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An Intuitive Design for the Dual Mode Adaptive Robust Controller Based on Indirect Control

Leonardo Teixeira[1], Josenalde Oliveira[2] and Aldayr Araujo[3]

Abstract— **In this paper it is proposed an indirect approach to the Dual Mode Adaptive Robust Controller (DMARC), which combines the typical transient and robustness properties of Variable Structure Systems, with a smooth control signal in steady-state, typical of conventional Adaptive Controllers, as Model Reference Adaptive Controller (MRAC). The aim of this indirect version, here named Indirect Dual Mode Adaptive Robust Controller (IDMARC), is to provide a more intuitive controller design, based on physical plant parameters, as resistances, inertia moments, capacitances, etc maintaining DMARC properties. In this paper, it will be presented a stability analysis for the proposed controller and simulations to an unstable second order plant.**

I. INTRODUCTION

Commonly a conventional linear controller is designed to act around a chosen operating point. But, any systems present variations or parametric uncertainties, demanding a new project for the controller to maintain the system performance. For these systems, adaptive control is a suitable strategy. The Model Reference Adaptive Control (MRAC) is one of the most used strategies of adaptive systems. Its performance is determined from a refernce model, which presents the desired response for a particular command signal. When the error between the plant output and the reference model output (output error) is zero, the matching condition is obtained.

Controllers based on parameter estimation, as the conventional MRAC [1], have good behavior in steady-state, but present slow and oscillating transient features. Moreover, this algorithm is not robust in the presence of unmodeled dynamics and/or external disturbances [2].

Other strategy used in control systems with large uncertainties is based in the Variable Structure Systems (VSS), specially on the known form as Sliding Mode Control (SMC). These systems, such as Variable Structure Model Reference Adaptive Control (VS-MRAC [3]), present fast transient and robustness in the presence of unmodeled dynamics and/or external disturbances, but require high frequency control action, difficulting their practical implementation.

Hsu and Costa [4], trying to obtain a combining solution between MRAC and VS-MRAC, developed the Binary Model Reference Adaptive Control (B-MRAC), where it was observed the adaptive scheme in the light of Binary Control Theory as devised by Emelyanov [5]. A specific feature of the binary algorithms proposed by Emelyanov is that the control generated by the controller is a continuous or piecewise-continuous function of time. This weakens the requirements imposed on actuators. In practice, B-MRAC consists of a high gain gradient adaptive law with projection [6]. The choice of a fixed adaptive gain defines how the B-MRAC behaves between MRAC and VS-MRAC.

In the Dual Mode Adaptive Robust Controller (DMARC), differently of the B-MRAC, the transition between the two control strategies can be made in real time depending on the output error [7]. The DMARC was implemented to control the speed of a three-phase induction motor using fuzzy logic (Mandani model [8]) to realize the transition cited above [9]. Mota and Araujo [10] used Takagi-Sugeno model [11] to realize the transition between MRAC and VS-MRAC, allowing the extraction of an analytical expression to control law, facilitating the development of stability analysis.

All the algorithms cited above are based on the direct adaptive control approach, in which the matching equations for the controller parameters are required to estimate their initial values (MRAC) or the relays amplitudes (VS-MRAC). A characteristic of this strategy is that for high order systems its expression may become complex.

As a possible and good solution one has the indirect approach, in which the coefficients of the plant polynomials and high frequency gain are represented by the vector θ_p^*. The online estimative $\theta_p(t)$ of θ_p^*, generated by an adaptive law, is used to calculate the controller parameter vector $\theta(t)$ at each time t, using the matching equations explicity in the algorithm [12].

The indirect version of MRAC is named Indirect MRAC (IMRAC) [12], and of VS-MRAC is named IVS-MRAC [13]. As a first attempt to combine the two control strategies using the indirect approach (IMRAC and IVS-MRAC), Teixeira *et al.* [14] developed the Indirect Binary Model Reference Adaptive Controller (IB-MRAC), ensuring global stability for the system in the presence of a persistently exciting reference. But, as B-MRAC, what defines the controller behavior (like IMRAC or IVS-MRAC) is a parameter definition on the design stage.

In this paper, we propose an algorithm here named Indirect Dual Mode Adaptive Robust Controller (IDMARC), which is an indirect version of DMARC. The goal now is to obtain the typicals transient and robustness properties of IVS-MRAC, with a smooth control signal in steady-state, typical

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[1] L. Teixeira is with Federal Institute of Education, Science and Technology of Rio Grande do Norte (IFRN), 59380-000, Currais Novos, RN, Brazil, `leonardo.teixeira@ifrn.edu.br`

[2] J. Oliveira is with Agricultural School of Jundiai, Federal University of Rio Grande do Norte (EAJ/UFRN), 59280-000, Macaba, RN, Brazil, `josenalde@eaj.ufrn.br`

[3] A. Araujo is with the Department of Electrical Engineering, Federal University of Rio Grande do Norte (DEE/UFRN), 59072-970, Natal, RN, Brazil, `aldayr@dca.ufrn.br`

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of IMRAC, with a transition in real time (depending of output error) and a design stage more intuitive, due to the indirect approach.

II. PROBLEM STATEMENT

Consider an unknown linear SISO (Single Input-Single Output) time-invariant plant, with transfer function

$$W(s) = k_p \frac{N_p(s)}{D_p(s)},$$

$$N_p(s) = s^{n-1} + \sum_{i=1}^{n-1} \beta_i s^{n-1-i}, \tag{1}$$

$$D_p(s) = s^n + \alpha_1 s^{n-1} + \sum_{i=1}^{n-1} \alpha_{i+1} s^{n-1-i},$$

input *u* and output *y*.

The reference model has input *r*, output y_m, and transfer function of the form

$$M(s) = k_m \frac{N_m(s)}{D_m(s)},$$

$$N_m(s) = s^{n-1} + \sum_{i=1}^{n-1} \beta_{m,i} s^{n-1-i}, \tag{2}$$

$$D_m(s) = s^n + \alpha_{m,1} s^{n-1} + \sum_{i=1}^{n-1} \alpha_{m,i+1} s^{n-1-i}.$$

The purpose is to determine a control law *u* such that the output error

$$e_0 = y - y_m \tag{3}$$

tends to zero for arbitrary initial conditions and arbitrary piecewise continuous uniformly bounded reference signals r(t).

The following usual assumptions are made:

a) The plant is completely observable and controllable with degree[D$_p$(s)] = n and degree[N$_p$(s)] = m, and relative degree n* = n - m = 1;
b) sgn(k$_p$) = sgn(k$_m$), positive for simplicity;
c) N$_p$(s) is Hurwitz - W(s) is minimum phase;
d) M(s) has the same relative degree as the plant and it is chosen to be Strictly Positive Real (SPR);
e) Only the plant input and output are used to generate *u*.

Consider the following input and output filters

$$\dot{v_1} = \Lambda v_1 + gu, \dot{v_2} = \Lambda v_2 + gy, \tag{4}$$

where $v_1, v_2 \in \Re^{n-1}$, $g = \begin{bmatrix} 0 & ... & 0 & \gamma \end{bmatrix}^T, \gamma > 0, \in \Re^{n-1}$ and $\Lambda \in \Re^{(n-1).(n-1)}$ is chosen in such a way that N$_m$(s) = det(sI-Λ).

Defining $\theta^T(t) = \begin{bmatrix} \theta_{v1}^T(t) & \theta_n(t) & \theta_{v2}^T(t) & \theta_{2n}(t) \end{bmatrix}$ as the vector of adaptive parameters, and $\omega^T = \begin{bmatrix} v_1^T & y & v_2^T & r \end{bmatrix}$ as the regressor vector, the control law is:

$$u = \theta^T(t)\omega(t). \tag{5}$$

If the plant W(s) is known, then an unique constant vector θ^* exists such that the transfer function of the closed-loop plant is M(s). When this vector is obtained, the matching condition is reached with $u = \theta^{*T}\omega$. When W(s) is unknown, $\theta(t)$ is adapted so that $e_0(t) \to 0$ as $t \to \infty$. Under some signal richness condition, one has $\theta(t) \to \theta^*$.

Let A, b, h^T be a minimal realization of the plant and $x \in \Re$ the corresponding state vector. Defining $X = \begin{bmatrix} x^T & {v_1}^T & {v_2}^T \end{bmatrix} \in \Re^{3n-2}$, the plant and filters can be represented on state space in the following form:

$$\begin{aligned} \dot{X} &= A_0 X + b_0 u, \\ y &= h_c^T X, \end{aligned} \tag{6}$$

where, $A_0 = \begin{bmatrix} A & 0 & 0 \\ 0 & \Lambda & 0 \\ gh^T & 0 & \Lambda \end{bmatrix}$, $b_0 = \begin{bmatrix} b \\ g \\ 0 \end{bmatrix}$, $h_c^T = \begin{bmatrix} h^T & 0 & 0 \end{bmatrix}$.

Adding $b_0\theta^{*T}\omega - b_0\theta^{*T}\omega$ to the right side of the $\dot{X}$, one has:

$$\begin{aligned} \dot{X} &= A_c X + b_c r + \frac{1}{\theta_{2n}^*} b_c \tilde{\theta}^T \omega \\ y &= h_c^T X \end{aligned} \tag{7}$$

where $A_c = A_0 + b_0 \begin{bmatrix} \theta_n^* h^T & \theta_{v1}^{*T} & \theta_{v2}^{*T} \end{bmatrix}$, $b_c = b_0\theta_{2n}^*$, and $\tilde{\theta} = \theta - \theta^*$.

Representing the reference model by state variables

$$\begin{aligned} \dot{X}_m &= A_c X_m + b_c r, \\ y_m &= h_c^T X_m, \end{aligned} \tag{8}$$

and defining the error vector $e \triangleq X - Xm$, it is possible to extract the error equation

$$\begin{aligned} \dot{e} &= A_c e + \frac{b_c}{\theta_{2n}^*}(u - \theta^{*T}\omega), \\ e_0 &= h_c^T e, \end{aligned} \tag{9}$$

knowing that $u - \theta^{*T}\omega = \tilde{\theta}^T\omega$.

On frequency domain,

$$e_0(s) = \frac{1}{\theta_{2n}^*} M(s)[u(s) - \theta^{*T} w(s)]. \tag{10}$$

In the indirect case, the plant parameters are estimated at each time *t*, represented by $\hat{\theta}_p(t) = \begin{bmatrix} \hat{k}_p(t) & \hat{\beta}^T(t) & \hat{\alpha}_1(t) & \hat{\alpha}^T(t) \end{bmatrix}$.

Knowing that

- $\hat{\beta}(t) \in \Re^{(n-1)}$ is a vector that has elements β_i, $i = n-1, ..., 1$ of $N_p(s)$ at time t;

- $\hat{\alpha}_1(t) \in \Re$ is the element α_1 of $D_p(s)$ at time t;
- $\hat{\alpha}(t) \in \Re^{n-1}$ is a vector that has elements α_{i+1}, $i = n-1, ...1$ of $D_p(s)$ at time t;
- $\beta_m \in \Re^{(n-1)}$ is a vector that has elements $\beta_{m,i}$, $i = n-1, ..., 1$ of $N_m(s)$;
- $\alpha_{m1} \in \Re$ is the element α_{m1} of $D_m(s)$;
- $\alpha_m \in \Re^{n-1}$ is a vector that has elements $\alpha_{m,i+1}$, $i = n-1, ...1$ of $D_m(s)$;

the controller parameters are calculated in this form [12]:

$$\begin{aligned} \theta_{v1}(t) &= \frac{\beta_m - \hat{\beta}(t)}{\gamma}, \\ \theta_n(t) &= \frac{\hat{\alpha}_1(t) - \alpha_{m1}}{\hat{k}_p(t)}, \\ \theta_{v2}(t) &= \frac{\hat{\alpha}(t) - \alpha_m + (\alpha_{m1} - \hat{\alpha}_1(t))}{\hat{k}_p(t)\gamma}, \\ \theta_{2n}(t) &= \frac{k_m}{\hat{k}_p(t)}, \end{aligned} \tag{11}$$

with $|\hat{k}_p(t)| \neq 0, \forall t \geq 0$.

Defining the parametric errors as

$$\begin{aligned} \tilde{k}_p &= \hat{k}_p - k_p, \tilde{\beta} = \hat{\beta} - \beta, \\ \tilde{\alpha}_1 &= \hat{\alpha}_1 - \alpha_1, \tilde{\alpha} = \hat{\alpha} - \alpha, \end{aligned} \tag{12}$$

and developing the error equation for indirect case [15], one has

$$\dot{e} = A_c e + \frac{b_c}{k_m}\left[\ \tilde{k}_p\zeta_p + k_p\tilde{\beta}^T\zeta_\beta + \tilde{\alpha}_1\zeta_1 + \tilde{\alpha}^T\zeta_\alpha\ \right], \tag{13}$$

where

$$\begin{aligned} \zeta_p &= \frac{\beta_m^T v_1}{\gamma_p} - u - \frac{\beta^T v_1}{\gamma_p}, \\ \zeta_{\beta i} &= \frac{-v_{1,i}}{\gamma}, i = 1, ..., n-1, \\ \zeta_1 &= y - \frac{\beta_m^T v_2}{\gamma_1}, \\ \zeta_{\alpha i} &= \frac{v_{2,i-1}}{\gamma}, i = 2, ..., n, \end{aligned} \tag{14}$$

with $\gamma > 0$, $\gamma_p > 0$ and $\gamma_1 > 0$.

III. DUAL MODE ADAPTIVE ROBUST CONTROLLER

As previously cited, the DMARC acts between the MRAC and the VS-MRAC, obtaining a fast transient, without oscillations, typical properties of VS-MRAC, with a smooth control signal in steady-state, typical of MRAC. Moreover, this transition between these strategies occurs in real time, depending on the output error in each instant.

This transition is possible due to a parameter μ on the adaptive law. It is represented by a gaussian function, expressed in (15) and showed in Figure (1). It is interesting to observe that as output error e_o grows, $\mu \to 0$, and the DMARC tends to VS-MRAC. Otherwise, as output error decreases, $\mu \to 1$, and the DMARC tends to MRAC. L is a parameter defined in design and it defines the action range of VS-MRAC in the system.

$$\mu = e^{\frac{-e_o^2}{L}} \tag{15}$$

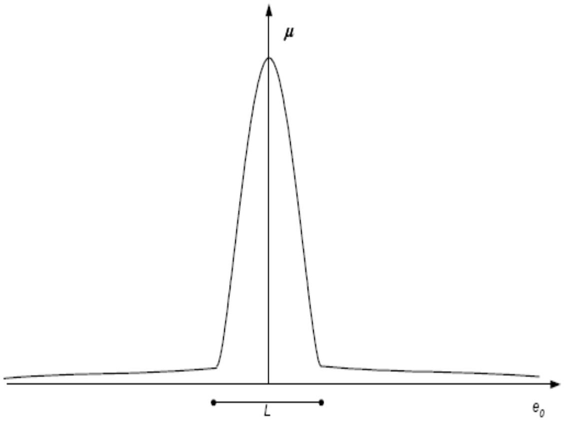

Fig. 1. Graphical representation of μ

According to Cunha *et al.* [7], one has the following adaptive law for the DMARC:

$$\mu\dot{\theta} = -\sigma\theta - (1-\mu)\sigma\Gamma e_o\omega - \mu\gamma e_o\omega, \tag{16}$$

where $\gamma > 0$, $\sigma > 0$ and $\Gamma = diag\left[\frac{\overline{\theta}_i}{|e_o\omega_i|}\right]$ with $\overline{\theta}_i > |\theta_i^*|$, $i = 1...2n$.

Performing some mathematical manipulations from (16), one has:

$$\dot{\theta} = -\frac{\sigma}{\mu}\theta - \frac{\sigma}{\mu}(1-\mu)\overline{\theta}sgn(e_o\omega) - \gamma e_o\omega, \tag{17}$$

with $\gamma > 0$ and $\sigma > 0$.

When $\mu \to 0$, the forgetting factor and the switched term tend to infinity and the adaptive term $-\gamma e_o\omega$ can be disregarded. So, DMARC acts like VS-MRAC. With this analysis, we can conclude that in the VS-MRAC the learning or adaptation is instantaneous. However, when $\mu \to 1$, the switched term tends to zero and DMARC acts like MRAC with forgetting factor [7].

The σ parameter is implemented as in (18).

$$\sigma = \left\{ \begin{array}{ll} 0, & \text{if } ||\theta|| < M_\theta \text{ and } \mu > \mu_p \\ \sigma_c, & \sigma_c > 0, \text{ otherwise} \end{array} \right. , \tag{18}$$

where μ_p is the minimum valid value of μ, $\mu_p > 0$, $\sigma_c > 0$ and σ_c must be chosen for $\frac{\sigma_c}{\mu_p}$ be sufficiently high to the case that $\mu \leqslant \mu_p$. From this, it is important to observe that when we find the $\frac{\sigma_c}{\mu_p}$ ideal relation, we can reduce the μ_p value indefinitely, since that σ_c is reduced in the same proportion. This fact is important in the stability proof of IDMARC.

IV. INDIRECT DUAL MODE ADAPTIVE ROBUST CONTROLLER

In Indirect Dual Mode Adaptive Robust Controller (IDMARC), goal of this paper, when the error between the reference model output and the plant output is sufficiently low, IDMARC tends to IMRAC. When the error increases, IDMARC tends to IVS-MRAC. The aim is to obtain the good transient properties (fast response without oscillations) of IVS-MRAC, a smooth control signal in steady-state of IMRAC over there a more intuitive design of controller due to direct relationship with physical parameters of plant (advantage of indirect approach).

Although it might arise a higher computational load, this is not a problem, because actually there are electronic devices which have as main features the possibility of dealing with high frequencies and therefore provide a faster processing, as Digital Signal Processor (DSP) and Field Programmable Gate Array (FPGA).

The main motivation to the development of this work was in the increasing use of IVS-MRAC in practical applications [16] [17].

The adaptive laws of IDMARC are based on DMARC [7], IMRAC [12] and IVS-MRAC [15], and are listed as following.

$$\begin{aligned}
&\dot{\hat{\beta}} = -\frac{\sigma_\beta}{\mu}\hat{\beta} - \frac{\sigma_\beta}{\mu}(1-\mu)\left[\overline{\beta} sgn(e_o\zeta_\beta sgn(k_p))\right] + \\
&+\Gamma_\beta e_o v_1 sgn(k_p), \\
&\dot{\hat{\alpha}}_1 = -\frac{\sigma_{\alpha_1}}{\mu}\hat{\alpha}_1 - \frac{\sigma_{\alpha_1}}{\mu}(1-\mu)\left[\overline{\alpha}_1 sgn(e_o\zeta_1)\right] - \gamma_1 e_o\zeta_1, \\
&\dot{\hat{\alpha}} = -\frac{\sigma_\alpha}{\mu}\hat{\alpha} - \frac{\sigma_\alpha}{\mu}(1-\mu)\left[\overline{\alpha} sgn(e_o\zeta_\alpha)\right] - \Gamma_\alpha e_o v_2, \\
&\text{if } (|\hat{k}_p| > k_0) \text{ or } (|\hat{k}_p| = k_0 \text{ and } e_o\zeta_p sgn(\hat{k}_p) \leq 0) \\
&\qquad \dot{\hat{k}}_p = -\tfrac{\sigma_{k_p}}{\mu}\hat{k}_p + \\
&\qquad +\tfrac{\sigma_{k_p}}{\mu}(1-\mu)\left[k_{p,nom} - \overline{k}_p sgn(e_o\zeta_p)\right] - \gamma_p e_o\zeta_p \\
&\text{otherwise} \\
&\qquad \dot{\hat{k}}_p = 0,
\end{aligned} \tag{19}$$

where k_0 is a lower bound for $\hat{k}_p$ (avoid numerical error), $\Gamma_\alpha = diag(\gamma_{\alpha i}), \gamma_{\alpha i} > 0$, i = 2...n, $\Gamma_\beta = diag(\gamma_{\beta i}), \gamma_{\beta i} > 0$, i = 1...n-1, $\gamma_p > 0$, μ is defined as in (15) and each plant parameter has a σ, detailed in (18). The σ_c was chosen equal for all parameters, as well as μ_p, such that the $\frac{\sigma_c}{\mu_p}$ will be sufficiently high for the case $\mu \leqslant \mu_p$.

A. Stability Proof

Theorem: Consider the system (6), output error (13) and adaptive laws (19). If the hypotheses (a-e) were satisfied, then:

(i) All the signals of closed loop system are uniformly bounded.
(ii) $||e(t)|| \to 0$ when $t \to \infty$.

Proof: Consider as Lyapunov Candidate Function, the following definite positive function:

$$\begin{aligned}
&V(e,\tilde{\theta}_p) = \frac{e^T P e}{2} + \\
&+\frac{1}{k_m}\left(\frac{\tilde{k}_p^2}{2\gamma_p} + \frac{\tilde{\beta}^T\Gamma_\beta^{-1}\tilde{\beta}k_p}{2\gamma} + \frac{\tilde{\alpha}_1^2}{2\gamma_1} + \frac{\tilde{\alpha}^T\Gamma_\alpha^{-1}\tilde{\alpha}}{2\gamma}\right),
\end{aligned} \tag{20}$$

where $\tilde{\theta}_p = \left[\begin{array}{cccc} \tilde{k}_p & \tilde{\beta}^T & \tilde{\alpha}_1 & \tilde{\alpha}^T \end{array}\right]$, $\Gamma_\alpha = \Gamma_\alpha^T > 0$, $\Gamma_\beta = \Gamma_\beta^T > 0$, $\gamma > 0$, $\gamma_p > 0$ e $\gamma_1 > 0$.

Calculating the derivative of (20), moreover considering the Kalman-Yakubovitch Lemma ($A_c^T P + PA_c = -2Q$, $Pb_c = h_c$, $P = P^T > 0$, $Q = Q^T > 0$) and knowing that $\dot{\tilde{\theta}}_p = \dot{\hat{\theta}}_p$, from (12), one has:

$$\begin{aligned}
&\dot{V}(e,\tilde{\theta}_p) = -e^T Q e + \\
&+\frac{e_o}{k_m}\left[\tilde{k}_p\zeta_p + k_p\tilde{\beta}^T\zeta_\beta + \tilde{\alpha}_1\zeta_1 + \tilde{\alpha}^T\zeta_\alpha\right] + \\
&+\frac{1}{k_m}\left(\frac{\tilde{k}_p\dot{\hat{k}}_p}{\gamma_p} + \frac{\tilde{\beta}^T\Gamma_\beta^{-1}\dot{\hat{\beta}}k_p}{\gamma} + \frac{\tilde{\alpha}_1\dot{\hat{\alpha}}_1}{\gamma_1} + \frac{\tilde{\alpha}^T\Gamma_\alpha^{-1}\dot{\hat{\alpha}}}{\gamma}\right).
\end{aligned} \tag{21}$$

Replacing $\dot{\hat{\theta}}_p$ as in (19) in the equation (21), and knowing that $\zeta_\beta = \frac{-v_1}{\gamma}$, $\zeta_\alpha = \frac{v_2}{\gamma}$, and $k_p > 0$ (for hypothesis):

$$\begin{aligned}
&\dot{V}(e,\tilde{\theta}_p) = -e^T Q e - \\
&-\frac{\sigma_{kp}\tilde{k}_p}{\mu k_m\gamma_p}\left\{\hat{k}_p - (1-\mu)\left[k_{p,nom} - \overline{k}_p sgn(e_o\zeta_p)\right]\right\} - \\
&-\frac{k_p\tilde{\beta}^T\Gamma_\beta^{-1}\sigma_\beta}{\mu k_m\gamma}\left\{\hat{\beta} + (1-\mu)\left[\overline{\beta} sgn(e_o\zeta_\beta)\right]\right\} - \\
&-\frac{\sigma_{\alpha 1}\tilde{\alpha}_1}{\mu k_m\gamma_1}\left\{\hat{\alpha}_1 + (1-\mu)\left[\overline{\alpha}_1 sgn(e_o\zeta_1)\right]\right\} - \\
&-\frac{\tilde{\alpha}^T\Gamma_\alpha^{-1}\sigma_\alpha}{\mu k_m\gamma}\left\{\hat{\alpha} + (1-\mu)\left[\overline{\alpha} sgn(e_o\zeta_\alpha)\right]\right\}.
\end{aligned} \tag{22}$$

In this moment it is required a term by term analysis for signal definition of $\dot{V}$.

Term of $\hat{\alpha}_1$:

$$\frac{\sigma_{\alpha 1}\tilde{\alpha}_1}{\mu k_m\gamma_1}\left\{\hat{\alpha}_1 + (1-\mu)\left[\overline{\alpha}_1 sgn(e_o\zeta_1)\right]\right\}$$

In this term, we know that $\mu > 0$, $k_m > 0$ and $\gamma_1 > 0$. Calculating the roots of $F(\hat{\alpha}_1) = (\hat{\alpha}_1 - \alpha_1{}^*)\{\hat{\alpha}_1 + (1-\mu)[\overline{\alpha}_1 sgn(e_o\zeta_1)]\}$ for $\mu_p \leqslant \mu \leqslant 1$, we obtain $-\overline{\alpha}_1$ and $\overline{\alpha}_1$ as larger roots in absolute value of $F(\hat{\alpha}_1)$, because $\overline{\alpha}_1 > |\alpha_1{}^*|$. So, $F(\hat{\alpha}_1) > 0\ \forall \hat{\alpha}_1$ such that $|\hat{\alpha}_1| > \overline{\alpha}_1$.

The parameter σ_{α_1} follow this rule:

$$\sigma_{\alpha 1} = \begin{cases} 0, & \text{if } |\hat{\alpha}_1| < M_{\alpha_1} = \overline{\alpha}_1 \text{ and } \mu > \mu_p \\ \sigma_c, & \sigma_c > 0, \text{ otherwise} \end{cases} \tag{23}$$

So, one has four possible cases for analysis as showed in Table (I).

TABLE I

CASES FOR VALUE DEFINITION OF σ_{α_1}

Case	Condition 1	Condition 2	σ_{α_1}
1	$\lvert\hat{\alpha}_1\rvert < \overline{\alpha}_1$	$\mu > \mu_p$	0
2	$\lvert\hat{\alpha}_1\rvert < \overline{\alpha}_1$	$\mu \leqslant \mu_p$	σ_c
3	$\lvert\hat{\alpha}_1\rvert \geqslant \overline{\alpha}_1$	$\mu > \mu_p$	σ_c
4	$\lvert\hat{\alpha}_1\rvert \geqslant \overline{\alpha}_1$	$\mu \leqslant \mu_p$	σ_c

In the first case, $\sigma_{\alpha_1} = 0$, therefore

$$\frac{\sigma_{\alpha 1}\tilde{\alpha}_1}{\mu k_m \gamma_1}\left\{\hat{\alpha}_1 + (1-\mu)\left[\overline{\alpha}_1 sgn(e_o\zeta_1)\right]\right\} = 0.$$

In the case (2), $\sigma_{\alpha_1} = \sigma_c > 0$. As it can be observed, in this case $\mu \leqslant \mu_p$, with μ_p negligible. So, the IDMARC adaptive law can be considered the same of the IVS-MRAC switched law: $\hat{\alpha}_1 = -\overline{\alpha}_1 sgn(e_o\zeta_1)$. This law make $\hat{\alpha}_1$ switches between $-\overline{\alpha}_1$ and $\overline{\alpha}_1$. Consequently, $F(\hat{\alpha}_1) = 0$, and the $\hat{\alpha}_1$ term becomes zero again.

In the cases (3) and (4), $\sigma_{\alpha_1} = \sigma_c > 0$ and $|\hat{\alpha}_1| \geqslant \overline{\alpha}_1$. Therefore, for these cases:

$$\frac{\sigma_{\alpha 1}\tilde{\alpha}_1}{\mu k_m \gamma_1}\left\{\hat{\alpha}_1 + (1-\mu)\left[\overline{\alpha}_1 sgn(e_o\zeta_1)\right]\right\} \geqslant 0.$$

So, considering the four possible cases, one has that the $\hat{\alpha}_1$ term is non-negative.

Performing the same procedure for $\hat{\beta}$, $\hat{\alpha}$ and $\hat{k}_p$ terms, we can assert that these terms are non-negative.

After this term by term analysis of (22), one has:

$$\dot{V}(e, \tilde{\theta}_p) \leqslant -e^T Q e \leqslant 0. \tag{24}$$

From (20) and (24), we can assert that the origin $[e, \tilde{\theta}_p] = [0, 0]$ is stable, so, $e \in L_\infty$ and $\tilde{\theta}_p \in L_\infty$. Consequently, $e_o \in L_\infty$ and $\theta_p \in L_\infty$. As $r \in L_\infty$ and M(s) is stable, $X_m \in L_\infty$ and $y_m \in L_\infty$.

The output error is expressed by $e_o = y - y_m$. Therefore, $y = e_o + y_m$. As $e_o \in L_\infty$ and $y_m \in L_\infty$, one has $y \in L_\infty$. Knowing that $e = X - X_m$, $e \in L_\infty$ and $X_m \in L_\infty$, then $X \in L_\infty$. Knowing that $X^T = [x^T v_1{}^T v_2{}^T]$, one has $x \in L_\infty$, $v_1 \in L_\infty$ and $v_2 \in L_\infty$.

Moreover, $\omega \in L_\infty$, because $\omega^T = \begin{bmatrix} v_1^T & y & v_2^T & r \end{bmatrix}$. The controller parameters vector θ is calculated in function of plant parameters vector θ_p. As $\theta_p \in L_\infty$ and $|\hat{k}_p| \geqslant k_0$ then $\theta \in L_\infty$ too. From the control law $u = \theta^T\omega$, one has $u \in L_\infty$. So, we prove the property *i*.

Observing the derivative equation of error ($\dot{e} = A_c e + \frac{b_c}{k_m}[\tilde{k}_p\zeta_p + k_p\tilde{\beta}^T\zeta_\beta + \tilde{\alpha}_1\zeta_1 + \tilde{\alpha}^T\zeta_\alpha]$), we note that $\dot{e} \in L_\infty$, because A_c has only eigenvalues with negative real term (M(s) stable), $\tilde{\theta}_p \in L_\infty$ and the variables $\zeta \in L_\infty$ too, because are calculated in function of uniformly bounded parameters.

As $V > 0$ and it has a minimum, and $\dot{V} \leqslant 0$, one has $\exists \lim\limits_{t\to\infty} V(t) = V_\infty$.

$\lambda_{min}(Q)\int_0^\infty ||e||^2\ dt \leqslant \int_0^\infty e^T Q e\ dt \leqslant -\int_0^\infty \dot{V}(t)dt = -\left[V(t)\right]_0^\infty = -V_\infty + V(0) \in L_\infty.$

With this, $e \in L_2$. As $e \in L_\infty$, $\dot{e} \in L_\infty$ and $e \in L_2$, one has by Barbalat Lemma:

$\lim\limits_{t\to\infty} e(t) = 0 \Longrightarrow \lim\limits_{t\to\infty} e_o(t) = 0$, that proves the property *ii*.

B. Simulation Results

With the goal of verifying the performance of the closed loop system with IDMARC, there was performed a simulation in an unstable second order plant, with relative degree $n^* = 1$. The plant and the reference model are represented below:

$$W(s) = \frac{s+1}{(s-1)^2}, M(s) = \frac{s+2}{(s+1)(s+3)}. \tag{25}$$

The IDMARC was tested in a scenario with fast reference changes and in the presence of disturbance in the plant input with the aim of verifying its performance regarding robustness. Those changes are detailed in Table (II). The initial estimatives for plant parameters were chosen next to their nominal values in (25): $\hat{\theta}_p(0) = \begin{bmatrix} \hat{k}_p(0) & \hat{\beta}(0) & \hat{\alpha}_1(0) & \hat{\alpha}(0) \end{bmatrix} = \begin{bmatrix} 1.2 & 1.2 & -1.8 & 1.2 \end{bmatrix}$. The filters were tuned as: $v_1(0) = v_2(0) = 0$, with $\Lambda = -\gamma$ ($\gamma = 2$) and $g = 2$. The other parameters were designed as following: $\overline{\beta} = 2.5$, $\overline{\alpha}_1 = 3.5$, $\overline{\alpha} = 3.5$, $\overline{k}_p = 1.2$, $k_{p,nom} = 1.6$, $\gamma_p = 6$, $\Gamma_\beta = 3$, $\Gamma_\alpha = 3$, $\gamma_{\alpha_1} = 3$, $L = 10^{-7}$, $h = 10^{-4}$, $\mu_p = 10^{-7}$ and $\sigma_c = 10^{-4}$. The result is showed in Figure (2). The parameter evolution can be verified in Figure (3).

TABLE II

REFERENCE AND DISTURBANCE CHANGES IN PLANT INPUT

Interval	Reference	Disturbance
$0 \leqslant t < 0.2$	r(t) = 1	d(t) = 0.8
$0.2 \leqslant t < 0.4$	r(t) = -1	d(t) = 0.0
$0.4 \leqslant t < 0.6$	r(t) = 1	d(t) = 0.8

From this result, we can observe a fast transient, without oscillations, typical of IVS-MRAC, with a smooth control signal in steady-state, typical of IMRAC, beyond the good performance of controller regarding robustness.

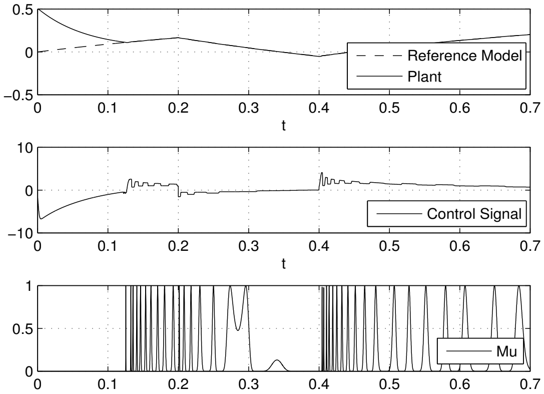

Fig. 2. IDMARC performance for the case of Table (II)

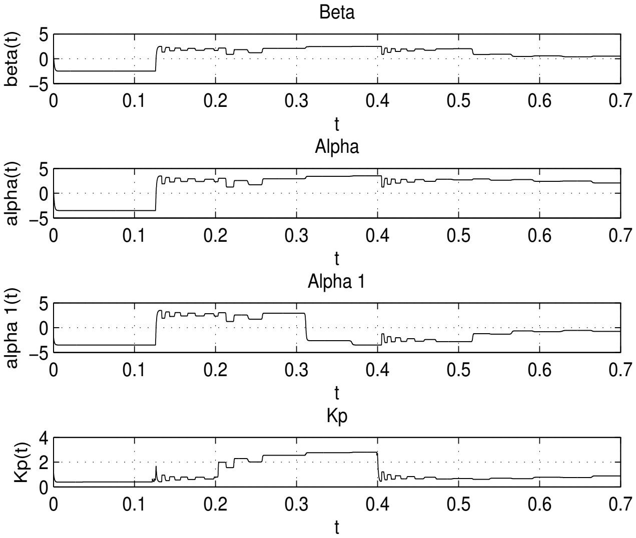

Fig. 3. Parameter adaptation with $\theta_p^{*T} = [\ k_p^* \ \beta^{*T} \ \alpha_1^* \ \alpha^{*T}\ ] = [\ 1\ 1\ -2\ 1\ ]$

When simulation starts, $\mu = \mu_p$, until the output error e_o becomes zero. Thenceforth, μ oscillates between μ_p and 1. This occurs because the IMRAC takes the control action ($\mu = 1$) and therefore parametric adaptation arises, generating an output oscillation. To overcome this effect, μ tends to decrease, and the IVS-MRAC takes over. Note that the control signal is directly influenced by the plant parameters evolution.

V. CONCLUSIONS

In this work it was presented an indirect approach to the Dual Mode Adaptive Robust Controller, here named IDMARC, which obtains a fast transient, without oscillations (IVS-MRAC properties), with a smooth control signal in steady-state (IMRAC property), over there a more intuitive design of controller due to direct relationship with the nominal plant parameters and their tolerance values.

Moreover, it was developed a stability proof of the system with the IDMARC, ensuring that all the signals of closed loop system are uniformly bounded and $||e(t)|| \to 0$ when $t \to \infty$, as well as the direct version (DMARC).

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