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Actuator Faults Reconstruction Using Reduced-Order Fuzzy Observer Structures

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Abstract— **This paper focuses on the principle for designing reduced-order fuzzy-observer-based actuator fault reconstruction for nonlinear systems. The problem addressed can be indicated as the generalized approach for a kind of reduced-order fuzzy observer design with special gain matrix structure that depends on the given matching conditions. Using the Lyapunov theory, design conditions are obtained and expressed in terms of linear matrix inequalities, and the terms for uniform estimation of the actuator faults are given.**

I. INTRODUCTION

In the observer-based sensor and actuator fault estimation in nonlinear systems, there are primarily used adaptive estimator structures (see e.g. [16], [20], [24]). An alternative is the Takagi-Sugeno (TS) fuzzy approximation of the nonlinear model equations. Using TS fuzzy model [14], a nonlinear system is represented by the fuzzy rules, each rule utilizes the local system dynamics by a linear model, and the nonlinear system is represented by a collection of the rules.

Observers based on TS fuzzy models are realized in the same structures as the linear observers [13], [15], and the design principles usually used techniques based on the linear matrix inequalities (LMI). Hence, research in TS fuzzy observers application in fault diagnosis is the subject of widely scattered publications (see e.g. [1], [11], [12]). Since the set of multi-structured observers have to be used to allow fault isolation [2], [3], [22], the fault estimation is recommended, especially for the actuator fault reconstruction [4], [8], [19], [23].

To estimate actuator faults, the adaptive state and actuator fault TS observers are used (see, e.g., [6], [21]). It should be pointed out that two set of the observer gains have to be designed using this approach and a fault estimation time is negatively affected by interaction of two observers dynamics. To exploit faster dynamics of reduced order TS observers, new simple approach was proposed for actuator fault estimation in [9]. The current paper extends this approach with respect to the systems with nonlinear control input as well as nonlinear fault input. The reduced-order observer design problem results in a convex LMI feasibility, in addition, the convergence analysis the actuator fault reconstruction scheme is developed to guarantee uniform estimation of actuator faults. The resulting numerical experience indicates that fault estimation can be designed following the presented approach.

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The remainder of this paper is organized as follows. Section II. and III. describe TS fuzzy model properties and the preliminaries. The design principle of the reduced-order TS observer is outlined in Section IV. and the actuator fault reconstruction, using reduced-order fuzzy observer, is given in Section V. In Section VI. one illustrative example is given and Section VII. draws some concluding remarks.

Throughout the paper, the following notations are used: $\boldsymbol{x}^T$, $\boldsymbol{X}^T$ denotes the transpose of the vector $\boldsymbol{x}$ and matrix $\boldsymbol{X}$, respectively, diag $[\,\cdot\,]$ denotes a block diagonal matrix, $\boldsymbol{X} = \boldsymbol{X}^T > 0$ (respectively $\boldsymbol{X} = \boldsymbol{X}^T < 0$) means that $\boldsymbol{X}$ is a symmetric positive definite matrix (respectively, symmetric negative definite matrix), the symbol $\boldsymbol{I}_n$ represents the n-th order unit matrix, $I\!R$ denotes the set of real numbers and $I\!R^{n\times r}$ denotes the set of all $n \times r$ real matrices.

II. TAKAGI-SUGENO FUZZY MODELS

The systems under consideration fall in a class of nonlinear dynamic systems, described as follows

$$\dot{\boldsymbol{q}}(t) = \mathbf{a}(\boldsymbol{q}(t)) + \mathbf{B}(\boldsymbol{q}(t))\boldsymbol{u}(t) + \mathbf{B}_f(\boldsymbol{q}(t))\boldsymbol{f}(t)) \quad (1)$$

$$\boldsymbol{y}(t) = \boldsymbol{C}\boldsymbol{q}(t) \quad (2)$$

where $\boldsymbol{q}(t) \in I\!R^n$, $\boldsymbol{u}(t)$, $\boldsymbol{f}(t) \in I\!R^r$, and $\boldsymbol{y}(t) \in I\!R^m$ are vectors of the state, input, actuator fault, and output variables, respectively, $\boldsymbol{C} \in I\!R^{m\times n}$ is a real matrix, $\mathbf{a}(\boldsymbol{q}(t)) \in I\!R^n$, $\mathbf{B}(\boldsymbol{q}(t)) \in I\!R^{n\times r}$, $\mathbf{B}_f(\boldsymbol{q}(t)) \in I\!R^{n\times r}$ are bounded nonlinear functions of $\boldsymbol{q}(t)$. It is assumed that $\mathbf{a}(\boldsymbol{q}(t))$ is bounded in associated sectors, $\mathbf{a}(0) = 0$, only actuator faults can occur, and if no actuator fault is occurred then $\boldsymbol{f}(t) = \mathbf{0},\ \forall t \geq 0$.

It is considered that the number of the nonlinear terms in the vector function $\mathbf{a}(\boldsymbol{q}(t))$ is p, and there exist the nonlinear sector functions $\{\mathrm{w}_{lj}(\theta_j(t)),\ l = 1,2,\ldots,p,\ j = 1,2,\ldots,k\}$ such that

$$\mathrm{w}_{lk}(\boldsymbol{\theta}(t)) = 1 - \sum_{j=1}^{k-1} \mathrm{w}_{lj}(\theta_j(t)) \quad (3)$$

where k is the number of sector functions, and

$$\boldsymbol{\theta}(t) = \left[\begin{array}{cccc} \theta_1(t) & \theta_2(t) & \cdots & \theta_q(t) \end{array}\right] \quad (4)$$

is the vector of premise variables. Thus, constructing the set of membership functions $\mathrm{w}_i(\boldsymbol{\theta}(t)) = \prod_{l=1|j}^{k} \mathrm{w}_{lj}(\theta_j(t))$, $i = 1,2,\ldots,s$, $s \in \langle 2^p, k^p\rangle$, from all combinations of the sector functions, the state of the system is inferred as follows

$$\dot{\boldsymbol{q}}(t) = \sum_{i=1}^{s} \mathrm{h}_i(\boldsymbol{\theta}(t))(\boldsymbol{A}_i\boldsymbol{q}(t) + \boldsymbol{B}_i\boldsymbol{u}(t) + \boldsymbol{B}_{fi}\boldsymbol{f}(t)) \quad (5)$$

$$\boldsymbol{y}(t) = \boldsymbol{C}\boldsymbol{q}(t) \tag{6}$$

where the system output is given by the relation (6), and

$$\mathrm{h}_i(\boldsymbol{\theta}(t)) = \mathrm{w}_i(\boldsymbol{\theta}(t)) / \sum_{i=1}^{s} \mathrm{w}_i(\boldsymbol{\theta}(t)) \tag{7}$$

is the averaging weight for the i-th rule. By definition, the membership functions satisfy the convex sum property

$$0 \leq \mathrm{h}_i(\boldsymbol{\theta}(t)) \leq 1, \ \sum_{i=1}^{s} \mathrm{h}_i(\boldsymbol{\theta}(t)) = 1 \ \forall i \in \langle 1, \dots, s \rangle \tag{8}$$

The above introduced, $\boldsymbol{A}_i \in I\!R^{n \times n}$ is the Jacobian matrix of $\boldsymbol{a}(\boldsymbol{r}_i)$, $\boldsymbol{B}_i \in I\!R^{n \times r}$, $\boldsymbol{B}_{fi} \in I\!R^{n \times r}$ are the matrices equal to $\mathbf{B}(\boldsymbol{r}_i)$, $\mathbf{B}_f(\boldsymbol{r}_i)$, respectively, where $\boldsymbol{r}_i$ is the vector of the i-th combination of the sector boundaries. It is supposed that TS model does not include parameter uncertainties, or external disturbances, all premise and output variables are measurable, a premise variable is the system state variable or a measurable external variable, but not any input variable.

III. Basic Preliminaries

Definition 1: Let $\boldsymbol{E} \in I\!R^{h \times h}$, $\mathrm{rank}(\boldsymbol{E}) = k < h$ be a rank deficient matrix. Then the null space $N_{\boldsymbol{E}}$ of $\boldsymbol{E}$ is the orthogonal complement of the row space of $\boldsymbol{E}$.

Lemma 1: If $\boldsymbol{E} \in I\!R^{h \times h}$, $\mathrm{rank}(\boldsymbol{E}) = k < h$, is a rank deficient matrix, an orthogonal complement $\boldsymbol{E}^{\perp}$ of $\boldsymbol{E}$ is

$$\boldsymbol{E}^{\perp} = \boldsymbol{E}^{\circ} \boldsymbol{U}_{E2}^T \tag{9}$$

where $\boldsymbol{U}_{E2}^T$ is the null space of $\boldsymbol{E}$ and $\boldsymbol{E}^{\circ}$ is an arbitrary matrix of appropriate dimension.

Proof: See, e.g., [7].

Lemma 2: If $\boldsymbol{M}$, $\boldsymbol{N}$ are matrices of appropriate dimension, and $\boldsymbol{X}$ is a symmetric positive definite matrix, then

$$\boldsymbol{M}^T\boldsymbol{X}\boldsymbol{N} + \boldsymbol{N}^T\boldsymbol{X}\boldsymbol{M} \leq \boldsymbol{N}^T\boldsymbol{X}\boldsymbol{N} + \boldsymbol{M}^T\boldsymbol{X}\boldsymbol{M} \tag{10}$$

Proof: Since $\boldsymbol{X} = \boldsymbol{X}^T > 0$, then

$$\big(\boldsymbol{X}^{-\frac{1}{2}}\boldsymbol{X}\boldsymbol{M} - \boldsymbol{X}^{\frac{1}{2}}\boldsymbol{N}\big)^T\big(\boldsymbol{X}^{-\frac{1}{2}}\boldsymbol{X}\boldsymbol{M} - \boldsymbol{X}^{\frac{1}{2}}\boldsymbol{N}\big) \geq 0 \tag{11}$$

$$\boldsymbol{M}^T\boldsymbol{X}\boldsymbol{X}^{-1}\boldsymbol{X}\boldsymbol{M} + \boldsymbol{N}^T\boldsymbol{X}\boldsymbol{N} - \boldsymbol{M}^T\boldsymbol{X}\boldsymbol{N} - \boldsymbol{N}^T\boldsymbol{X}\boldsymbol{M} \geq 0 \tag{12}$$

respectively. It is evident that (12) implies (10). ■

Lemma 3: If $\boldsymbol{X}$ is a symmetric positive definite matrix, $\boldsymbol{Y}_i$ are matrices of appropriate dimension, $h_i \geq 0$ is a real scalar and s is a positive integer, then for $\sum_{j=1}^{s} h_j > 0$

$$\Big(\sum_{i=1}^{s} h_i \boldsymbol{Y}_i^T\Big)\boldsymbol{X}\Big(\sum_{i=1}^{s} h_i \boldsymbol{Y}_i\Big) \leq \sum_{i=1}^{s} h_i \boldsymbol{Y}_i^T\boldsymbol{X}\boldsymbol{Y}_i \Big(\sum_{j=1}^{s} h_j\Big) \tag{13}$$

Proof: Since with (13) it can write

$$\Big(\sum_{i=1}^{s} h_i \boldsymbol{Y}_i^T\Big)\boldsymbol{X}\Big(\sum_{i=1}^{s} h_i \boldsymbol{Y}_i\Big) = \sum_{i=1}^{s}\sum_{j=1}^{s} h_i h_j \boldsymbol{Y}_i^T\boldsymbol{X}\boldsymbol{Y}_j \tag{14}$$

then, using (10) with $\boldsymbol{M}^T = \boldsymbol{Y}_i^T$, $\boldsymbol{N} = \boldsymbol{Y}_j$, it is

$$\begin{aligned} &\sum_{i=1}^{s}\sum_{j=1}^{s} \tfrac{1}{2} h_i h_j \boldsymbol{Y}_i^T\boldsymbol{X}\boldsymbol{Y}_j + \sum_{i=1}^{s}\sum_{j=1}^{s} \tfrac{1}{2} h_i h_j \boldsymbol{Y}_j^T\boldsymbol{X}\boldsymbol{Y}_i \leq \\ &\leq \sum_{i=1}^{s}\sum_{j=1}^{s} \tfrac{1}{2} h_i h_j \big(\boldsymbol{Y}_i^T\boldsymbol{X}\boldsymbol{Y}_i + \boldsymbol{Y}_j^T\boldsymbol{X}\boldsymbol{Y}_j\big) = \\ &= \sum_{i=1}^{s} h_i \Big(\sum_{j=1}^{s} \tfrac{1}{2} h_j \boldsymbol{Y}_j^T\boldsymbol{X}\boldsymbol{Y}_j\Big) + \sum_{j=1}^{s} h_j \Big(\sum_{i=1}^{s} \tfrac{1}{2} h_i \boldsymbol{Y}_i^T\boldsymbol{X}\boldsymbol{Y}_i\Big) \end{aligned} \tag{15}$$

Evidently, (15) implies (13). ■

Lemma 4: (Congruence transform) Let the system output matrix $\boldsymbol{C}$ is of full raw rank, $rank\,\boldsymbol{C} = m$, then there exists a new coordinate system such that $\boldsymbol{C}$ takes the structure $\boldsymbol{C}_a = \begin{bmatrix} \boldsymbol{I}_m & \boldsymbol{0} \end{bmatrix}$

Proof: (compare, e.g., [17]) Applying singular value decomposition (SVD) to $\boldsymbol{C}$ gives

$$\boldsymbol{C} = \boldsymbol{U}\begin{bmatrix} \boldsymbol{S} & \boldsymbol{0} \end{bmatrix}\boldsymbol{V}^T = \boldsymbol{U}\boldsymbol{S}\begin{bmatrix} \boldsymbol{I}_m & \boldsymbol{0} \end{bmatrix}\boldsymbol{V}^T \tag{16}$$

where rows of $\boldsymbol{U}^T \in I\!R^{m \times m}$ are the left singular vectors of $\boldsymbol{C}$, and columns of $\boldsymbol{V} \in I\!R^{n \times n}$ are the right singular vectors of $\boldsymbol{C}$, all ordered in such way to be associated with the singular values of $\boldsymbol{C}$, written as diagonal elements of $\boldsymbol{S} \in I\!R^{m \times m}$,

$$\boldsymbol{S} = \mathrm{diag}\begin{bmatrix} \sigma_1 & \cdots & \sigma_m \end{bmatrix}, \ \sigma_1 \geq \cdots \geq \sigma_m > 0 \tag{17}$$

Using the notations

$$\boldsymbol{W}^{-1} = \boldsymbol{U}\boldsymbol{S}, \quad \boldsymbol{T}_a = \boldsymbol{V}^T, \quad \boldsymbol{C}_a = \begin{bmatrix} \boldsymbol{I}_m & \boldsymbol{0} \end{bmatrix} \tag{18}$$

$$\boldsymbol{T}_a^T = \begin{bmatrix} \boldsymbol{T}_{a1}^T & \boldsymbol{T}_{a2}^T \end{bmatrix}, \quad \boldsymbol{T}_{a1}^T \in I\!R^{n \times m} \tag{19}$$

then (16) implies

$$\boldsymbol{C} = \boldsymbol{W}^{-1}\boldsymbol{C}_a\boldsymbol{T}_a, \quad \boldsymbol{C}_a = \boldsymbol{W}\boldsymbol{C}\boldsymbol{T}_a^{-1} \tag{20}$$

This concludes the proof. ■

Note, if $\boldsymbol{C}$ is of rank m, $\boldsymbol{W} \in I\!R^{m \times m}$ is a regular matrix, and $\boldsymbol{T}_a \in I\!R^{n \times n}$ is a matrix such that $\boldsymbol{T}_a^{-1} = \boldsymbol{T}_a^T = \boldsymbol{V}$.

Lemma 5: Using the transform (20), each linear submodel of (5), (6) can be partitioned such that

$$\begin{aligned} \boldsymbol{q}_{ai}^{\bullet}(t) &= \begin{bmatrix} \boldsymbol{q}_{a1i}^{\bullet}(t) \\ \boldsymbol{q}_{a2i}^{\bullet}(t) \end{bmatrix} = \begin{bmatrix} \boldsymbol{A}_{a11i} & \boldsymbol{A}_{a12i} \\ \boldsymbol{A}_{a21i} & \boldsymbol{A}_{a22i} \end{bmatrix}\begin{bmatrix} \boldsymbol{q}_{a1}(t) \\ \boldsymbol{q}_{a2}(t) \end{bmatrix} + \\ &+ \begin{bmatrix} \boldsymbol{B}_{a1i} \\ \boldsymbol{B}_{a2i} \end{bmatrix}\boldsymbol{u}(t) + \begin{bmatrix} \boldsymbol{B}_{af1i} \\ \boldsymbol{B}_{af2i} \end{bmatrix}\boldsymbol{f}(t) \end{aligned} \tag{21}$$

$$\dot{\boldsymbol{q}}_a(t) = \sum_{i=1}^{s} h_i(\boldsymbol{\theta}(t))\boldsymbol{q}_{ai}^{\bullet}(t), \quad \boldsymbol{y}(t) = \boldsymbol{W}^{-1}\boldsymbol{v}(t) \tag{22}$$

$$\boldsymbol{v}(t) = \begin{bmatrix} \boldsymbol{I}_m & \boldsymbol{0} \end{bmatrix}\boldsymbol{q}_a(t) = \begin{bmatrix} \boldsymbol{I}_m & \boldsymbol{0} \end{bmatrix}\begin{bmatrix} \boldsymbol{q}_{a1}(t) \\ \boldsymbol{q}_{a2}(t) \end{bmatrix} \tag{23}$$

$$\boldsymbol{q}_a^T(t) = \begin{bmatrix} \boldsymbol{q}_{a1}^T(t) & \boldsymbol{q}_{a2}^T(t) \end{bmatrix}, \quad \boldsymbol{q}_a(t) = \boldsymbol{T}_a\boldsymbol{q}(t) \tag{24}$$

where $\boldsymbol{q}_{a1i}(t) \in I\!R^{m}$, $\boldsymbol{q}_{a2i}(t) \in I\!R^{n-m}$, $\boldsymbol{B}_{a1i} \in I\!R^{m \times r}$, $\boldsymbol{B}_{af1i} \in I\!R^{m \times r}$, respectively.

Proof: Substituting (20) into (6) gives

$$\boldsymbol{y}(t) = \boldsymbol{W}^{-1}\boldsymbol{C}_a\boldsymbol{T}_a\boldsymbol{q}(t) = \boldsymbol{W}^{-1}\boldsymbol{C}_a\boldsymbol{q}_a(t) \tag{25}$$

Thus, using

$$\boldsymbol{v}(t) = \boldsymbol{C}_a\boldsymbol{q}_a(t) \tag{26}$$

(25) implies (22), and with (19), (20), then (26) implies (23).

Inserting (24) in (5), it can be obtained

$$\dot{\boldsymbol{q}}_a(t) = \sum_{i=1}^{s} h_i(\boldsymbol{\theta}(t))(\boldsymbol{A}_{ai}\boldsymbol{q}_a(t) + \boldsymbol{B}_{ai}\boldsymbol{u}(t) + \boldsymbol{B}_{afi}\boldsymbol{f}(t)) \tag{27}$$

$$\boldsymbol{A}_{ai} = \boldsymbol{T}_a\boldsymbol{A}_i\boldsymbol{T}_a^{-1}, \ \boldsymbol{B}_{ai} = \boldsymbol{T}_a\boldsymbol{B}_i, \ \boldsymbol{B}_{afi} = \boldsymbol{T}_a\boldsymbol{B}_{fi} \tag{28}$$

and partitioning accordingly to (23), (28) implies (21). ■

Proposition 1: (matching condition) The actuator fault input matrices $\boldsymbol{B}_{afi}$, and the output matrix $\boldsymbol{C}_a$ satisfies the conditions: $\operatorname{rank}\boldsymbol{B}_{afi} = r,\ \forall i,\ \operatorname{rank}\boldsymbol{C}_a > \operatorname{rank}\boldsymbol{B}_{afi}$, respectively, and the matrix $\boldsymbol{B}_{afi}$ takes the structure

$$\boldsymbol{B}_{afi} = \boldsymbol{C}_a^T \boldsymbol{B}_{af1i} \tag{29}$$

The matching condition, given in Proposition 1. seems to be restrictive theoretically, but for many practical systems it is satisfied. In addition, comparing with the static decoupling control principle [18], the condition reflects inserting at least one redundant sensor into the sensor structure.

IV. Reduced Order TS Fuzzy Observer

Theorem 1: Considering the affine TS fuzzy system (5), (6), then the reduced TS fuzzy observer takes the form

$$\dot{\boldsymbol{p}}_{2e}(t) = \sum_{i=1}^{s} h_i(\boldsymbol{\theta}(t))\boldsymbol{q}^{\circ}_{2ei}(t) \tag{30}$$

$$\boldsymbol{q}^{\circ}_{2ei}(t) = \boldsymbol{A}_{aei}\boldsymbol{p}_{2e}(t) + \boldsymbol{A}_{avi}\boldsymbol{v}(t) + \begin{bmatrix}-\boldsymbol{J}_i & \boldsymbol{I}_{n-m}\end{bmatrix}\boldsymbol{B}_{ai}\boldsymbol{u}(t) \tag{31}$$

$$\boldsymbol{q}_{a2e}(t) = \boldsymbol{p}_{2e}(t) + \sum_{i=1}^{s} h_i(\boldsymbol{\theta}(t))\boldsymbol{J}_i\boldsymbol{v}(t) \tag{32}$$

$$\boldsymbol{A}_{avi} = \boldsymbol{A}_{a21i} - \boldsymbol{J}_i\boldsymbol{A}_{a11i} + (\boldsymbol{A}_{a22i} - \boldsymbol{J}_i\boldsymbol{A}_{a12i})\sum_{j=1}^{s} h_j(\boldsymbol{\theta}(t))\boldsymbol{J}_j \tag{33}$$

$$\boldsymbol{A}_{aei} = \boldsymbol{A}_{a22i} - \boldsymbol{J}_i\boldsymbol{A}_{a12i} \tag{34}$$

and $\boldsymbol{J}_i \in I\!R^{(n-m)\times m},\ i = 1, 2, \dots, s$ is the set of gains.

Proof: Since (21) can be partitioned as

$$\boldsymbol{A}_{a12i}\boldsymbol{q}_{a2}(t) = \boldsymbol{q}^{\bullet}_{a1i}(t) - \boldsymbol{A}_{a11i}\boldsymbol{v}(t) - \boldsymbol{B}_{a1i}\boldsymbol{u}(t) \tag{35}$$

$$\boldsymbol{q}^{\bullet}_{a2i}(t) = \boldsymbol{A}_{a21i}\boldsymbol{v}(t) + \boldsymbol{A}_{a22i}\boldsymbol{q}_{a2}(t) + \boldsymbol{B}_{a2i}\boldsymbol{u}(t) \tag{36}$$

then the TS fuzzy observer can be defined as follows

$$\dot{\boldsymbol{q}}_{a2e}(t) = \sum_{i=1}^{s} h_i(\boldsymbol{\theta}(t))\boldsymbol{q}^{\star}_{a2ei}(t) \tag{37}$$

$$\begin{aligned}\boldsymbol{q}^{\star}_{a2ei}(t) &= \boldsymbol{A}_{a21i}\boldsymbol{v}(t) + \boldsymbol{A}_{a22i}\boldsymbol{q}_{a2e}(t) + \boldsymbol{B}_{a2i}\boldsymbol{u}(t) + \\ &+ \boldsymbol{J}_i\big(\boldsymbol{q}^{\bullet}_{a1i}(t) - \boldsymbol{A}_{a11i}\boldsymbol{v}(t) - \boldsymbol{B}_{a1i}\boldsymbol{u}(t) - \boldsymbol{A}_{a12i}\boldsymbol{q}_{a2e}(t)\big)\end{aligned} \tag{38}$$

where $\boldsymbol{q}_{a2e}(t) \in I\!R^{n-m}$ is an estimation of the unmeasurable part of $\boldsymbol{q}_a(t)$ and $\boldsymbol{J}_i,\ i = 1, 2, \dots, s,\ \boldsymbol{J}_i \in I\!R^{(n-m)\times m}$, is the set of the observer gain matrices. Now, (37), (38) can be rewritten as

$$\dot{\boldsymbol{q}}_{a2e}(t) - \sum_{i=1}^{s} h_i(\boldsymbol{\theta}(t))\boldsymbol{J}_i\boldsymbol{q}^{\bullet}_{a1i}(t) = \sum_{i=1}^{s} h_i(\boldsymbol{\theta}(t))\boldsymbol{q}^{\diamond}_{a2i}(t) \tag{39}$$

$$\begin{aligned}\boldsymbol{q}^{\diamond}_{a2i}(t) &= \boldsymbol{B}_{a2i}\boldsymbol{u}(t) + \boldsymbol{A}_{a21i}\boldsymbol{v}(t) + \\ &+ \boldsymbol{A}_{a22i}\left(\begin{array}{c}\boldsymbol{q}_{a2e}(t) - \sum_{j=1}^{s} h_j(\boldsymbol{\theta}(t))\boldsymbol{J}_j\boldsymbol{v}(t) + \\ + \sum_{j=1}^{s} h_j(\boldsymbol{\theta}(t))\boldsymbol{J}_j\boldsymbol{v}(t)\end{array}\right) + \\ &+ \boldsymbol{J}_i\left(\begin{array}{c}-\boldsymbol{A}_{a11i}\boldsymbol{v}(t) - \boldsymbol{B}_{a1i}\boldsymbol{u}(t) - \\ -\boldsymbol{A}_{a12i}\left(\begin{array}{c}\boldsymbol{q}_{a2e}(t) - \sum_{j=1}^{s} h_j(\boldsymbol{\theta}(t))\boldsymbol{J}_j\boldsymbol{v}(t) + \\ + \sum_{j=1}^{s} h_j(\boldsymbol{\theta}(t))\boldsymbol{J}_j\boldsymbol{v}(t)\end{array}\right)\end{array}\right)\end{aligned} \tag{40}$$

Defining the new state variable

$$\boldsymbol{p}_{2e}(t) = \boldsymbol{q}_{a2e}(t) - \sum_{i=1}^{s} h_i(\boldsymbol{\theta}(t))\boldsymbol{J}_i\boldsymbol{v}(t) \tag{41}$$

then (41) implies (32), and defining the left side of (39) as

$$\dot{\boldsymbol{p}}_{2e}(t) = \dot{\boldsymbol{q}}_{a2e}(t) - \sum_{i=1}^{s} h_i(\boldsymbol{\theta}(t))\boldsymbol{J}_i\boldsymbol{q}^{\bullet}_{a1i}(t) \tag{42}$$

it can be obtained

$$\dot{\boldsymbol{p}}_{2e}(t) = \sum_{i=1}^{s} h_i(\boldsymbol{\theta}(t))\boldsymbol{q}^{\circ}_{2ei} \tag{43}$$

$$\begin{aligned}\boldsymbol{q}^{\circ}_{2ei} &= (\boldsymbol{A}_{a22i} - \boldsymbol{J}_i\boldsymbol{A}_{a12i})\boldsymbol{p}_{2e}(t) + \\ &+ (\boldsymbol{B}_{a2i} - \boldsymbol{J}_i\boldsymbol{B}_{a1i})\boldsymbol{u}(t) + (\boldsymbol{A}_{a21i} - \boldsymbol{J}_i\boldsymbol{A}_{a11i})\boldsymbol{v}(t) + \\ &+ (\boldsymbol{A}_{a22i} - \boldsymbol{J}_i\boldsymbol{A}_{a12i})\sum_{j=1}^{s} h_j(\boldsymbol{\theta}(t))\boldsymbol{J}_j\boldsymbol{v}(t)\end{aligned} \tag{44}$$

Since

$$\boldsymbol{B}_{a2i} - \boldsymbol{J}_i\boldsymbol{B}_{a1i} = \begin{bmatrix}-\boldsymbol{J}_i & \boldsymbol{I}_{n-m}\end{bmatrix}\boldsymbol{B}_{ai} \tag{45}$$

with (33), (34) and (45) then (43), (44) implies (30), (31).

It is evident that

$$\boldsymbol{v}(t) = \boldsymbol{q}_{a1}(t) = \boldsymbol{p}_{a1}(t) \tag{46}$$

This concludes the proof. ■

Theorem 2: The reduced order TS fuzzy observer (30), (31) is asymptotically stable if there exists a symmetric positive definite matrix $\boldsymbol{P}^{\circ} \in I\!R^{(n-m)\times(n-m)}$ and matrices $\boldsymbol{W}^{\circ}_i \in I\!R^{(n-m)\times m},\ i = 1, 2, \dot{s}, s$ such that

$$\boldsymbol{P}^{\circ} = \boldsymbol{P}^{\circ T} > 0 \tag{47}$$

$$\boldsymbol{A}^T_{a22i}\boldsymbol{P}^{\circ} + \boldsymbol{P}^{\circ}\boldsymbol{A}_{a22i} - \boldsymbol{A}^T_{a12i}\boldsymbol{W}^{\circ T}_i - \boldsymbol{W}^{\circ}_i\boldsymbol{A}_{a12i} < 0 \tag{48}$$

If the above conditions hold, the set of the observer gain matrices is given as

$$\boldsymbol{J}_i = (\boldsymbol{P}^{\circ})^{-1}\boldsymbol{W}^{\circ}_i \tag{49}$$

Proof: Using (30), (31), it can be rewritten

$$\begin{aligned}&\dot{\boldsymbol{p}}_{2e}(t) = \\ &= \sum_{i=1}^{s} h_i(\boldsymbol{\theta}(t))(\boldsymbol{A}_{aei}\boldsymbol{p}_{2e}(t) + \boldsymbol{A}_{avi}\boldsymbol{v}(t) + (\boldsymbol{B}_{a2i} - \boldsymbol{J}_i\boldsymbol{B}_{a1i})\boldsymbol{u}(t))\end{aligned} \tag{50}$$

and, with (33), the autonomous part of (50) takes the form

$$\dot{\boldsymbol{p}}_{2e}(t) = \sum_{i=1}^{s} h_i(\boldsymbol{\theta}(t))(\boldsymbol{A}_{a22i} - \boldsymbol{J}_i\boldsymbol{A}_{a12i})\boldsymbol{p}_{2e}(t) \tag{51}$$

Defining the quadratic positive definite Lyapunov function

$$v(\boldsymbol{p}_{2e}(t)) = \boldsymbol{p}^T_{2e}(t)\boldsymbol{P}^{\circ}\boldsymbol{p}_{2e}(t) \tag{52}$$

where $\boldsymbol{P}^{\circ} = \boldsymbol{P}^{\circ T} > 0,\ \boldsymbol{P}^{\circ} \in I\!R^{(n-m)\times(n-m)}$ then, after evaluation of derivative with respect to t, it is obtained

$$\dot{v}(\boldsymbol{p}_{2e}(t)) = \boldsymbol{p}^T_{2e}(t)\boldsymbol{P}^{\circ}\dot{\boldsymbol{p}}_{2e}(t) + \dot{\boldsymbol{p}}^T_{2e}(t)\boldsymbol{P}^{\circ}\boldsymbol{p}_{2e}(t) < 0 \tag{53}$$

Substituting (51) into (53) gives

$$\dot{v}(\boldsymbol{p}_{2e}(t)) = \boldsymbol{p}^T_{2e}(t)\sum_{i=1}^{s} h_i(\boldsymbol{\theta}(t))\big(\boldsymbol{P}^{\circ}\boldsymbol{A}_{aei} + \boldsymbol{A}^T_{aei}\boldsymbol{P}^{\circ}\big)\boldsymbol{p}_{2e}(t) < 0 \tag{54}$$

respectively. Thus, (54) is negative, if there exist a set of matrices $\boldsymbol{J}_i,\ i=1,2,\dots,s$, and a matrix $\boldsymbol{P}^{\circ}$ such that

$$(\boldsymbol{A}_{a22i}-\boldsymbol{J}_i\boldsymbol{A}_{a12i})^T\boldsymbol{P}^{\circ}+\boldsymbol{P}^{\circ}(\boldsymbol{A}_{a22i}-\boldsymbol{J}_i\boldsymbol{A}_{a12i})<0 \quad (55)$$

for all i. Setting

$$\boldsymbol{P}^{\circ}\boldsymbol{J}_i=\boldsymbol{W}_i^{\circ} \quad (56)$$

(55) implies (48). ■

Theorem 3: The stability condition of the autonomous part of (51) is the same as the asymptotic stability condition of the error reference model

$$\dot{\boldsymbol{e}}_{aq2}(t)=\sum_{i=1}^{s}h_i(\boldsymbol{\theta}(t))(\boldsymbol{A}_{a22i}-\boldsymbol{J}_i\boldsymbol{A}_{a12i})\boldsymbol{e}_{aq2}(t) \quad (57)$$

where the estimation error of the unmeasurable part of state variables is

$$\boldsymbol{e}_{aq2}(t)=\boldsymbol{q}_{a2}(t)-\boldsymbol{q}_{a2e}(t) \quad (58)$$

Proof: Substituting (32) in (58) gives

$$\boldsymbol{e}_{a2}(t)=\boldsymbol{q}_{a2}(t)-\boldsymbol{p}_{2e}(t)-\sum_{i=1}^{s}h_i(\boldsymbol{\theta}(t))\boldsymbol{J}_i\boldsymbol{v}(t) \quad (59)$$

Defining, in analogy with (30)-(32), the reference variable $\boldsymbol{p}_2(t)$ as follows

$$\boldsymbol{p}_2(t)=\boldsymbol{q}_{a2}(t)-\sum_{i=1}^{s}h_i(\boldsymbol{\theta}(t))\boldsymbol{J}_i\boldsymbol{v}(t) \quad (60)$$

$$\dot{\boldsymbol{p}}_2(t)=\sum_{i=1}^{s}h_i(\boldsymbol{\theta}(t))\boldsymbol{q}_{2i}^{\circ}(t) \quad (61)$$

$$\boldsymbol{q}_{2i}^{\circ}(t)=\boldsymbol{A}_{aei}\boldsymbol{p}_2(t)+\boldsymbol{A}_{avi}\boldsymbol{v}(t)+\left[-\boldsymbol{J}_i\ \boldsymbol{I}_{n-m}\right]\boldsymbol{B}_{ai}\boldsymbol{u}(t) \quad (62)$$

and substituting (60) in (59) gives

$$\boldsymbol{e}_{aq2}(t)=\boldsymbol{p}_2(t)-\boldsymbol{p}_{2e}(t)=\boldsymbol{e}_{p2}(t),\quad \dot{\boldsymbol{e}}_{aq2}(t)=\dot{\boldsymbol{e}}_{p2}(t) \quad (63)$$

Thus, inserting (30), (31) and (61), (62) into (63) results in

$$\dot{\boldsymbol{e}}_{aq2}(t)=\sum_{i=1}^{s}h_i(\boldsymbol{\theta}(t))\boldsymbol{A}_{aei}(\boldsymbol{p}_2(t)-\boldsymbol{p}_{2e}(t)) \quad (64)$$

and with (34), (63) then (64) implies (57).

Since (51) and (57) are associated with the same system matrix $\boldsymbol{A}_{aei}$, this concludes the proof. ■

Note, the time derivative (42) yields by the definition.

Using the equivalency of the stability conditions, an actuator fault estimation structure based on reduced-order TS fuzzy observer can be discussed.

V. Estimation of Actuator Faults

To obtain an actuator fault estimation structure based on reduced-order TS fuzzy observer, the matching condition (29) has to be satisfied.

Theorem 4: The estimation error dynamics of the reduced order TS fuzzy observer (30), (31) is not affected by actuator faults if, with (29), there exists a symmetric positive definite matrix $\boldsymbol{P}^{\circ}\in I\!R^{(n-m)\times(n-m)}$ such that

$$\boldsymbol{P}^{\circ}\boldsymbol{J}_i=\boldsymbol{B}_{af1i}^{\perp} \quad (65)$$

where $\boldsymbol{B}_{af1i}^{\perp}$ is the orthogonal complement to $\boldsymbol{B}_{af1i}$, and the orthogonal complements exist for all $i=1,2,\dots,s$.

Proof: The system with an actuator fault is described as

$$\begin{aligned}&\dot{\boldsymbol{q}}_{af}(t)=\\&=\sum_{i=1}^{s}h_i(\boldsymbol{\theta}(t))(\boldsymbol{A}_{ai}\boldsymbol{q}_{afi}(t)+\boldsymbol{B}_{ai}\boldsymbol{u}(t)+\boldsymbol{B}_{afi}\boldsymbol{f}(t)))\end{aligned} \quad (66)$$

and the dynamics of the error (63) can be rewritten as

$$\begin{aligned}\dot{\boldsymbol{e}}_{aq2}(t)&=\sum_{i=1}^{s}h_i(\boldsymbol{\theta}(t))\boldsymbol{A}_{aei}\boldsymbol{e}_{aq2}(t)+\\&+\sum_{i=1}^{s}h_i(\boldsymbol{\theta}(t))\left[\,-\boldsymbol{J}_i\quad \boldsymbol{I}_{n-m}\,\right]\boldsymbol{B}_{afi}\boldsymbol{f}(t)\end{aligned} \quad (67)$$

Defining the quadratic positive definite Lyapunov function

$$v(\boldsymbol{e}_{aq2}(t))=\boldsymbol{e}_{aq2}^T(t)\boldsymbol{P}^{\circ}\boldsymbol{e}_{aq2}(t) \quad (68)$$

where $\boldsymbol{P}^{\circ}=\boldsymbol{P}^{\circ T}>0$, $\boldsymbol{P}^{\circ}\in I\!R^{(n-m)\times(n-m)}$ then, after evaluation of derivative of (67) with respect to t, it is obtained

$$\dot{v}(\boldsymbol{e}_{aq2}(t))=\dot{\boldsymbol{e}}_{aq2}^T(t)\boldsymbol{P}^{\circ}\boldsymbol{e}_{aq2}(t)+\boldsymbol{e}_{aq2}^T(t)\boldsymbol{P}^{\circ}\dot{\boldsymbol{e}}_{aq2}(t) \quad (69)$$

From the expression (67) it follows that

$$\begin{aligned}&\dot{v}(\boldsymbol{e}_{aq2}(t))=\\&=\boldsymbol{e}_{aq2}^T(t)\sum_{i=1}^{s}h_i(\boldsymbol{\theta}(t))(\boldsymbol{P}^{\circ}\boldsymbol{A}_{aei}+\boldsymbol{A}_{aei}^T\boldsymbol{P}^{\circ})\boldsymbol{e}_{a2}(t)+\\&+\boldsymbol{e}_{aq2}^T(t)\boldsymbol{P}^{\circ}\sum_{i=1}^{s}h_i(\boldsymbol{\theta}(t))\left[-\boldsymbol{J}_i\ \boldsymbol{I}_{n-m}\right]\boldsymbol{B}_{afi}\boldsymbol{u}_f(t)+\\&+\boldsymbol{u}_f^T(t)\sum_{i=1}^{s}h_i(\boldsymbol{\theta}(t))\boldsymbol{B}_{afi}^T\left[-\boldsymbol{J}_i\ \boldsymbol{I}_{n-m}\right]^T\boldsymbol{P}^{\circ}\boldsymbol{e}_{aq2}(t)<0\end{aligned} \quad (70)$$

and with respect to (29), it can be set

$$\boldsymbol{P}^{\circ}\left[-\boldsymbol{J}_i\ \boldsymbol{I}_{n-m}\right]\boldsymbol{B}_{afi}=\boldsymbol{P}^{\circ}\left[-\boldsymbol{J}_i\boldsymbol{B}_{af1i}\ \boldsymbol{0}_{n-m}\right]=\boldsymbol{0} \quad (71)$$

which results in the equality

$$\boldsymbol{P}^{\circ}\boldsymbol{J}_i\boldsymbol{B}_{af1i}=\boldsymbol{0}\quad \forall i \quad (72)$$

Evidently, (72) will be satisfied if (65) is satisfied. ■

Theorem 5: The estimation error dynamic (67) is asymptotically stable, if there exists a symmetric positive definite matrix $\boldsymbol{P}^{\circ}\in I\!R^{(n-m)\times(n-m)}$ such that for $i=1,2,\dots,s$

$$\boldsymbol{P}^{\circ}=\boldsymbol{P}^{\circ T}>0 \quad (73)$$

$$\boldsymbol{A}_{a22i}^T\boldsymbol{P}^{\circ}+\boldsymbol{P}^{\circ}\boldsymbol{A}_{a22i}-\boldsymbol{A}_{a12i}^T\boldsymbol{B}_{af1i}^{\perp T}-\boldsymbol{B}_{af1i}^{\perp}\boldsymbol{A}_{a12i}<0 \quad (74)$$

where $\boldsymbol{B}_{af1i}^{\perp}$ is the orthogonal complement to $\boldsymbol{B}_{af1i}$.

If the above conditions hold, then

$$\boldsymbol{J}_i=(\boldsymbol{P}^{\circ})^{-1}\boldsymbol{B}_{af1i}^{\perp} \quad (75)$$

Proof: Satisfying (71) then (34), (70) implies

$$\boldsymbol{e}_{aq2}^T(t)\sum_{i=1}^{s}h_i(\boldsymbol{\theta}(t))\left(\boldsymbol{P}^{\circ}\boldsymbol{A}_{aei}+\boldsymbol{A}_{aei}^T\boldsymbol{P}^{\circ}\right)\boldsymbol{e}_{aq2}(t)<0 \quad (76)$$

It is evident that (76) is negative if for all i it is

$$(\boldsymbol{A}_{a22i}-\boldsymbol{J}_i\boldsymbol{A}_{a12i})^T\boldsymbol{P}^{\circ}+\boldsymbol{P}^{\circ}(\boldsymbol{A}_{a22i}-\boldsymbol{J}_i\boldsymbol{A}_{a12i})<0 \quad (77)$$

Using (65) then (77) implies (48). ■

Corollary 1: Using (75), it is

$$\begin{aligned} &\left[\begin{array}{cc} -\boldsymbol{J}_i & \boldsymbol{I}_{n-m} \end{array}\right] \boldsymbol{B}_{afi} = \\ &= (\boldsymbol{P}^\circ)^{-1} \left[\begin{array}{cc} -\boldsymbol{B}_{af1i}^{\perp} & \boldsymbol{P}^\circ \end{array}\right] \boldsymbol{C}_a^T \boldsymbol{B}_{af1i} = \\ &= (\boldsymbol{P}^\circ)^{-1} \left[\begin{array}{cc} -\boldsymbol{B}_{af1i}^{\perp}\boldsymbol{B}_{af1i} & \boldsymbol{0} \end{array}\right] = \boldsymbol{0} \end{aligned} \tag{78}$$

The equality given above implies that the reduced order fuzzy observer equation (31) is not affected by actuator faults.

Theorem 6: Designed with respect to $\boldsymbol{P}^\circ$ satisfying (47)-(49), the reduced order TS fuzzy observer (30), (31) estimates the actuator faults with uniform convergence.

Proof: Since (19), (22) implies

$$\boldsymbol{q}_e(t) = \boldsymbol{T}_a^T \boldsymbol{q}_{ae}(t) = \boldsymbol{T}_{a2}^T \boldsymbol{q}_{a2e}(t) + \boldsymbol{T}_{a1}^T \boldsymbol{v}(t) \tag{79}$$

substituting (23) in (79) leads to

$$\boldsymbol{q}_e(t) = \boldsymbol{T}_{a2}^T \boldsymbol{p}_{2e}(t) + (\boldsymbol{T}_{a1}^T + \boldsymbol{T}_{a2}^T \boldsymbol{J}) \boldsymbol{W} \boldsymbol{y}(t) \tag{80}$$

$$\dot{\boldsymbol{q}}_e(t) = \sum_{i=1}^{s} h_i (\boldsymbol{A}_i \boldsymbol{q}_e(t) + \boldsymbol{B}_i \boldsymbol{u}(t) + \boldsymbol{B}_{afi} \boldsymbol{f}_e(t)) \tag{81}$$

respectively. For simplicity, here and hereafter, $h_i(\boldsymbol{\theta}(t))$ is written only as h_i. Thus, writing (81) as follows

$$\sum_{i=1}^{s} h_i \boldsymbol{B}_{afi} \boldsymbol{f}_e(t) = \sum_{i=1}^{s} h_i (\dot{\boldsymbol{q}}_e(t) - \boldsymbol{A}_i \boldsymbol{q}_e(t) - \boldsymbol{B}_i \boldsymbol{u}(t)) \tag{82}$$

multiplying the left-hand side of (82) by $h_j \boldsymbol{B}_{afj}^T$, and applying the average summation, gives

$$\begin{aligned} &\sum_{j=1}^{s} \sum_{i=1}^{s} h_j h_i \boldsymbol{B}_{afj}^T \boldsymbol{B}_{afi} \boldsymbol{f}_e(t) = \\ &= \Big(\sum_{j=1}^{s} h_j \boldsymbol{B}_{afj}^T\Big) \sum_{i=1}^{s} h_i \big(\dot{\boldsymbol{q}}_e(t) - \boldsymbol{A}_i \boldsymbol{q}_e(t) - \boldsymbol{B}_i \boldsymbol{u}(t)\big) \end{aligned} \tag{83}$$

Using (13), then (83) can be written as

$$\begin{aligned} &\Big(\sum_{i=1}^{s} h_i\Big) \sum_{j=1}^{s} h_j \boldsymbol{B}_{afj}^T \boldsymbol{B}_{afj} \boldsymbol{f}_e(t) \le \\ &\le \Big(\sum_{j=1}^{s} h_j \boldsymbol{B}_{afj}^T\Big) \sum_{i=1}^{s} h_i \big(\dot{\boldsymbol{q}}_e(t) - \boldsymbol{A}_i \boldsymbol{q}_e(t) - \boldsymbol{B}_i \boldsymbol{u}(t)\big) \end{aligned} \tag{84}$$

Thus, since $\sum_{j=1}^{s} h_j(\boldsymbol{\theta}(t)) = 1$, it can be set

$$\boldsymbol{f}_e(t) \le \Big(\sum_{j=1}^{s} h_j \boldsymbol{B}_{afj}^{\ominus 1}\Big) \sum_{i=1}^{s} h_i \big(\dot{\boldsymbol{q}}_e(t) - \boldsymbol{A}_i \boldsymbol{q}_e(t) - \boldsymbol{B}_i \boldsymbol{u}(t)\big) \tag{85}$$

where $\boldsymbol{B}_{afj}^{\ominus 1} = (\boldsymbol{B}_{afj}^T \boldsymbol{B}_{afj})^{-1} \boldsymbol{B}_{afj}^T$ is the Moore-Penrose pseudoinverse of $\boldsymbol{B}_{afj}$.

Explaining the reference $\boldsymbol{f}(t)$ as follows

$$\boldsymbol{f}(t) \le \Big(\sum_{j=1}^{s} h_j \boldsymbol{B}_{afj}^{\ominus 1}\Big) \sum_{i=1}^{s} h_i \big(\dot{\boldsymbol{q}}(t) - \boldsymbol{A}_i \boldsymbol{q}(t) - \boldsymbol{B}_i \boldsymbol{u}(t)\big) \tag{86}$$

then, with $\boldsymbol{e}_f(t) = \boldsymbol{f}_e(t) - \boldsymbol{f}(t)$, it yields

$$\boldsymbol{e}_f(t) \le \Big(\sum_{j=1}^{s} h_j \boldsymbol{B}_{afj}^{\ominus 1}\Big) \sum_{i=1}^{s} h_i (\boldsymbol{A}_i \boldsymbol{e}_q(t) - \dot{\boldsymbol{e}}_q(t)) \tag{87}$$

Owing to the observer error asymptotic convergence, $\boldsymbol{e}_f(t)$ uniformly converges to zero. ■

Note that $\dot{\boldsymbol{q}}_e(t)$ has to be computed numerically from (80), since (36) is affected by actuator faults, and so cannot be used for evaluation of the first state vector component derivative.

VI. Illustrative Example

The description of the hydrostatic transmission is taken from [5], and is used for input/output signal generation. The system dynamics is represented by the state-space model

$$\begin{aligned} \dot{q}_1(t) &= -a_{11} q_1(t) + b_{11} u_1(t) \\ \dot{q}_2(t) &= -a_{22} q_2(t) + b_{22} u_2(t) \\ \dot{q}_3(t) &= a_{31} q_1(t) p(t) - a_{33} q_3(t) - a_{34} q_2(t) q_4(t) \\ \dot{q}_4(t) &= a_{43} q_2(t) q_3(t) - a_{44} q_4(t) \end{aligned}$$

where $q_1(t)$ is the normalized hydraulic pump angle, $q_2(t)$ is the normalized hydraulic motor angle, $q_3(t)$ is the pressure difference [bar], $q_4(t)$ is the hydraulic motor speed [rad/s], $u_1(t)$ is the normalized control signal of the hydraulic pump, $u_2(t)$ is the normalized control signal of the hydraulic motor, and the external signal $p(t)$ represents speed of hydraulic pump [rad/s]. It is supposed that the external variable $p(t)$ and all state variables except $q_3(t)$ are measurable, and the model parameters are

$$\begin{array}{lll} a_{11} = 7.6923 & a_{22} = 4.5455 & a_{33} = 7.6054{*}10^{-4} \\ a_{31} = 0.7877 & a_{34} = 0.9235 & b_{11} = 1.8590{*}10^{3} \\ a_{43} = 12.1967 & a_{44} = 0.4143 & b_{22} = 1.2879{*}10^{3} \end{array}$$

Since the variables $p(t) \in \langle c_1, c_2 \rangle = \langle 105, 300 \rangle$ and $q_2(t) \in \langle d_1, d_2 \rangle = \langle 0, 1 \rangle$ are bounded on the prescribed sectors, the vector of premise variables was chosen as

$$\boldsymbol{\theta}(t) = \left[\begin{array}{cc} \theta_1(t) & \theta_2(t) \end{array}\right] = \left[\begin{array}{cc} q_2(t) & p(t) \end{array}\right]$$

The set of normalized membership functions, constructed from all combination of the next sector functions

$$w_{11}(q_2(t)) = \frac{b_1 - q_2(t)}{b_1 - b_2}, \; w_{12}(q_2(t)) = 1 - w_{11}(q_2(t))$$

$$w_{21}(p(t)) = \frac{c_1 - p(t)}{c_1 - c_2}, \; w_{22}(p(t)) = 1 - w_{21}(p(t))$$

implies the TS fuzzy system parameters

$$\boldsymbol{A}_i = \begin{bmatrix} -a_{11} & 0 & 0 & 0 \\ 0 & -a_{22} & 0 & 0 \\ a_{31} c_k & 0 & -a_{33} & -a_{34} d_l \\ 0 & 0 & a_{43} d_l & -a_{44} \end{bmatrix}$$

$$\boldsymbol{B} = \boldsymbol{B}_f = \begin{bmatrix} b_{11} & 0 \\ 0 & b_{22} \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \; \boldsymbol{C} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Using the results of SVD of $\boldsymbol{C}$, the transform matrices are

$$\boldsymbol{U} = \boldsymbol{S} = \boldsymbol{W}^{-1} = \boldsymbol{I}_3, \; \boldsymbol{V} = \boldsymbol{T}_a = \operatorname{diag}\left[\begin{array}{cc} \boldsymbol{I}_2 & \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \end{array}\right]$$

With respect to (9), the scalar LMI variable P° was found by using the convex optimization techniques, with $\boldsymbol{B}_{a1}^{\perp}$ defined as a structured LM variable of the form

$$\boldsymbol{B}_{a1}^{\perp} = \left[\begin{array}{ccc} 0 & 0 & Z \end{array}\right], \quad Z \in I\!R$$

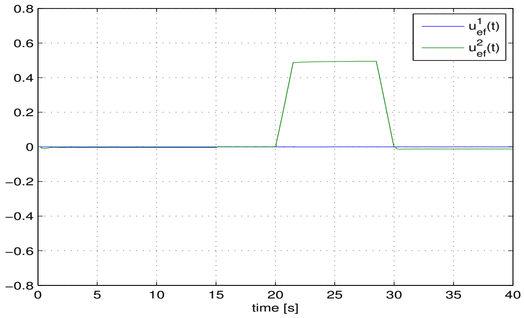

Fig. 1. The reconstruction of the second actuator fault

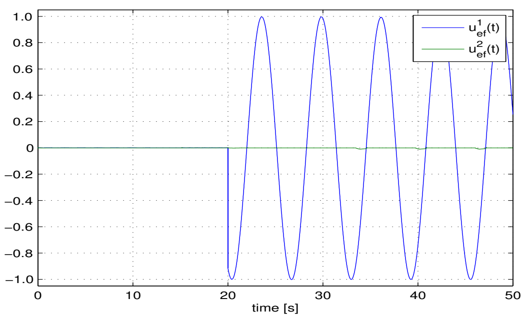

Fig. 2. The reconstruction of the second actuator signal oscillation

and solving (73), (74) with respect to the LMI variables P°, Z using Self–Dual–Minimization (SeDuMi) package for Matlab. the reduced observer gain design problem was feasible with the results

$$P^{\circ} = 1.0832, \; \boldsymbol{B}_{a1}^{\perp} = \begin{bmatrix} 0 & 0 & 0.0410 \end{bmatrix}, \; \boldsymbol{J} = \begin{bmatrix} 0 & 0 & 0.0378 \end{bmatrix}$$

The simulation for the fault reconstruction performance was done using the stabilizing state control without fault compensation [10]. Figure 1 and 2 present the signals u_{fe}^{1}, u_{fe}^{2}, obtained from (85) as a reconstruction of the fault. To suppress the effect of the 1% noise the five point stencil for the first derivative computation was used.

VII. Concluding Remarks

Generalized design method of a reduced-order observer-based fault estimation scheme is developed, as augmentation of unknown observers synthesis for the nonlinear systems described by TS fuzzy model. This is achieved by manipulation of observer stability with respect to the proposed matching condition. Design conditions are derived in terms of LMI, to manipulate the reducer-order observer stability. Because of the specific observer gain matrix structure, the estimated unmeasurable part of the system state is free of actuators faults. By examining the estimated state vector it is presented that, using a numerical realization of time derivative of the state, the actuator fault signals can be faithfully reconstructed.

References

[1] M. Chadli, "State and an LMI approach to design observer for unknown inputs Takagi-Sugeno fuzzy models", *Asian J. Control*, vol.12, no.4, pp. 524530, 2010.

[2] J. Chen and R.J. Patton, *Robust Model-Based Fault Diagnosis for Dynamic Systems*, Norwell: Kluwer Academic Publishers, 1999.

[3] S. Ding, *Model-Based Fault Diagnosis Techniques. Design Schemes, Algorithms, and Tools*, Berlin: Springer-Verlag, 2008.

[4] Z. Gao, X. Shi, and S.X. Ding, "Fuzzy state/disturbance observer design for T-S fuzzy systems with application to sensor fault estimation", *IEEE Trans. Systems, Man, and Cybernetics, Part B: Cybernetics*, vol. 38, no. 3, pp. 875-880, 2008.

[5] P. Gerland, D. Gross, N. Schulte, and A. Kroll, "Robust adaptive fault detection using global state information and application to mobile working machines", in *Proc. Conf. Control and Fault-Tolerant Systems (SysTol'10)*, pp. 813-818, Nice, France, 2010.

[6] D. Ichalal, B. Marx, J. Ragot, and D. Maquin, Design of observers for Takagi-Sugeno discrete-time systems with unmeasurable premise variables, in *Proc. 5th Workshop Advanced Control and Diagnosis (ACD 2007)*, Grenoble, France. 2007.

[7] D. Krokavec and A. Filasová, *Dynamic Systems Diagnosis*, Košice, Slovakia: Elfa, 2007. (in Slovak)

[8] D. Krokavec and A. Filasová, "Reducer-order fuzzy-observer-based actuator fault reconstruction for a class of nonlinear systems", *Proc. 6th IASTED Int. Conf. Computational Intelligence and Bioinformatics (CIB'2011)*, pp. 61-68, Pittsburgh, PE, USA, 2011.

[9] D. Krokavec and A. Filasová, "An approach to reconstruction of actuator faults for a class of nonlinear systems", *Prepr. 8th IFAC Symp. Fault Detection, Supervision and Safety of Technical Processes (SAFEPROCESS'2012)*, pp. 216-221, Mexico City, Mexico, 2012.

[10] D. Krokavec and A. Filasová, "Optimal fuzzy control for a class of nonlinear systems". *Math. Problems in Engineering*, vol. 2012, Article ID 481942, 29 pages, 2012.

[11] Z. Lendek, T.M. Guerra, R. Babuška, and B. De Schutter, *Stability Analysis and Nonlinear Observer Design Using Takagi-Sugeno Fuzzy Models*, Berlin: Springer-Verlag, 2010.

[12] A.M. Nagy Kiss, B. Marx, G. Mourot, G. Schutz, and J. Ragot, "Observers design for uncertain Takagi-Sugeno systems with unmeasurable premise variables and unknown inputs. Application to a wastewater treatment plant", *J. Process Control*, vol. 21, no. 7. pp. 1105-1114, 2011.

[13] K.M. Passino and S. Yurkovich, *Fuzzy Control*, Berkeley: Addison-Wesley Longman, 1998.

[14] T. Takagi and M. Sugeno, "Fuzzy identification of systems and its applications to modeling and control", *IEEE Trans. Systems, Man, and Cybernetics*, vol. 15, no. 1, pp. 116-132, 1985.

[15] K. Tanaka and H.O. Wang, *Fuzzy Control Systems Design and Analysis. A Linear Matrix Inequality Approach*, New York: John Wiley & Sons, 2001.

[16] H. Wang and S. Daley, "Actuator fault diagnosis. An adaptive observer-based technique", *IEEE Trans. Automatic Control*, vol. 41, nol. 7, pp. 1073-1078, 1996.

[17] S.H. Wang and R. Zachery, "Singular value decomposition of system input-output matrix and its symmetry property", Coputers Elect. Eng., vol. 22, no. 3, pp. 231-234, 1996.

[18] Q.G. Wang, *Decoupling Control*, Springer-Verlag, Berlin, 2003.

[19] D. Xu, B. Jiang, and P. Shi, "Nonlinear actuator fault estimation observer. An inverse system approach via a TS fuzzy model", *Int. J. Appl. Math. Comput. Sci.*, vol. 22, no. 1, pp. 183-196, 2012.

[20] X.G. Yan and C. Edwards, "Nonlinear robust fault reconstruction and estimation using a sliding mode observer", *Automatica*, vol. 43, no. 9, pp. 1605-1614, 2007.

[21] X. Zhang, M.M. Polycarpou, and T. Parisini, "Fault diagnosis of a class of nonlinear uncertain systems with Lipschitz nonlinearities using adaptive estimation", *Automatica* vol. 46, no. 2, pp. 290-299, 2010.

[22] F. Zhu, "State estimation and unknown input reconstruction via both reduced-order and high-order sliding mode observers", *J. Process Control*, vol. 22, no. 1, pp. 296-302, 2012.

[23] F. Zhu and F. Cen, "Full-order observed-based actuator fault detection and reduced-order observer-based fault reconstruction for a clas of uncertain nonlinear systems", *J. Process Control*, vol. 20, no. 10, pp. 1141-1149, 2010.

[24] A. Zolghadri, D. Henry, and M. Monsion, "Design of nonlinear observers for fault diagnosis. A case study", *Control Engineering Practice*, vol. 4, no. 11, pp. 1535-1544, 1996.