

Continuous-Time and Discrete-Time Switched H_∞ State Feedback Controllers: Application for a Robust Steering Vehicle Control

Lghani Menhour, Damien Koenig and Brigitte d'Andréa-Novel

Abstract—Two control laws for switched uncertain linear systems are proposed: firstly, a continuous-time switched H_∞ state feedback control is given, and secondly, a discrete-time H_∞ state feedback control. These controllers are designed using a common Lyapunov function approach and switched Lyapunov function approach used for switched continuous-time and switched discrete-time linear systems respectively. All sufficient conditions of the existence of these controllers are proved and given in term of LMIs. These conditions are provided for an arbitrary switching rule. The lateral control via steering vehicle control is implemented to control simultaneously lateral deviation and yaw motion of the vehicle. In fact, a validation of these two controllers is performed using experimental data acquired by laboratory vehicle under high lateral accelerations.

Keywords: Switched control, Continuous-Time and Discrete-Time Switched linear systems, stability, H_∞ -filtering, LMI, vehicle application, steering vehicle control.

I. INTRODUCTION

The modeling of real complex systems is very hard task. Consequently, the available models are uncertain and their validity domain is local. For this problem, several solutions like fuzzy systems and switched systems are proposed. The switched systems are a class of hybrid systems and have been introduced and developed [2], [8], [4], [15], [10], [7], [9], [11], [6]. In fact, switched systems are defined by a finite number of local models and switching rules. The switching rules are used to determine the corresponding local model. The stability and design of switched systems are also treated in literature (see, [8], [15], [11] and the references therein).

Switched systems are defined by a set of dynamical models (nonlinear and/or linear). The stability analysis of two kinds of switched systems under arbitrary switching signals are treated: by using common Lyapunov function approach [8], [15], [10], [11], and by using switched Lyapunov function approach [2], [4], [7], [5]. Other switched systems are also treated like switched linear and nonlinear descriptor systems [7], [6] and switched systems with state delays [5].

In this study, two switched control laws for continuous-time and discrete-time switched systems are considered. These controllers use H_∞ norm [5], [16], a common quadratic

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Lyapunov function approach for continuous-time case [8], [15], [10], [11] and switched Lyapunov function for discrete-time case [2], [4], [7], [5] are respectively considered. The sufficient conditions of the existence of the switched H_∞ controllers are given. All these conditions are demonstrated in Lyapunov sense and expressed in term of linear matrix inequality. The control approaches developed here are tested on a vehicle dynamics. More precisely, the switched controllers are used to control the lateral deviation and yaw motion via steering angle. In fact, lateral vehicle control is widely treated in literature [1], [14], [3] and the references cited therein. The most part these studies assume that the vehicle models are exactly known. This assumption is not always checked. In fact, the exact knowledge of some vehicle parameters is not easy task. In order to overcome parametric uncertainties, the multi-model or switched models representation can be used. For these reasons, the switched systems are used here.

This paper is organized as follows. Section II describes the Linear Two Wheels Vehicle Model (L2WVM) and problem statement. Section III presents the design methods of continuous-time and discrete-time switched H_∞ state feedback controllers. The simulation results of these controllers using the experimental data acquired by laboratory vehicle are illustrated in section IV. Concluding remarks and perspectives are presented in section V.

II. VEHICLE MODEL AND STEERING CONTROL PROBLEM

A. Bicycle vehicle model

The uncertain and disturbed bicycle vehicle model used here is composed of lateral and yaw motions (see table I for notations). The state space representation is:

$$\begin{cases} \dot{x}(t) = A(t)x(t) + B(t)u(t) + F\omega(t) \\ y(t) = Cx(t) \end{cases} \quad (1)$$

where

$$A(t) = \begin{bmatrix} -\frac{2C_f+2C_r}{mV_x(t)} & -\frac{2C_fL_f-2C_rL_r}{mV_x(t)} & 0 & \frac{2C_f+2C_r}{m} \\ -\frac{2C_fL_f-2C_rL_r}{I_zV_x(t)} & -\frac{2C_fL_f^2+2C_rL_r^2}{I_zV_x(t)} & 0 & \frac{2C_fL_f-2C_rL_r}{I_z} \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix},$$

$$x(t) = \begin{bmatrix} \dot{Y}(t) & \dot{\psi}(t) & Y(t) & \psi(t) \end{bmatrix}^T, u = \delta, C = I_n, B^T = \begin{bmatrix} \frac{2C_f}{m} & \frac{2L_fC_f}{I_z} & 0 & 0 \end{bmatrix} \text{ and } F^T = \begin{bmatrix} -g & 0 & 0 & 0 \end{bmatrix}$$

The lateral forces are modeled as proportional to sideslip angles ($F_{yf} = C_f\alpha_f$ and $F_{yr} = -C_r\alpha_r$) of each axle as follows:

$$F_{yf} = C_f \left(\delta - \beta - \frac{L_f\dot{\psi}}{V_x} \right), F_{yr} = -C_r \left(\beta - \frac{L_r\dot{\psi}}{V_x} \right) \quad (2)$$

Here, $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^p$ is the control input, $y(t) \in \mathbb{R}^m$ is the output, and $\omega(t) = \phi_r(t) \in \mathbb{R}^{n_\omega}$ is the disturbance input that satisfies $\omega \in L_2[0, \infty)$. A , B , C and F are system matrices with appropriate size.

TABLE I
NOTATIONS

Symbol	Variable name
V_x	longitudinal speed [km/h]
$\psi, \dot{\psi}$	yaw rate [rad/s] and yaw angle [rad]
Y, \dot{Y}	lateral deviation [m] and its derivative [m/s]
δ, ϕ_r	wheel steer angle [deg] and road bank angle [rad]
F_{yi}	lateral force in the wheel coordinate [N]
F_{yf}, F_{yr}	front and rear lateral forces in the vehicle coordinate [N]
L_f, L_r	distances from the CoG to the front and rear axles [m]
I_z	yaw moment of inertia [Kg.m ²]
C_f, C_r	front and rear cornering stiffnesses [N/rad]
α_f, α_r	front and rear tire slip angle [rad]
g, m	acceleration due to gravity [m/s ²] and vehicle mass [kg]
$(\cdot)^T$	stands for the transpose matrix
$(\cdot) > 0$	denotes a symmetric positive definite matrix.

B. Problem formulation

Model (1) is affected by several uncertainties. In fact, when the vehicle is subjected to high dynamic loads, the cornering stiffnesses C_f and C_r become coupled and nonlinear functions of sideslip angles, longitudinal slip ratio, vertical forces, camber angle and other dynamical parameters. Moreover, model (1) depends also on the longitudinal speed $V_x(t)$. Unfortunately, if we take into account all parameter variations, model (1) becomes nonlinear. Then, design control and estimation algorithms becomes a difficult task. In order to take into account parametric uncertainties and keep simple models, we define switching rules. Then, model (1) can be viewed as an uncertain switched continuous-time system:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^M \alpha_i(t) [A_i x(t) + B_i u(t) + F_i \omega(t)] \\ y(t) = \sum_{i=1}^M \alpha_i(t) C_i x(t) \end{cases} \quad (3)$$

The function $\alpha_i(t)$ is the switching signal

$$\alpha_i : \mathbb{R}^+ \longrightarrow \{0, 1\} \quad \sum_{i=1}^M \alpha_i(t) = 1, \quad t \in \mathbb{R}^+ \quad (4)$$

The discrete-time system corresponding to model (3) using the first order Euler approximation at frequency 200Hz is

$$\begin{cases} x(k+1) = \sum_{i=1}^M \alpha_i(k) [\bar{A}_i x(k) + \bar{B}_i u(k) + \bar{F}_i \omega(k)] \\ y(k) = \sum_{i=1}^M \alpha_i(k) \bar{C}_i x(k) \end{cases} \quad (5)$$

For model (5), the switching signal $\alpha_i(k)$ is as follows:

$$\alpha_i : Z^+ \longrightarrow \{0, 1\} \quad \sum_{i=1}^M \alpha_i(k) = 1, \quad k \in Z^+ = \{0, 1, \dots\} \quad (6)$$

In the sequel, the two switched state feedback control problems for systems (3) and (5) are addressed.

Problem 1: Consider the following H_∞ continuous-time state feedback controller for switched system (3)

$$u(t) = - \sum_{i=1}^M \alpha_i(t) K_i x(t) \quad (7)$$

where the gains $K_i \in \mathbb{R}^{p \times n}$ are computed such that the following conditions are ensured:

- C1. the closed-loop system $\dot{x}(t) = \sum_{i=1}^M \alpha_i(t) (A_i - B_i K_i) x(t)$ is globally asymptotically stable when $\omega(k) = 0$;
- C2. the transfer function $H_{y\omega}(p) = \sum_{i=1}^M C_i (pI - (A_i - B_i K_i))^{-1} F_i$, from $\omega(t)$ to $y(t)$ satisfies the H_∞ constraint $\|H_{y\omega}(p)\|_\infty < \gamma$ for positive scalar γ .

Problem 2: Consider the following H_∞ discrete-time state feedback controller for switched discrete-time system (5)

$$u(k) = - \sum_{i=1}^M \alpha_i(k) \bar{K}_i x(k) \quad (8)$$

where the gains $\bar{K}_i \in \mathbb{R}^{p \times n}$ are computed such that the following conditions are ensured:

- C3. the closed-loop system $x(k+1) = \sum_{i=1}^M \alpha_i(k) (\bar{A}_i - \bar{B}_i \bar{K}_i) x(k)$ is asymptotically stable when $\omega(k) = 0$;
- C4. the transfer function $\bar{H}_{y\omega}(z) = \sum_{i=1}^M \bar{C}_i (zI - (\bar{A}_i - \bar{B}_i \bar{K}_i))^{-1} \bar{F}_i$, from $\omega(k)$ to $y(k)$ satisfies the H_∞ constraint $\|\bar{H}_{y\omega}(z)\|_\infty < \gamma$ for positive scalar γ .

III. SWITCHED H_∞ CONTROLLERS

In this section, we present two state feedback controllers for switched linear systems: firstly, a switched control law for switched continuous-time systems using a common Lyapunov function method and secondly, a control law for switched discrete-time systems using switched Lyapunov functions method.

A. Switched H_∞ State Feedback Control: Continuous-Time Case

The main objective is to compute the gain of state feedback control (7) to control system (3) in order that the following closed-loop system

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^M \alpha_i(t) (A_i - B_i K_i) x(t) + \sum_{i=1}^M \alpha_i(t) F_i \omega(t) \\ y(t) = \sum_{i=1}^M \alpha_i(t) C_i x(t) \end{cases} \quad (9)$$

is stable and satisfies C2. For this, we establish the following theorem.

Theorem 1: Suppose that for $i \in \{1, \dots, M\}$, the pair (A_i, B_i) is controllable. If there exist a constant $\gamma > 0$, a common symmetric positive definite matrix $X \in \mathbb{R}^{n \times n}$ and matrices $\Gamma_i \in \mathbb{R}^{m \times n}$ such that the following inequality is satisfied for $i \in \{1, \dots, M\}$:

$$\begin{bmatrix} XA_i^T + A_i X - \Gamma_i^T B_i^T - B_i \Gamma_i & F_i & X C_i^T \\ * & -\gamma^2 I & 0 \\ * & * & -I \end{bmatrix} < 0 \quad (10)$$

then, the gains of the switched H_∞ state feedback controller are given by $K_i = \Gamma_i X^{-1} = \Gamma_i P$.

Proof: To establish sufficient conditions for the existence of (7) such that the closed-loop system satisfies C1 and C2, let us consider the following Lyapunov function:

$$V(x, t) = x^T(t) P x(t) \quad (11)$$

A sufficient condition is related to the derivative of Lyapunov function which must satisfy the following inequality

$$\dot{V}(t) + y^T(t) y(t) - \gamma^2 \omega^T(t) \omega(t) < 0 \quad (12)$$

The time derivative \dot{V} along of (9) gives

$$\begin{aligned} & \sum_{i=1}^M \alpha_i(t) [(A_i - B_i K_i)x(t) + F_i \omega(t)]^T P x(t) \\ & + \sum_{i=1}^M \alpha_i(t) x^T(t) P [P(A_i - B_i K_i)x(t) + F_i \omega(t)] \\ & + y^T(t)y(t) - \gamma^2 \omega^T(t)\omega(t) < 0 \end{aligned} \quad (13)$$

For all possible switches, we consider the case $\alpha_i(t) = 1$, $\alpha_{l \neq i}(t) = 0$, (13) becomes

$$\begin{aligned} & x^T(t)(A_i - B_i K_i)^T P x(t) + \omega^T(t) F_i^T P x(t) \\ & + x^T(t) P (A_i - B_i K_i)x(t) + x^T(t) P F_i \omega(t) \\ & + y^T(t)y(t) - \gamma^2 \omega^T(t)\omega(t) < 0 \end{aligned} \quad (14)$$

Now, (14) can rewrite as follows:

$$\begin{bmatrix} x(t) \\ \omega(t) \end{bmatrix}^T \begin{bmatrix} \sigma_i + C_i^T C_i & P F_i \\ F_i^T P & -\gamma^2 I \end{bmatrix} \begin{bmatrix} x(t) \\ \omega(t) \end{bmatrix} < 0 \quad (15)$$

where $\sigma_i = (A_i - B_i K_i)^T P + P(A_i - B_i K_i)$. It follows that $\dot{V} < 0$ for any nonzero vector $\begin{bmatrix} x(t) & \omega(t) \end{bmatrix}$ if

$$\begin{bmatrix} \sigma_i & P F_i \\ F_i^T P & -\gamma^2 I \end{bmatrix} + \begin{bmatrix} C_i^T \\ 0 \end{bmatrix} I \begin{bmatrix} C_i & 0 \end{bmatrix} < 0 \quad (16)$$

which is equivalent, by Schur complement, to

$$\begin{bmatrix} (A_i - B_i K_i)^T P + P(A_i - B_i K_i) & P F_i & C_i^T \\ * & -\gamma^2 I & 0 \\ * & * & -I \end{bmatrix} < 0 \quad (17)$$

Now, pre- and post-multiplying (17) by $Z = \text{diag}(X = P^{-1}, I, I)$, (17) becomes

$$\begin{bmatrix} X(A_i - B_i K_i)^T + (A_i - B_i K_i)X & F_i & X C_i^T \\ * & -\gamma^2 I & 0 \\ * & * & -I \end{bmatrix} < 0 \quad (18)$$

Substituting $\Gamma_i = K_i X$ into (18), (10) is obtained. \blacksquare

B. Switched H_∞ State Feedback Control: Discrete-Time Case

The second objective is to determine the gains of discrete-time state feedback control (8) for system (5) in order to stabilize the following closed-loop system

$$\begin{aligned} x(k+1) &= \sum_{i=1}^M \alpha_i(k) [(\bar{A}_i - \bar{B}_i \bar{K}_i)x(k) + \bar{F}_i \omega(k)] \\ y(k) &= \sum_{i=1}^M \alpha_i(k) \bar{C}_i x(k) \end{aligned} \quad (19)$$

and satisfies C4. For this, we establish the following second theorem.

Theorem 2: Suppose that for $(i, j) \in \{1, \dots, M\}^2$, the pair (\bar{A}_i, \bar{B}_i) is controllable. If there exist a constant $\gamma > 0$, symmetric positive definite matrices $X_i \in \mathbb{R}^{n \times n}$ and matrices $U_i \in \mathbb{R}^{m \times n}$ such that the following inequality is satisfied for $(i, j) \in \{1, \dots, M\}^2$:

$$\begin{bmatrix} -X_i & 0 & X_i \bar{A}_i^T - U_i^T \bar{B}_i^T & X_i \bar{C}_i^T \\ * & -\gamma^2 I & \bar{F}_i^T & 0 \\ * & * & -X_j & 0 \\ * & * & * & -I \end{bmatrix} < 0 \quad (20)$$

then, the gains of discrete-time switched state feedback controller are given by $\bar{K}_i = U_i X_i^{-1}$.

Proof: To give sufficient conditions for the existence of (8) such that the closed-loop system (19) satisfies C3 and C4, let us consider the following switched Lyapunov functions:

$$V(x, k) = \sum_{i=1}^M \alpha_i(k) x^T(k) P_i x(k) \quad (21)$$

For this problem, a sufficient condition is related to the existence of a switched Lyapunov function V_k such that the following inequality should be verified

$$V(k+1) - V(k) + y^T(k)y(k) - \gamma^2 \omega^T(k)\omega(k) < 0 \quad (22)$$

Then, we compute the difference $V(k+1) - V(k)$ along (19), so that (22) becomes

$$\begin{aligned} & \sum_{j=1}^M \alpha_j(k+1) x^T(k+1) P_j x(k+1) \\ & - \sum_{i=1}^M \alpha_i(k) x^T(k) P_i x(k) + y^T(k)y(k) \\ & - \gamma^2 \omega^T(k)\omega(k) < 0 \end{aligned} \quad (23)$$

To take into account all possible switching, the following switching rule is considered:

$$\begin{cases} \alpha_i(k) = 1 \text{ and } \alpha_{l \neq i}(k) = 0 \\ \alpha_j(k+1) = 1 \text{ and } \alpha_{l \neq j}(k+1) = 0 \end{cases}$$

Then, (19) and (23) become respectively

$$\begin{cases} x(k+1) = (\bar{A}_i - \bar{B}_i \bar{K}_i)x(k) + \bar{F}_i \omega(k) \\ y(k) = \bar{C}_i x(k) \end{cases} \quad (24)$$

$$x^T(k+1) P_j x(k+1) - x^T(k) P_i x(k) + y^T(k)y(k) - \gamma^2 \omega^T(k)\omega(k) < 0 \quad (25)$$

From (24) and (25), we obtain

$$(25) \Leftrightarrow \begin{bmatrix} (\bar{A}_i - \bar{B}_i \bar{K}_i)x(k) + \bar{F}_i \omega(k) \\ (\bar{A}_i - \bar{B}_i \bar{K}_i)x(k) + \bar{F}_i \omega(k) \\ -x^T(k) P_i x(k) + y^T(k)y(k) \\ -\gamma^2 \omega^T(k)\omega(k) \end{bmatrix}^T P_j \times \quad (26)$$

which can be rewritten as

$$\begin{bmatrix} x(k) & \omega(k) \end{bmatrix} \times \begin{bmatrix} \Pi_{i,j} - P_i & (\bar{A}_i - \bar{B}_i \bar{K}_i)^T P_j \bar{F}_i \\ \bar{F}_i^T P_j (\bar{A}_i - \bar{B}_i \bar{K}_i) & -\gamma^2 I + \bar{F}_i^T P_j \bar{F}_i \end{bmatrix} \begin{bmatrix} x(k) \\ \omega(k) \end{bmatrix} \quad (27)$$

where $\Pi_{i,j} = (\bar{A}_i - \bar{B}_i \bar{K}_i)^T P_j (\bar{A}_i - \bar{B}_i \bar{K}_i) + \bar{C}_i^T \bar{C}_i$. It follows that $\Delta V(x, k) < 0$ for any nonzero vector $\begin{bmatrix} x(k) & \omega(k) \end{bmatrix}$ if

$$\begin{bmatrix} \Pi_{i,j} - P_i & (\bar{A}_i - \bar{B}_i \bar{K}_i)^T P_j \bar{F}_i \\ \bar{F}_i^T P_j (\bar{A}_i - \bar{B}_i \bar{K}_i) & -\gamma^2 I + \bar{F}_i^T P_j \bar{F}_i \end{bmatrix} < 0 \quad (28)$$

which is equivalent to

$$\begin{bmatrix} (\bar{A}_i - \bar{B}_i \bar{K}_i) & \bar{F}_i \\ C_i & 0 \end{bmatrix}^T \begin{bmatrix} P_j & 0 \\ 0 & I \end{bmatrix} \times \begin{bmatrix} (\bar{A}_i - \bar{B}_i \bar{K}_i) & \bar{F}_i \\ C_i & 0 \end{bmatrix} + \begin{bmatrix} -P_i & 0 \\ 0 & -\gamma^2 I \end{bmatrix} < 0 \quad (29)$$

using the Schur complement, (29) can be rewritten as

$$\begin{bmatrix} -P_i & 0 & (\bar{A}_i - \bar{B}_i \bar{K}_i)^T & \bar{C}_i^T \\ * & -\gamma^2 I & \bar{F}_i^T & 0 \\ * & * & -P_j^{-1} & 0 \\ * & * & * & -I \end{bmatrix} < 0 \quad (30)$$

Now, consider the matrices $Z_i = \text{diag}(P_i^{-1}, I, I, I)$, the LMIs (30) becomes:

$$Z_i^T \begin{bmatrix} -P_i & 0 & (\bar{A}_i - \bar{B}_i \bar{K}_i)^T & \bar{C}_i^T \\ * & -\gamma^2 I & \bar{F}_i^T & 0 \\ * & * & -P_j^{-1} & 0 \\ * & * & * & -I \end{bmatrix} Z_i < 0 \quad (31)$$

which is equivalent to

$$\begin{bmatrix} -P_i^{-1} & 0 & P_i^{-1}(\bar{A}_i - \bar{B}_i \bar{K}_i)^T & P_i^{-1} \bar{C}_i^T \\ * & -\gamma^2 I & \bar{F}_i^T & 0 \\ * & * & -P_j^{-1} & 0 \\ * & * & * & -I \end{bmatrix} < 0 \quad (32)$$

Now, let us define the change of variables $X_i = P_i^{-1}$ and $X_j = P_j^{-1}$, (32) becomes

$$\begin{bmatrix} -X_i & 0 & X_i(\bar{A}_i - \bar{B}_i \bar{K}_i)^T & X_i \bar{C}_i^T \\ * & -\gamma^2 I & \bar{F}_i^T & 0 \\ * & * & -X_j & 0 \\ * & * & * & -I \end{bmatrix} < 0 \quad (33)$$

Substituting $U_i = \bar{K}_i X_i$ into (33), (20) is obtained. ■

IV. SIMULATION RESULTS USING EXPERIMENTAL DATA

In this section, a validation of the two control laws is performed using experimental data. These data are used to construct the reference trajectories for lateral deviation and yaw motion of the vehicle. The main experimental data used for our simulations are: driver steering angle, longitudinal and lateral speeds, yaw rate. The measurements are acquired by laboratory car with acquisition device operating at frequency 200 Hz.

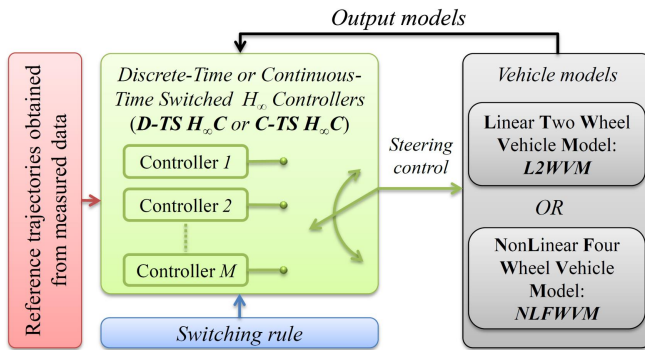


Fig. 1. A diagram block of continuous-time or discrete-time switched H_∞ steering vehicle controllers coupled with two vehicle models.

Figure 1 shows three specific parts: reference path, two switched H_∞ steering vehicle controllers and two vehicle models (L2WVM and NLFWVM). As mentioned previously, the real data are used in our validation to compute the reference trajectories, then, we compare the measurements with the obtained results vehicle models controlled with switched controllers according to the diagram of figure 1.

The simulation tests of continuous-time and discrete-time switched H_∞ controllers (D-TSH $_\infty$ C and C-TSH $_\infty$ C) are conducted under high lateral accelerations and using a

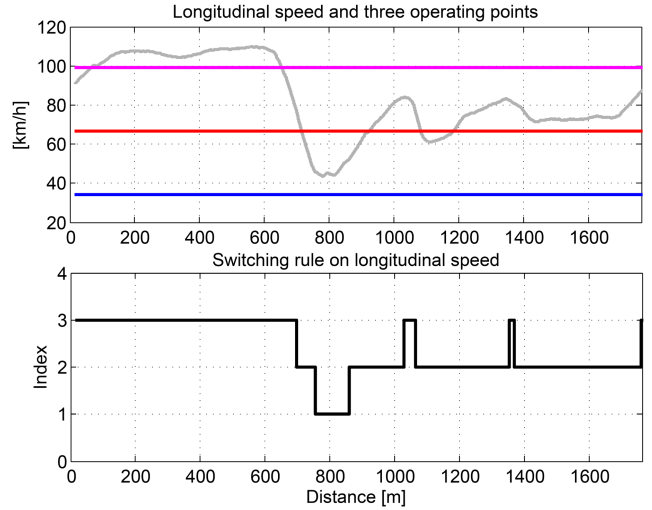


Fig. 2. Measured longitudinal speed with three operating points and switching rule

10DoF Non Linear Four Wheels Vehicle Model (NLFWM). This model is used as a complete vehicle simulator and is composed by longitudinal V_x , lateral V_y and vertical V_z translational motions, roll ϕ , pitch θ and yaw ψ rotational motions and dynamical models of the four wheels. To simulate extreme driving situations with high lateral accelerations using NLFWM, the coupled nonlinear tire model of Pacejka [13] is used. This tire model takes into account the coupling of longitudinal slip ratio, lateral sideslip angles, vertical forces and camber angle.

In order to show the performances of two switched controllers, only the longitudinal speed V_x variation is considered. Three values of V_x are chosen which are $V_{x1} = 33\text{km/h}$, $V_{x2} = 66\text{km/h}$ and $V_{x3} = 99\text{km/h}$ (see upper part of figure 2). Consequently, three local models are obtained, then, $M = 3$ and $(i, j) \in \{1, 2, 3\}^2$. Then, the stability of two switched H_∞ state feedback controllers (7) and (8) are guaranteed by the resolution of 3 LMIs of theorem 1 and of 9 LMIs of theorem 2 in order to obtain a common Lyapunov matrix P and three Lyapunov matrices (X_1 , X_2 and X_3) respectively. It should be pointed out that the LMIs were solved using YALMIP software [12].

The results presented here are obtained in two steps: firstly, two controllers are coupled with two L2WVMs (for lateral variables see figures 3 and 4, for trajectories and tracking errors see figures 8 and 9), secondly, two controllers are tested using two NLFWVMs (for longitudinal and lateral variables see figures 5, 6, then, for trajectories and tracking errors see 7, and figures 8 and 9). The closed-loop results obtained using D-TSH $_\infty$ C + L2WVM, D-TSH $_\infty$ C + NLFWVM, C-TSH $_\infty$ C + L2WVM and C-TSH $_\infty$ C + NLFWVM are similar to the measured ones, this, for any given vehicle model (L2WVM or NLFWVM). Notice that the measurements are presented by solid gray curves and simulated results are presented by dash-dot blue curves and dash black curves. The main dynamical variables plotted on these figures are: lateral accelerations, yaw rates, longitudinal accelerations,

longitudinal speed, X-Y trajectories.

Figure 9 shows the performances results of two controllers in terms of trajectories tracking. These errors are less than $0.2m$ for lateral deviation errors and $2deg$ on yaw angle errors. These errors are quite small and satisfactory for any vehicle model used. However, the trajectories tracking obtained with D-TSH $_{\infty}$ C are less than those obtained with C-TSH $_{\infty}$ C, this may be due to the use of common Lyapunov function for design of C-TSH $_{\infty}$ C.

Figures 4 and 7 show the steering control angles computed by D-TSH $_{\infty}$ C and C-TSH $_{\infty}$ C which are similar to the measured ones. This observation remains the same for any given vehicle model (L2WVM or NLFWVM).

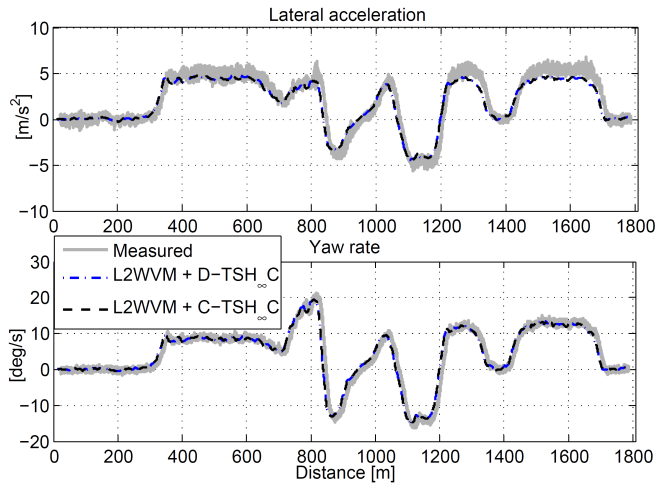


Fig. 3. lateral acceleration and yaw rate: measurements and computed by L2WVM + D-TSH $_{\infty}$ C and L2WVM + C-TSH $_{\infty}$ C

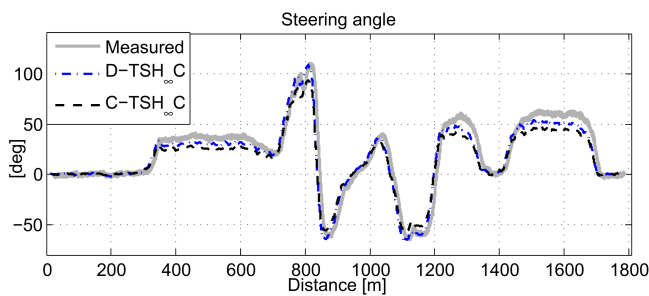


Fig. 4. Steering angle: Measured and computed by D-TSH $_{\infty}$ C + L2WVM and C-TSH $_{\infty}$ C + L2WVM

The unknown input attenuation properties can be observed on the transfer functions of subsystems $((A_3 - B_3K_3), F_3, C_3)$ for continuous-time case and $((\bar{A}_3 - \bar{B}_3\bar{K}_3), \bar{F}_3, \bar{C}_3)$ for discrete-time case, between ω to y given in figures 10 and 11. Notice that the UI attenuation properties of figures 10 and 11 are obtained for longitudinal speed equal to $V_{x3} = 99km/h$. Similar attenuation properties are obtained for two other values $V_{x1} = 33km/h$ and $V_{x2} = 66km/h$.

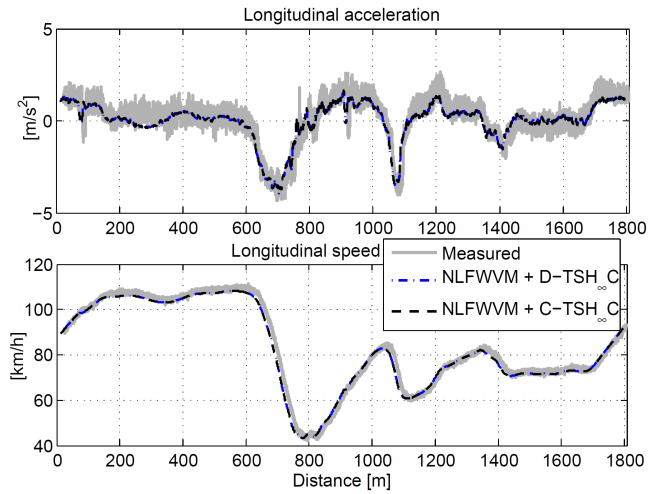


Fig. 5. Longitudinal acceleration and longitudinal speed: measurements and computed by NLFWVM + D-TSH $_{\infty}$ C and NLFWVM + C-TSH $_{\infty}$ C

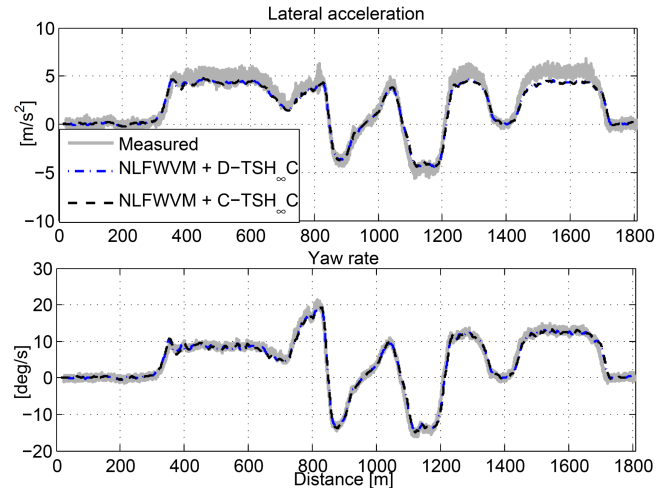


Fig. 6. lateral acceleration and yaw rate: measurements and computed by NLFWVM + D-TSH $_{\infty}$ C and NLFWVM + C-TSH $_{\infty}$ C

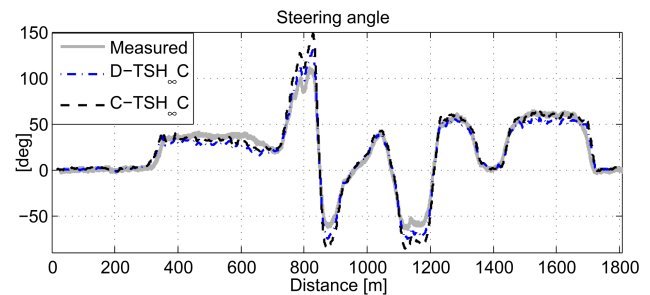


Fig. 7. Steering angle: Measured and computed by D-TSH $_{\infty}$ C + NLFWVM and C-TSH $_{\infty}$ C + NLFWVM

V. CONCLUSIONS AND FUTURE WORK

Two robust steering vehicle control laws based on continuous-time and discrete-time switching H $_{\infty}$ controllers are proposed. Using common Lyapunov function and switched Lyapunov function, sufficient conditions for the existence of two controllers are obtained in terms of linear matrix inequalities (LMIs). The gains of these controllers

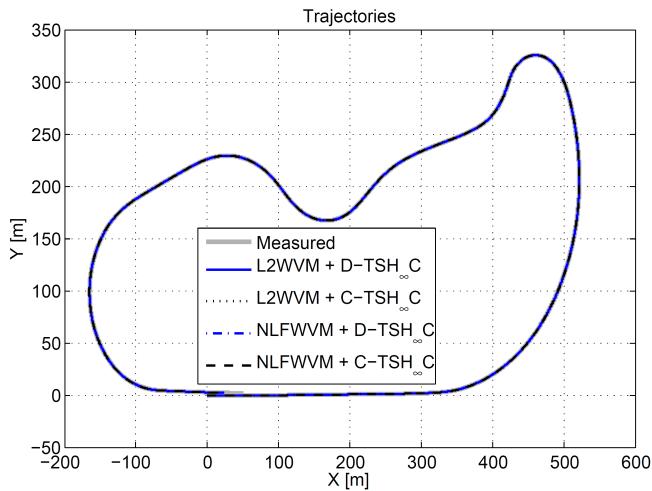


Fig. 8. Trajectories: Reference and computed by L2WVM+D-TSH_∞C, NLFWVM+D-TSH_∞C, L2WVM+C-TSH_∞C and NLFWVM+C-TSH_∞C

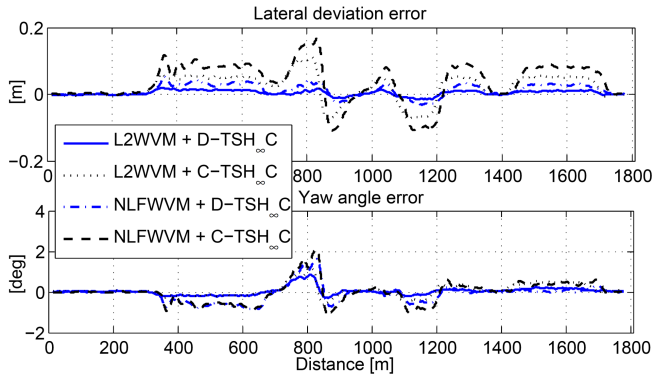


Fig. 9. Tracking trajectory errors on lateral deviation and yaw angle of L2WVM+D-TSH_∞C, NLFWVM+D-TSH_∞C, L2WVM+C-TSH_∞C and NLFWVM+C-TSH_∞C

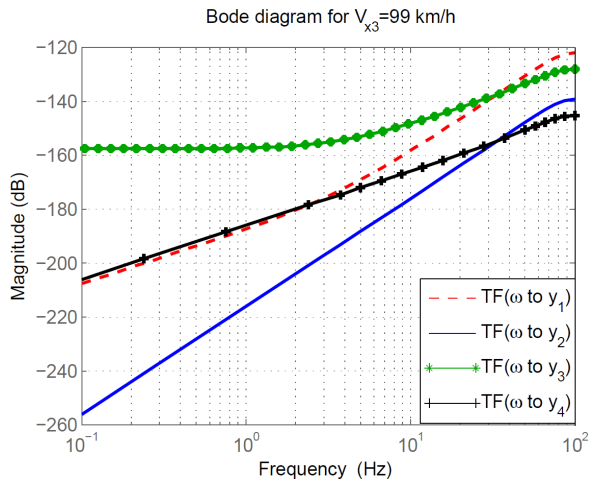


Fig. 10. Transfer function of system $((A_i - B_1 K_i), F_i, C_i)$ for $V_x = 99 \text{ km/h}$, between ω to y

are obtained by solving of a set of linear matrix inequalities (LMIs). The proposed approaches are tested on vehicle steering control using the experimental data. These simulations and robustness tests are performed under high lateral acceleration and using two kinds of vehicle models.

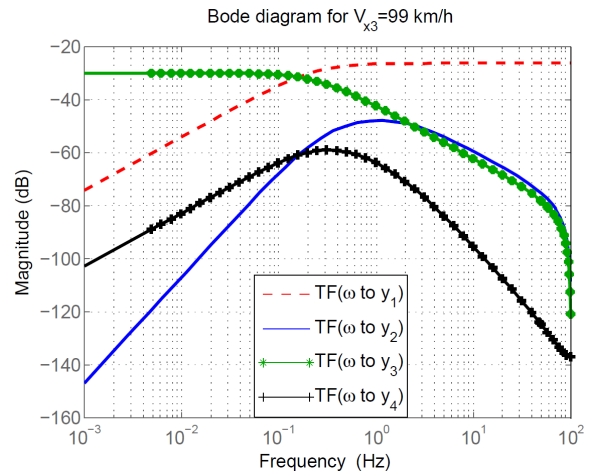


Fig. 11. Transfer function of system $((\bar{A}_i - \bar{B}_i \bar{K}_i), \bar{F}_i, \bar{C}_i)$ for $V_x = 99 \text{ km/h}$, between ω to y

To improve the robustness of two controllers, it will be interesting to define a switching rule on tire cornering stiffnesses C_f and C_r , in order to solve partially the linearity assumption of lateral tire forces.

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