of trade-off curves

where $\xi(k)$ corresponds to the modeling error given by Gaussian white noise and the variance σ_ξ^2 , and d denotes the dead-time. If it is difficult to obtain the accurate dead-time, d is changed within envisaged range $\{d_{\min}, d_{\min}+1, \dots, d_{\max}-1, d_{\max}\}$, and d which minimizes the estimation error is decided as the estimated dead-time \hat{d} . Plant parameters α_1 , β_0 and β_1 in (16) are estimated using the following recursive least squares method.

$$\hat{\theta}(j) = \hat{\theta}(j-1) + \frac{\Gamma(j-1)\psi(j-1)}{1 + \psi^T(j-1)\Gamma(j-1)\psi(j-1)}\varepsilon(j) \quad (17)$$

$$\Gamma(j) = \Gamma(j-1) - \frac{\Gamma(j-1)\psi(j-1)\psi^T(j-1)\Gamma(j-1)}{1 + \psi^T(j-1)\Gamma(j-1)\psi(j-1)} \quad (18)$$

$$\varepsilon(j) = \Delta y(j) - \hat{\theta}^T(j-1)\psi(j-1)$$

$$\hat{\theta}(j) = [\hat{\alpha}_1(j) \ \hat{\beta}_0(j) \ \hat{\beta}_1(j)]^T$$

$$\psi(j-1) = [-\Delta y(j-1) \ \Delta u(j-d-1) \ \Delta u(j-d-2)]^T$$

$$j = k - N + 1, k - N + 2, \dots, k - 1, k$$

$$d = d_{\min}, d_{\min} + 1, \dots, d_{\max}$$

The parameters in (1) are estimated as follows [14], [17].

$$\hat{T}(t) = -\frac{T_s}{\log(-\hat{\alpha}_1(t))} \quad (19)$$

$$\hat{K}(t) = \frac{\hat{\beta}_0(t) + \hat{\beta}_1(t)}{1 + \hat{\alpha}_1(t)} \quad (20)$$

$$\hat{L}(t) = \left(\frac{\hat{\beta}_1(t)}{\hat{\beta}_0(t) + \hat{\beta}_1(t)} + \hat{d}(t) \right) T_s \quad (21)$$

In order to prevent frequent parameter updating, parameter estimation (17) is executed only when the modeling-assessment index (22) is satisfied. On the other hand, the plant parameters are not updated if this index is not satisfied.

$$|\eta(k)| \geq \gamma\sigma_\varepsilon \quad (22)$$

$$\eta(k) = \Delta y(k) - \hat{\theta}^T(k)\psi(k-1) \quad (23)$$

where σ_ε is the standard deviation of estimation error $\varepsilon(k)$, and γ is a parameter corresponding to the standard value of the modeling error. It is assumed that the prediction error approximately depends on the Gaussian distribution, and γ is set between 3.0 and 5.0 from a statistical point of view [14].

From the designed PID control law and the plant model (16), a closed-loop system is calculated as follows:

$$y(k) = \frac{z^{-(d+1)}\beta(z^{-1})C(z^{-1})}{T(z^{-1})}r(k) + \frac{1}{T(z^{-1})}\xi(k) \quad (24)$$

$$T(z^{-1}) = \Delta\alpha(z^{-1}) + z^{-(k+1)}\beta(z^{-1})C(z^{-1}) \quad (25)$$

In the steady state, when the compensator is designed such that the plant output converges to its reference input, the following relations are obtained:

$$e(k) = -\frac{1}{T(z^{-1})}\xi(k) \quad (26)$$

$$\Delta u(k) = -\frac{C(z^{-1})}{T(z^{-1})}\xi(k) \quad (27)$$

In Section IV-C, the variances of (26) and (27) are evaluated, and λ and N_2 are decided.

C. Algorithm

The proposed controller is updated as follows:

- 1) The minimum and maximum values of the search ranges of λ and N_2 are given, and N for parameter estimation is selected.
- 2) The plant parameters are estimated using (17) and (18), and the standard deviation σ_ε of estimation error $\varepsilon(k)$ is calculated.
- 3) By changing λ and N_2 , variances $E[e^2(k)]$ and $E[(\Delta u(k))^2]$ are calculated using the H_2 norm.

$$E[e^2(k)] = \left\| -\frac{1}{T(z^{-1})} \right\|_2^2 \sigma_\xi^2 \quad (28)$$

$$E[(\Delta u(k))^2] = \left\| -\frac{C(z^{-1})}{T(z^{-1})} \right\|_2^2 \sigma_\xi^2 \quad (29)$$

where σ_ε is used instead of σ_ξ because σ_ξ is unknown.

- 4) λ and N_2 are decided such that the variance of the control error is equal to or less than the acceptable maximum value and the variance of the difference in the control input is minimized. Using the decided values of λ and N_2 , the PID parameters are updated using (13) ~ (15).
- 5) $k = k + 1$
- 6) Based on the estimated parameters calculated in Step 2, $\eta(k)$ is calculated. If (22) is satisfied, return to Step 2. Otherwise, return to Step (5).

V. NUMERICAL EXAMPLE

Consider the following model:

$$G(s) = \frac{1}{20s + 1}e^{-10s} \quad (30)$$

where the sampling interval is 2[s]. This system is disturbed by Gaussian white noise with variance 10^{-3} . In this simulation, the control parameters were fixed from the start to 200 steps, where $k_c = 1$, $T_I = 6$ and $T_D = 0$, and the proposed method was applied after that. In the proposed method, the design parameters are set to be $N_1 = 6$ and $N_u = 1$, and the research ranges of λ and N_2 are $[0, 20]$ and $[6, 20]$, respectively. Moreover, λ and N_2 are decided such that the variance of the control error is not more than 2.0×10^{-1} .

The simulation results are shown in Fig. 2 ~ Fig. 9. Based on the output results shown in Fig. 2, the vibration width of the plant output can be suppressed after step 200. Fig. 3 shows that the improvement in the deviation of the control input is slight in order to suppress the variance of the control error from step 200 to step 765. However, after step 766, the deviation of the control input is substantially reduced although the control input does not deteriorated (Fig. 4).

The identified parameters of $\alpha(z^{-1})$ and $\beta(z^{-1})$ are plotted in Fig. 5 and Fig. 6. Before step 200, α_1 , β_0 and β_1 are 0 because the parameters are estimated after step 200.

The proportional gain, the integral time and the derivative time at each step are shown in Fig. 7, and all the values change at step 200 and step 766 in order to improve the control performance. The selected values of the PID parameters at step 200 are 1.01, 8.52 and 2.29, respectively, and their values at step 766 are updated to be 0.249, 5.48 and 1.61, respectively.

Fig. 8 and Fig. 9 show the trade-off curves at step 200 and step 766, respectively. Based on Fig. 8, N_2 and λ were selected to be 13 and 13, respectively, and at step 766, N_2 and λ were changed to be 6 and 9, respectively.

VI. CONCLUSION

In the present study, a new design method for a PID control system was proposed. The proposed PID controller is based on generalized predictive control (GPC), and the design parameters of GPC are decided such that the user-desired control performance is attained. In the conventional performance-adaptive design method, the number of variable design parameters is 1. On the other hand, since maximum predictive horizon N_2 and weighting factor λ are varied, the achievable performance can be broadened.

VII. ACKNOWLEDGMENTS

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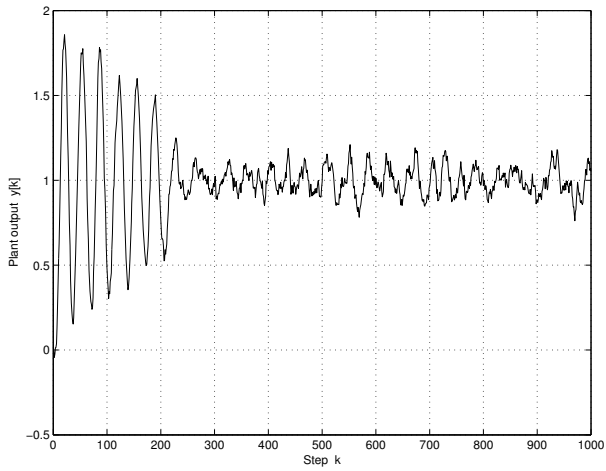


Fig. 2. Plant output

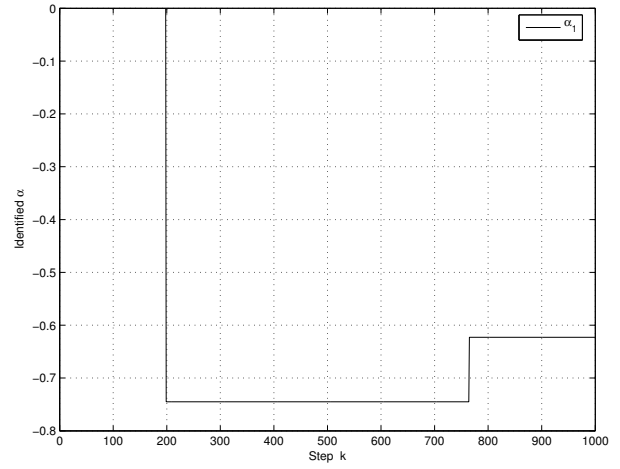


Fig. 5. Identified $\alpha[z^{-1}]$

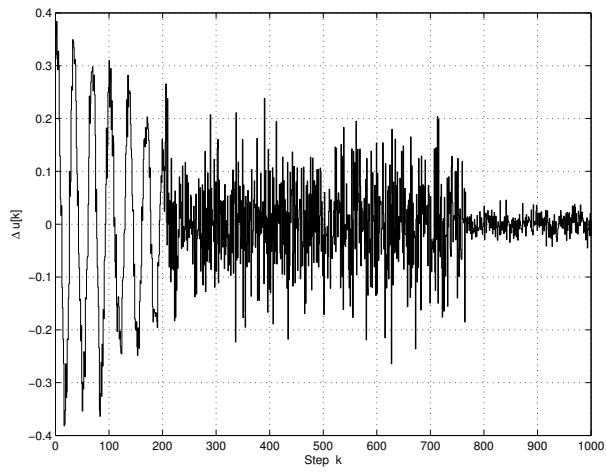


Fig. 3. Difference in the control input

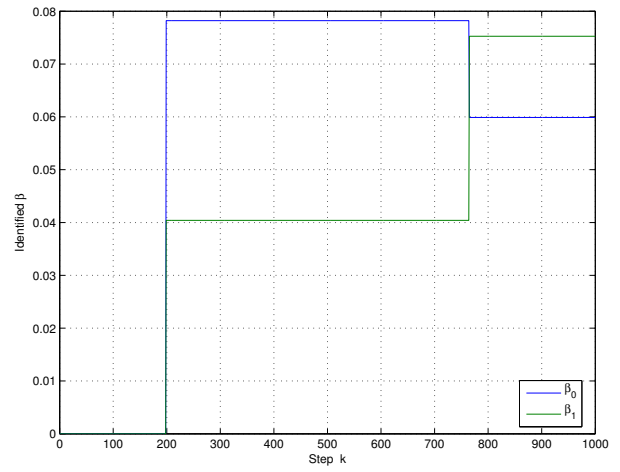


Fig. 6. Identified $\beta[z^{-1}]$

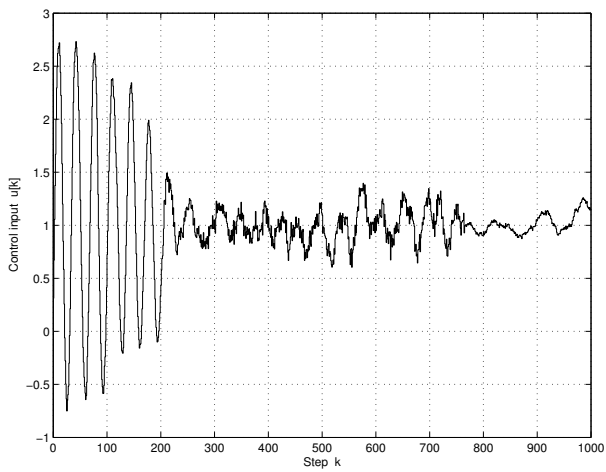


Fig. 4. Control input

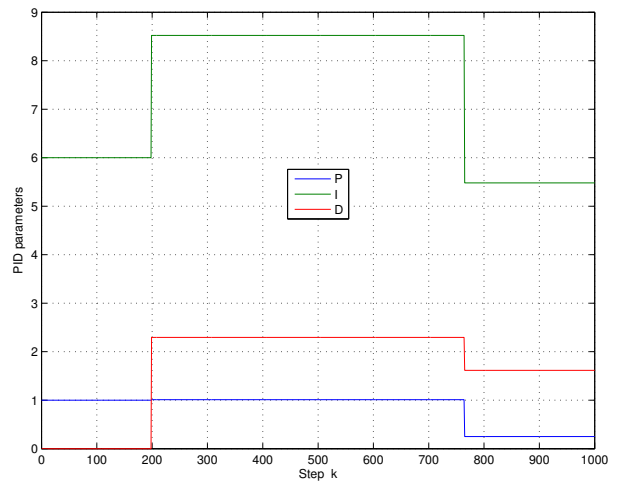


Fig. 7. PID parameters

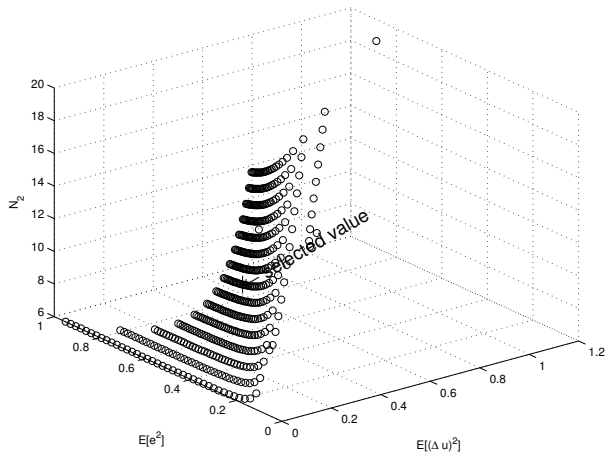


Fig. 8. Trade-off curves at step 200

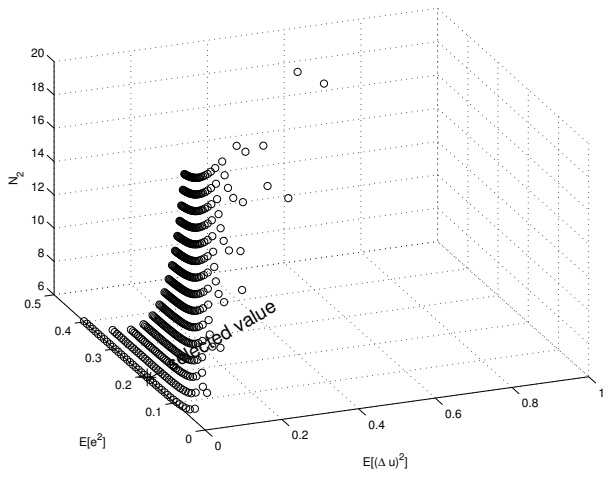


Fig. 9. Trade-off curves at step 766