

Model Predictive Control of semi-active and active suspension systems with available road preview

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Abstract—Semi-active and active suspensions influencing the vertical dynamics of vehicles enable improvement of ride comfort and handling characteristics compared to vehicles with passive suspensions. If the road height profile in front of the car is measured using vehicle sensors, more efficient control strategies can be applied. First, regarding vehicles with continuously variable dampers: A Model Predictive Controller incorporating the nonlinear constraints of the damper characteristic is set up and compared to a controller using linear constraints. The approximate linear constraints are obtained by a prediction of passive vehicle behavior over the preview horizon using a linearized model. Additionally, a controller without input constraints and clipping of the damper force is applied. This results in a quadratic program without constraints, which can be solved efficiently. The result of applying the optimal force without constraints is also evaluated, which corresponds to an ideal high bandwidth actuator. Next, two controllers for vehicles with a low bandwidth active suspension and variable dampers are proposed. While the first approach optimizes only the actuator displacement combined with the damper's soft characteristic, the second approach optimizes both the damper force and the actuator displacement. Simulation results of the controllers and the active and semi-active suspensions over real road height profiles are presented.

I. INTRODUCTION

A. Motivation

An important goal in vehicle development is constantly improving ride comfort and handling characteristics. Active and semi-active suspensions between the wheels and the vehicle body allow for a reduction in vehicle body acceleration and, hence, improved ride comfort. Ride safety and enhanced handling characteristics can be quantified by reduced dynamic wheel load fluctuations. Vehicles with semi-active suspensions consisting of continuously variable dampers are available on the market. Vehicles with low bandwidth active suspensions up to about 5 Hz combined with variable dampers are also considered in this paper. Current developments in the automotive industry are often based on recognition of the environment using sensors. Several advanced driver assistance systems, which are scanning the surroundings of the vehicle, are already available in modern cars. These sensors can also be used to measure the road height profile in front of the car, as illustrated in Fig. 1. With this additional information, more efficient controllers for semi-active and active suspensions are feasible. In this



Fig. 1. Vehicle with active suspension and preview information

paper the road height profile in front of the car is assumed to be known and possible controllers are discussed.

B. Preview suspension controllers in the literature

The common approach for preview active suspension controllers is the optimal control approach based on a quarter-car or half-car model (e.g. [1],[2],[3], [4], [5], [6], [7], [8], [9]). This results in a Linear-Quadratic Regulator (LQR) as state feedback and a preview feedforward term. In [10] a semi-active suspension with preview is considered and an approach using soft constraints in the cost function is compared to an approach with hard constraints in the optimization formulation. In contrast to [10], in this paper constraints on damper force are represented by a nonlinear characteristic dependent on the relative velocity of the damper.

A LQR cannot explicitly incorporate constraints. Since actuator displacement and actuator displacement rate of an active suspension system and the damper force of variable dampers are limited, Model Predictive Control is applied in this paper. In [11], [12] and [13] an MPC for preview active suspension systems based on a quarter-car model and a half-car model is applied. A Model Predictive Controller for low bandwidth active suspension systems based on a full-car model was already proposed by the authors in [14].

In [15] Hybrid MPC for semi-active suspension without road preview is proposed and a quarter car model is used; [16] applies an LQR in a vehicle with semi-active suspension using the clipped-optimal approach; [17] evaluates the potential of a low bandwidth active suspension system with variable dampers using a LQR.

C. New approach

Preview controllers for semi-active suspensions taking into account nonlinear damper characteristics and preview controllers for low bandwidth active suspension systems combined with variable dampers have not yet been considered in the literature. Damper force of a continuously

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variable damper is constrained by a nonlinear characteristic which is dependent on the relative damper velocity. A Model Predictive Controller, incorporating the nonlinear constraints, is proposed in this paper as a reference. Furthermore, two approaches using linear Model Predictive Control are proposed in order to carry out the optimization with less computational effort. The first approach simulates the passive vehicle over the given preview road height profile using linearized damper characteristics. Hence, at each time step an approximation for the relative damper velocities over the preview horizon is obtained. This approximation is used to calculate constraints for the damper force over the preview horizon. A third controller is proposed, which calculates an optimal damper force without consideration of constraints and the resulting force is clipped to the limits of the damper characteristic. This approach is well known for semi-active suspensions without preview using a LQR. The resulting quadratic program without constraints can be solved efficiently.

Furthermore, preview model predictive controllers for vehicles with low bandwidth active suspensions and variable dampers are proposed. The first controller optimizes the active suspension combined with the passive soft mode of the damper. The second controller optimizes both the actuator and the damper force. Therefore, the passive vehicle is simulated over the preview horizon and constraints for the damper forces over the preview horizon are obtained. Additionally, constraints on actuator displacement and displacement rate of the active suspension are incorporated.

II. VEHICLE MODEL

A full-car model of vertical dynamics with seven degrees of freedom is used as controller model, shown in Fig. 2. There are three degrees of freedom of the vehicle body, the heave (z_b), pitch (φ) and roll (θ) mode. The vertical displacements of the wheels are denoted $z_{w,ii}$, where throughout the paper $ii \in \{fl, fr, rl, rr\}$ stands for front/rear left/right. Furthermore, vectors denote the variable on each wheel written in a column, e.g. $\underline{f} = (f_{fl} \ f_{fr} \ f_{rl} \ f_{rr})^T$. The tire is modeled as a spring and $z_{r,ii}$ denotes the vertical road disturbance under every tire. One strut is depicted in detail, but the same model is used for the others. The strut consists of a variable damper with damper force f_{ii} and an actuator in series to the primary spring. Low bandwidth actuators with a frequency up to about 5 Hz are considered. The actuator is described by its displacement s_{ii} . To support the vehicle body in a steady-state without losing energy, a spring parallel to the actuator is necessary, as depicted.

The derivation of the equations of motion was already presented in [14]. This leads to a state space formulation with fourteen states:

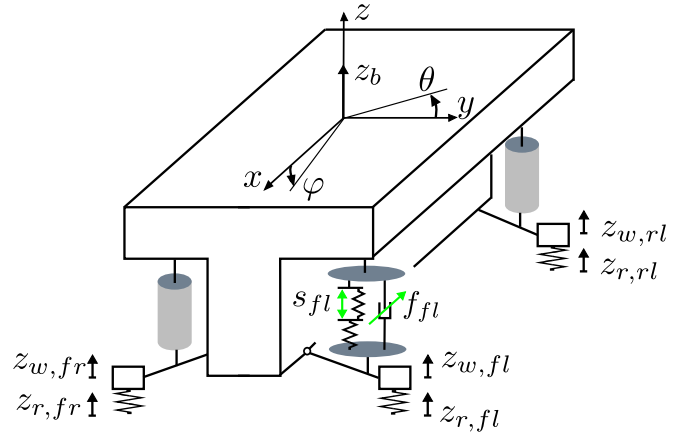


Fig. 2. Vehicle model

$$\begin{aligned} \dot{\underline{x}}(t) &= \mathbf{A}\underline{x}(t) + \mathbf{B}_u\underline{u}(t) + \mathbf{B}_w\underline{w}(t) \\ \underline{y}(t) &= \mathbf{C}\underline{x}(t) + \mathbf{D}_u\underline{u}(t) + \mathbf{D}_w\underline{w}(t) \\ \underline{x} &= (z_b \ \varphi \ \theta \ z_w \ \dot{z}_b \ \dot{\varphi} \ \dot{\theta} \ \dot{z}_w)^T \\ \underline{u} &= (s \ \underline{f})^T \\ \underline{w} &= \underline{z}_r \end{aligned} \quad (1)$$

The controlled inputs of the system are damper force and actuator displacement. Road unevenness is a known disturbance to the system. The output for the controller design consists of the three vehicle body accelerations $\underline{y} = (\ddot{z}_b \ \ddot{\varphi} \ \ddot{\theta})^T$.

III. SEMI-ACTIVE SUSPENSIONS

In this section vehicles with semi-active suspensions will be considered. Damper force of variable dampers can be adjusted according to a characteristic dependent on relative damper velocity, as shown in Fig. 3. Positive relative damper velocity corresponds to the rebound characteristic of the damper. Since semi-active suspensions are considered in this section, the actuator displacements s in (1) are set to zero and spring constants are chosen appropriately for a vehicle with a semi-active suspension.

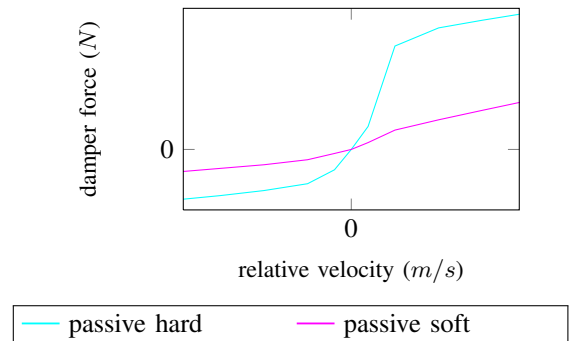


Fig. 3. Constraints on damper force of a variable damper

A. Model Predictive Controller for semi-active suspensions

A zero-order hold discretization of the vehicle model (1) is conducted. The output $\underline{y} = (\ddot{z}_b \quad \dot{\varphi} \quad \ddot{\theta})^T$ can be predicted as a function of the initial state $\underline{x}[k]$, the initial control variable $\underline{u}[k]$, the road height information over the prediction horizon \hat{w} , and the control variables over the prediction horizon \hat{u} :

$$\hat{y} = \hat{\Phi}\underline{x}[k] + \hat{\Gamma}_{u,k}\underline{u}[k] + \hat{\Gamma}_u\hat{u} + \hat{\Gamma}_w\hat{w} \quad (2)$$

$$\hat{y} = \begin{pmatrix} \underline{y}[k+1] \\ \vdots \\ \underline{y}[k+p] \end{pmatrix} \quad \hat{u} = \begin{pmatrix} \underline{u}[k+1] \\ \vdots \\ \underline{u}[k+p] \end{pmatrix} \quad \hat{w} = \begin{pmatrix} \underline{w}[k] \\ \vdots \\ \underline{w}[k+p] \end{pmatrix}$$

The matrices $\hat{\Phi}$, $\hat{\Gamma}_{u,k}$, $\hat{\Gamma}_u$ and $\hat{\Gamma}_w$ can be calculated from the state space matrices from (1), p denotes the prediction horizon. This output over the prediction horizon is inserted in a cost function:

$$\min_{\hat{u}} (\hat{y}^T \mathbf{Q} \hat{y} + \hat{u}^T \mathbf{R} \hat{u}) \quad (3)$$

Omitting the terms not dependent on the optimization variable leads to the following expression:

$$\min_{\hat{u}} \frac{1}{2} \hat{u}^T (\hat{\Gamma}_u^T \mathbf{Q} \hat{\Gamma}_u + \mathbf{R}) \hat{u} + (\underline{x}[k]^T \hat{\Phi}^T \mathbf{Q} \hat{\Gamma}_u + \underline{u}[k]^T \hat{\Gamma}_{u,k}^T \mathbf{Q} \hat{\Gamma}_u + \hat{w}^T \hat{\Gamma}_w^T \mathbf{Q} \hat{\Gamma}_u) \hat{u} \quad (4)$$

This is equivalent to a quadratic program:

$$\min_{\hat{u}} \frac{1}{2} \hat{u}^T \mathbf{H} \hat{u} + g^T \hat{u} \quad (5)$$

s.t. $\hat{u} \in \mathcal{U}$

This optimization calculates optimal damper forces over a preview horizon at each time step. Using an inverse damper characteristic, a corresponding current for the desired damper force is calculated and the output current for the variable damper is obtained.

The input constraints on damper force are a nonlinear function of vehicle states as shown in Fig. 3. Three different approaches to consider the constraints are proposed:

- 1) The nonlinear constraints on damper force are incorporated. Therefore, relative damper velocities are predicted as a function of the initial values, road profile over the preview horizon, and damper force, which is the optimization variable. This expression is equivalent to (2) with the relative velocities as output \underline{y} . Using the measured, nonlinear damper characteristic, represented by a polygonal line (Fig. 3), constraints on the optimization variable are obtained. This results in a linear quadratic optimization problem (5) with nonlinear constraints $c(\hat{u}) < 0$. The optimization is solved at each time step with the Matlab function `fmincon`. This nonlinear model predictive control is carried out as reference solution.

- 2) In order to obtain a linear optimization with less computational effort, it is proposed to predict the relative damper velocities \underline{v}_d of a passive vehicle. Therefore, a constant linear damper force is chosen appropriately and an approximation for relative damper velocities is predicted via (2) with $\underline{y} = \underline{v}_d$. Constraints for the damper force over the prediction horizon can be calculated using the damper characteristic. A linear optimization problem (5) with linear constraints $\hat{u}_{min} \leq \hat{u} \leq \hat{u}_{max}$ is obtained. If the calculated damper force is outside the characteristic, a maximum or minimum damper force for the given relative velocity is applied. This is known in the literature as the clipped-optimal approach for semi-active suspensions.
- 3) The third approach is to calculate optimal control variables without input constraints. If the obtained damper force is outside the feasible region, the force is clipped and maximum or minimum damper force is applied. Without constraints, the solution of the optimization (5) is $\mathbf{H}\hat{u} = -g$. Since the Hessian Matrix is constant in the present formulation, an inversion can be conducted offline. Hence, the optimal control variables can also be calculated via a matrix multiplication $\hat{u} = -\mathbf{H}^{-1}g$.

B. Road height profile

The following simulations are performed over the real road height profile of a rough road, see Fig. 4. The road height profile under the left and the right wheel is the same and delayed for the rear wheels. The vehicle is driving at a constant speed of 60 km/h.

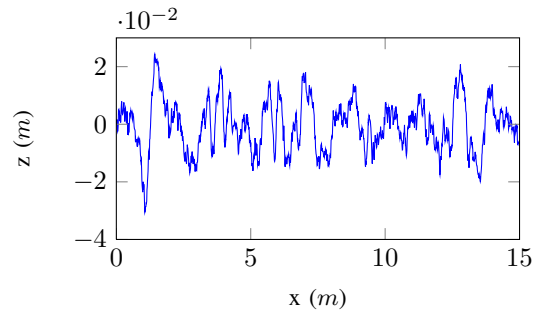


Fig. 4. Real road profile for simulations

C. Simulation results with plant model equals controller model

Three model-based controllers were proposed. One controller incorporates the nonlinear damper characteristic. The second controller is predicting passive vehicle behavior and obtaining approximated constraints; afterwards the force is clipped if outside the feasible region. The third controller is calculating an optimal force without input constraints and clipping the obtained force to the feasible region. To compare the controllers and to avoid effects of the simplified controller model, first simulation results with the controller

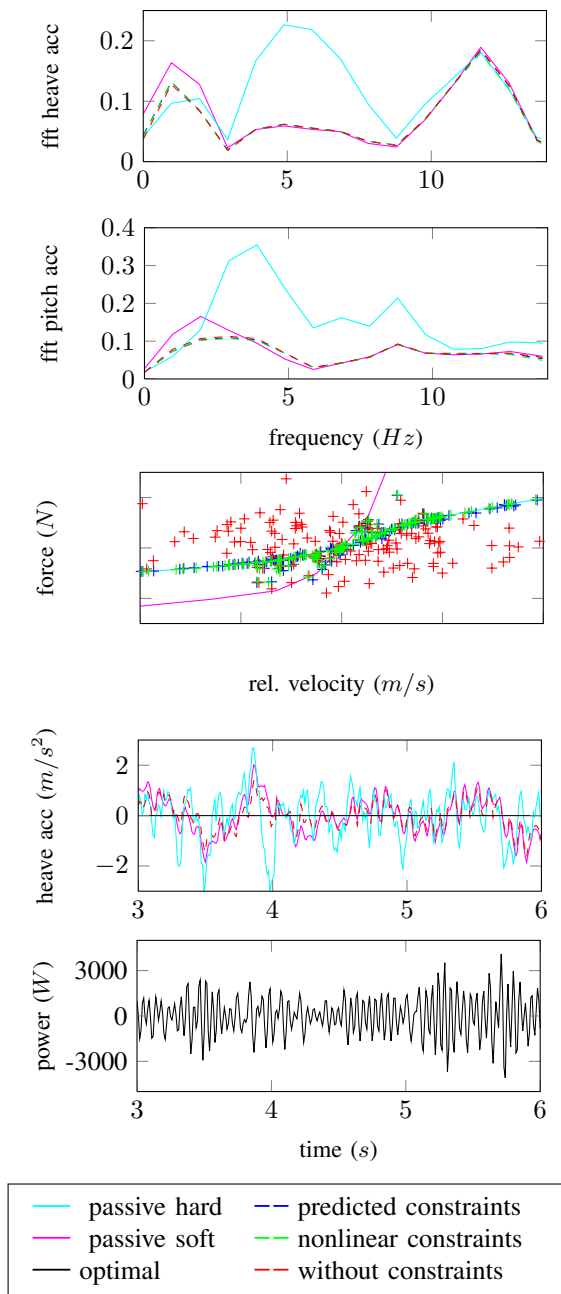


Fig. 5. Simulation results with vehicle model equals controller model

model (2) as the simulation model are presented. The first two diagrams of Fig. 5 indicate a reduced heave and pitch acceleration using the proposed controllers in comparison with the passive soft or passive hard characteristic of the uncontrolled damper. It can be observed that the performance of the three controllers is nearly the same. The third diagram depicts the calculated force of the three controllers for the front left actuator, before the force is clipped. The controller incorporating the exact nonlinear constraints calculates forces within the feasible region of the damper and no clipping is necessary, as shown. The second controller calculates forces, which are sometimes slightly outside the feasible range, due to the linearization. The third controller

without constraints calculates forces in the whole region. As shown in the first two diagrams, the clipping of these forces give results which mirror an exact incorporation of the constraints. The fourth diagram shows the heave acceleration as a function of time. Heave acceleration can be eliminated if the optimal force without constraints is applied and controller model equals simulation model. Therefore, a high bandwidth actuator between wheel and vehicle body is necessary.

In the fifth diagram, a simplified calculation of the power consumption of the system is depicted. The power is calculated as force multiplied by relative velocity considering all four actuators, $p = \sum f_{ii} \cdot v_{d,ii}$. This is a simplified calculation, since power loss and inertia of the actuators is not considered. The mean values of the curves are 4 W for the optimal control and about -250 W for the passive hard, passive soft and the controlled damper. This shows that a high bandwidth actuator, which can apply the optimal control signal, ideally consumes no energy, since the applied power is recovered. Furthermore the power of passive dampers is always negative, since energy is dissipated.

D. Simulation results with a validated vehicle model

In this section, the same simulation is performed with a validated vehicle model, instead of the simplified controller model as plant.

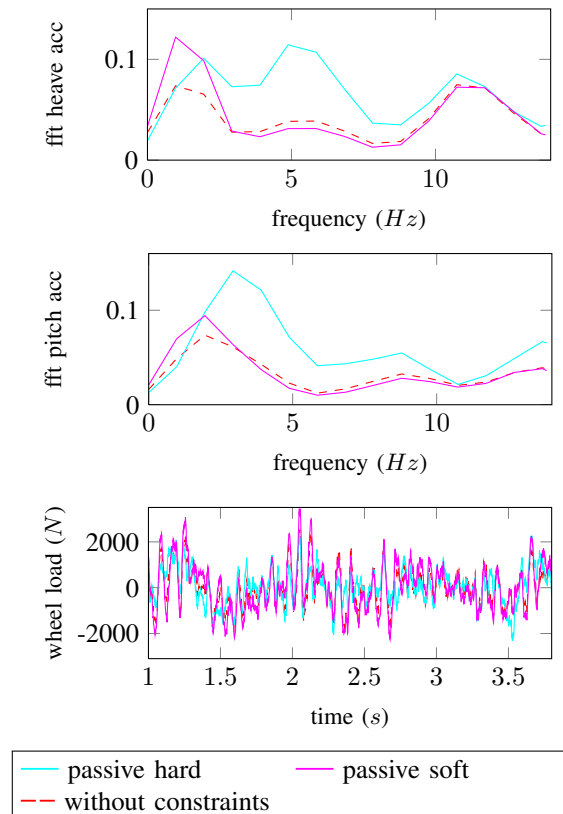


Fig. 6. Simulation results with validated vehicle model

The results are shown in Fig. 6. The third controller is compared to the passive soft and passive hard mode of the dampers. The controller reduces heave and pitch acceleration

in comparison to passive dampers, as shown, and hence ride comfort is improved. Ride safety and handling characteristics were evaluated and therefore dynamic wheel load fluctuations are depicted in the third diagram. The passive soft damper characteristic results in the highest dynamic wheel load fluctuation and hence a reduced road holding. This can influence ride safety during a turn. The proposed controller results in slightly higher peaks of dynamic wheel load fluctuation than when in the passive hard damper mode. Therefore, the controller can be adapted while turning.

IV. ACTIVE SUSPENSION WITH VARIABLE DAMPER

In this section a vehicle with a low bandwidth actuator, in series with the spring, combined with a continuously variable damper, as depicted in Fig. 2, is considered. Therefore, appropriate spring constants for a vehicle with an active suspension are used. The constraints of the variable dampers are shown in Fig. 3. The constraints of the active suspension consist of a limited actuator displacement and a limited actuator displacement rate.

One possibility is to calculate an optimal force for each wheel and to divide this force into the two controlled elements, the variable damper and the actuator. However, it is difficult to incorporate the constraints of the damper and the actuator at the same time, thus they are considered as separate optimization variables.

A. Proposed controllers

Since low bandwidth actuators up to about 5 Hz are considered, but the eigenfrequency of the wheels is about 12 Hz, the actuator cannot influence wheel dynamics. Therefore, wheel dynamics are eliminated in the model (1) by introducing a steady state tire deflection. This was already shown in [14] and results in the following state space formulation of the reduced model:

$$\begin{aligned}\dot{\underline{x}}^r(t) &= \mathbf{A}^r \underline{x}^r(t) + \mathbf{B}_u^r \underline{u}(t) + \mathbf{B}_w^r \underline{w}^r(t) \\ \underline{y}^r(t) &= \mathbf{C}^r \underline{x}^r(t) + \mathbf{D}_u^r \underline{u}(t) + \mathbf{D}_w^r \underline{w}^r(t) \\ \underline{x}^r &= (z_b \quad \varphi \quad \theta \quad \dot{z}_b \quad \dot{\varphi} \quad \dot{\theta})^T \\ \underline{u} &= (\underline{s} \quad \underline{f})^T \\ \underline{w}^r &= (z_r \quad \dot{z}_r)^T\end{aligned}\quad (6)$$

Both the road height profile and the first derivative of the road height profile must be known. Applying equations (2) to (4) results in a quadratic program based on the reduced model:

$$\begin{aligned}\min_{\hat{u}} \quad & \frac{1}{2} \hat{u}^T \mathbf{H}^r \hat{u} + g^{rT} \hat{u} \\ \text{s.t.} \quad & \hat{u} \in \mathcal{U}\end{aligned}\quad (7)$$

Two control approaches for a vehicle with an active suspension and a continuously variable damper are proposed:

- 1) The passive soft characteristic of the damper results in good ride comfort except for the low frequency

range (see Fig. 5). Given that the actuator can influence this frequency range, the damper is set to the passive soft characteristic and only the actuator displacement is optimized. To obtain a linear controller model for linear MPC, the vehicle model (6) is formulated with a linearized damper characteristic. This leads to $\underline{u}^* = \underline{s}$. The optimization (7) is solved incorporating the constraints on actuator displacement $\underline{s}_{min} \leq \underline{u}^* \leq \underline{s}_{max}$ and actuator displacement rate $\Delta \underline{s}_{min} \leq \Delta \underline{u}^* \leq \Delta \underline{s}_{max}$. The passive soft damper characteristic results in good ride comfort. While turning, the damper characteristic has to be adapted to ensure ride safety.

- 2) The second approach is to optimize both damper force and actuator displacement in order to further improve ride comfort. Therefore, the input of the system (6) is $\underline{u} = (\underline{s} \quad \underline{f})^T$. To obtain linear constraints, the approach proposed in chapter III-A is used. Passive vehicle behavior is predicted using the vehicle model (1) with a linearized damper characteristic and the actuator equal zero. The prediction is used to calculate an approximation of the relative velocities of the dampers. The constraints for the damper force over the preview horizon are obtained via the characteristic in Fig. 3. Furthermore, constraints on the actuator displacement and actuator displacement rate are incorporated.

B. Simulation results

In this section simulation results of the two proposed controllers for a vehicle with a low bandwidth active suspension and variable dampers are presented. The seven degrees of freedom vehicle model (1) is used as the simulation model. The first controller optimizes actuator displacements and the damper force for the simulation model is calculated via the passive soft characteristic of Fig. 3. The second controller optimizes both, damper force and actuator displacement.

The first two diagrams of Fig. 7 show the improved ride comfort of an active suspension compared to the same vehicle with actuator displacement equal to zero and the passive soft or passive hard characteristic of the damper. They also show that optimization of the damper force and the actuator results in a slightly better reduction of vehicle body accelerations than the passive soft damper adjustment with optimization of the actuator. The third diagram shows the actuator displacement of the two compared controllers, and the fourth diagram depicts the damper current. High current corresponds to the soft characteristic and low current to the hard characteristic of the damper. It is visible that the optimized damper force corresponds mainly to the soft characteristic and occasionally the damper is stiffened.

V. CONCLUSION

Model Predictive approaches for vehicles with semi-active and active suspensions were discussed. A semi-active suspension controller incorporating the nonlinear constraints of

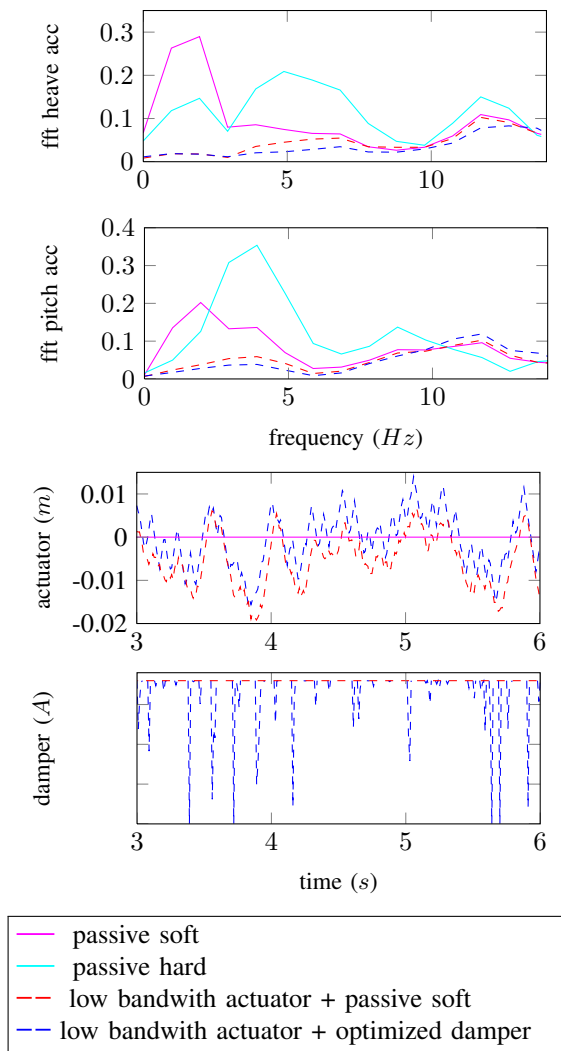


Fig. 7. Simulation results for a vehicle with a low bandwidth active suspension and variable dampers

variable dampers was compared to Model Predictive Controllers using approximated linear input constraints. Simulation results with the simulation model equal to the controller model and with a validated vehicle model were presented. The linear MPC results in the same performance as the nonlinear reference solution. Performance and power consumption of a system with ideal high bandwidth actuators were evaluated. Furthermore, controllers for a vehicle with low bandwidth actuators in series with the spring and combined with continuously variable dampers were discussed and compared. A system with a low bandwidth actuator can further improve ride comfort. If there is also a continuously variable damper, an optimization of both actuator and damper force results in the best performance.

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