

Dead-Beat Sliding Mode Control of Perishable Inventories with Transportation Losses and Multiple Suppliers

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Abstract—In this paper a new dead-beat sliding mode warehouse management strategy is proposed. The strategy ensures full customers' demand satisfaction and prevents from exceeding the warehouse capacity. Furthermore, it ensures that the ordered quantities of goods are always non-negative and upper bounded. The proposed strategy not only explicitly takes into account decay of goods stored in the warehouse (perishing inventories) but it also accounts for transportation losses which take place on the way from suppliers to the warehouse.

I. INTRODUCTION

THE control theoretic approach to the management of logistic processes, and in particular to the problem of supply chain management has recently become an important research subject. A good overview of the techniques used in the field and the obtained results can be found in [1 – 4]. The first application of the control theory methods to the management of logistic processes was reported in the early 1950s when Simon [5] applied servomechanism control algorithm to find an efficient strategy of goods replenishment in continuous time, single product inventory control systems. A few years later the discrete time servomechanism control algorithm for the purpose of efficient goods replenishment has been proposed [6]. Since that time numerous solutions have been presented, and therefore, further in this section we are able to mention only a few, arbitrarily selected examples of solutions proposed over the last decades. In [7] and [8] autoregressive moving average (ARMA) system structure has been applied in order to model uncertain demand. Then in [9] and [10] model predictive control of supply chain has been proposed and in [11] a robust controller for the continuous-time system with uncertain processing time and delay has been designed by minimising H_∞ -norm. However, practical implementation of the strategy described in [11] requires application of numerical methods in order to obtain the control law parameters, which limits its analytical tractability.

In [12] lead-time delay is explicitly taken into account and represented by additional state variables in the state space description. This approach results in the optimal

controller designed by minimisation of quadratic performance index. A similar approach is applied in [13] where an LQ optimal sliding mode controller is designed. However, both papers [12] and [13] are concerned with conventional, non-deteriorating inventories only. An extension of the results presented in [12] to the case of perishable inventories is given in [14] and an LQ optimal sliding mode controller for supply chains with deteriorating stock is proposed in [15]. However, none of the papers [12 – 15] takes into account transportation losses (or in other words goods decay during the order procurement time). Therefore, in this paper we consider perishable inventories and we explicitly account for the ordered goods losses during the non-negligible lead time.

In this paper we also consider a periodic-review inventory system with perishable goods replenished from multiple supply sources. However, in contrast to the previously published results we consider not only losses which take place when the commodity is stored in the warehouse, but also those which happen during the supply process, i.e. the losses on the way from the supplier to the warehouse. We propose a discrete time representation of the supply chain dynamics and we apply discrete time sliding mode methodology [16 – 26] to design the controller for the considered system. The controller design objective is on one hand to fully satisfy the imposed demand, and on the other, to minimize at the same time the on-hand inventory volume. Since the demand may vary quite rapidly, we determine the sliding hyperplane so that the proposed discrete-time sliding-mode controller ensures the dead-beat system performance. Hence the closed-loop system is stabilized and its error converges to zero in the shortest possible time. This approach results in good dynamics of the closed loop system and its fast reaction to the unpredictable variations of demand. Moreover, the sliding mode controller proposed further in this paper leads to chattering free system operation. The controller is determined analytically in a closed form, which allows us to state and formally prove important properties of proposed inventory policy. First, we prove that the designed management policy always generates strictly positive and upper bounded order quantities, which is an important issue from the practical point of view. Next, we define the warehouse capacity which provides enough space for all incoming shipments, and finally we show that all the imposed demand is fully satisfied, which guarantees 100% service level.

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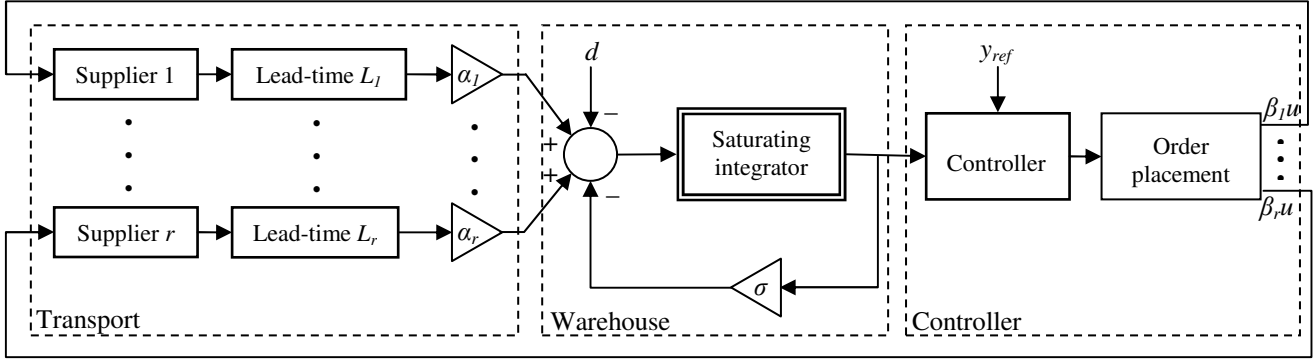


Fig. 1. System model.

II. PROBLEM STATEMENT

In this paper we consider a periodic review production-inventory system, where distribution centre, replenished from multiple supply sources, provides products for customers or another production stage. The model of the analysed inventory system is presented in Fig. 1. The analysed inventory system is subject to an *a priori* unknown, bounded, time-varying demand. The main objective of this work is to design a stable supply policy, which will maximize demand satisfaction from the resources available at the distribution centre. The design procedure proposed in the paper not only explicitly takes into account the delay (lead-time) between placing of an order at the suppliers and goods arrival at the distribution centre, but it also directly accounts for the on-hand stock deterioration and commodity losses in the supply process, i.e. the losses which take place during the lead-time.

The stock replenishment orders $u(kT)$ are placed at regular time instants kT , where T is the review period of the considered process and $k = 0, 1, 2, \dots$. The particular value of each order is calculated on the basis of the current stock level $y(kT)$, the stock reference level y_{ref} and the order history. We assume, that replenishment order $u(kT)$ can be split among r supply options. As a consequence, in each time instant, β_i of the total order is placed at supplier i ($i = 1, \dots, r$), where β_i is a fraction from interval $[0, 1]$ satisfying

$$\sum_{i=1}^r \beta_i = 1. \quad (1)$$

Each nonzero order placed at the supplier is realized with lead-time delay L_i , which is a multiple of the review period. Thus, $L_i = n_i T$, where n_i is a positive integer. Without the loss of generality, we may order the supply alternatives according to their lead time as follows

$$L_1 \leq L_2 \leq \dots \leq L_r. \quad (2)$$

The imposed demand (the amount of goods requested from inventory in period k) is modelled as an unknown, bounded function of time $0 \leq d(kT) \leq d_{max}$. It is worth to

notice, that this definition of the demand is quite general and makes the presented approach fairly universal. According to this definition the following two situations may occur:

- If there is a sufficient amount of goods in the warehouse, then the imposed demand is fully satisfied.
- If the imposed demand is greater than the amount of goods available at the on-hand stock and in arriving shipments, then only some part of the demand is satisfied. Hence, additional demand is lost, as we assume that the sales are not backordered.

Let $h(kT)$ denote the amount of goods sold to customers or sent to retailers in the distribution network at time instant kT . Then

$$0 \leq h(kT) \leq d(kT) \leq d_{max}. \quad (3)$$

Stock balance equation for the considered system with perishable inventory has the following form

$$y[(k+1)T] = \rho y(kT) + u_R(kT) - h(kT), \quad (4)$$

where $u_R(kT)$ is the order received at time kT . The fraction of perishable stock which remains in the warehouse after each review period is represented by $\rho = 1 - \sigma$. Incoming shipments also deteriorate during transportation process. Consequently, the fractions of ordered goods which arrive at the warehouse are represented by α_i ($i = 1, \dots, r$), where

$$0 < \alpha_i \leq 1. \quad (5)$$

Thus, the order received at time kT is expressed by

$$u_R(kT) = \sum_{i=1}^r \alpha_i \beta_i u[(k - n_i)T]. \quad (6)$$

Furthermore, we assume that the warehouse is initially empty, i.e. $y(kT) = 0$ for $k < 0$, and the first order is placed at the time instant $kT = 0$. Due to the lead-time delay, the first order arrives at the warehouse at the time instant n_1 , and $y(kT) = 0$ for any $k \leq n_1$. Taking into account our

assumptions, initial conditions and (6), the stock level for any $k > 0$ may be expressed as

$$\begin{aligned} y(kT) &= \sum_{i=1}^r \alpha_i \beta_i \sum_{j=0}^{k-1} \rho^{k-1-j} u[(j-n_i)T] - \sum_{j=0}^{k-1} \rho^{k-1-j} h(jT) \\ &= \sum_{i=1}^r \alpha_i \beta_i \sum_{j=0}^{k-n_i-1} \rho^{k-n_i-1-j} u(jT) - \sum_{j=0}^{k-1} \rho^{k-1-j} h(jT) \end{aligned} \quad (7)$$

In order to save on notation, in the remainder of the paper we will use k as the independent variable in place of kT .

Let us consider the following discrete time state space representation of the analysed inventory system

$$\begin{aligned} \mathbf{x}(k+1) &= \mathbf{A}\mathbf{x}(k) + \mathbf{b}u(k) + \mathbf{v}h(k), \\ y(k) &= \mathbf{q}^T \mathbf{x}(k), \end{aligned} \quad (8)$$

where $\mathbf{x}(k) = [x_1(k) \ x_2(k) \ \dots \ x_n(k)]^T$ is the state vector, $x_1(k) = y(k)$ is the on-hand stock level at time instant k and $x_j(k) = u(k-n+j-1)$ for any $j = 2, \dots, n$ represents delayed input signal u . Furthermore, \mathbf{A} is $n \times n$ state matrix, \mathbf{b} , \mathbf{v} , and \mathbf{q} are $n \times 1$ vectors

$$\mathbf{A} = \begin{bmatrix} \rho & a_2 & a_3 & \dots & a_n \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1 \\ 0 & 0 & 0 & \dots & 0 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} -1 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}, \mathbf{q} = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}. \quad (9)$$

The system order is equal to $n = n_r + 1 = L_r/T + 1$ and it depends on the review period and lead-time L_r of the supplier with the largest delay. The elements a_i ($i = 2, \dots, n$) in the first row of the state matrix \mathbf{A} , are calculated as follows

$$a_i = \sum_{j=n_i-n+1}^n \alpha_j \beta_j. \quad (10)$$

The desired system state vector is defined as

$$\mathbf{x}_d^T = [1 \quad (1-\rho)/\Omega \quad \dots \quad (1-\rho)/\Omega] y_{ref}, \quad (11)$$

where y_{ref} denotes the reference stock level, and

$$\Omega = \sum_{j=1}^r \alpha_j \beta_j = \sum_{j=2}^n a_j. \quad (12)$$

Since the main objective of the controller design procedure is to obtain stable supply policy, it is necessary to stabilize the first state variable at the reference level. Therefore, when choosing the desired state vector, it is necessary to take into

account that the commodities perish at the rate $1-\rho$ when kept in the warehouse as well the fact that they decay during the transportation process at the rate proportional to Ω . Substituting the proposed desired system state vector into the state equation, one can verify that in the steady state, the on-hand stock is refilled by incoming shipments at the rate equal to $y_{ref}(1-\rho)/\Omega$.

III. PROPOSED INVENTORY MANAGEMENT POLICY

In this section we present the controller design procedure for the considered multi-supplier inventory system (8) – (9) with perishable goods and transportation losses. The procedure is based on the discrete time sliding mode approach. In part A of this section, the choice of the sliding hyperplane ensuring dead-beat performance is described. Next, in part B we formulate and prove the most important properties of the proposed inventory management policy.

A. Dead-Beat Sliding-Mode Controller Design

For the sliding mode controller design purpose we introduce a sliding hyperplane described by

$$s(k) = \mathbf{c}^T \mathbf{e}(k) = 0, \quad (13)$$

where $\mathbf{c} = [c_1 \ c_2 \ \dots \ c_n]^T$ is such a vector that $\mathbf{c}^T \mathbf{b} \neq 0$. Parameters c_1, c_2, \dots, c_n will be determined further in this section. The closed-loop system error may be expressed as $\mathbf{e}(k) = \mathbf{x}_d - \mathbf{x}(k)$. Substituting (8) into equation $\mathbf{c}^T \mathbf{e}(k+1) = 0$ we obtain the following control law

$$u(k) = (\mathbf{c}^T \mathbf{b})^{-1} \mathbf{c}^T [\mathbf{x}_d - \mathbf{A}\mathbf{x}(k)]. \quad (14)$$

One can easily notice from (14), that the controller performance depends on the choice of the sliding plane parameters c_1, c_2, \dots, c_n . Using (9) we can rewrite (14) as

$$\begin{aligned} u(k) &= c_n^{-1} y_{ref} \left[c_1 + (1-\rho) / \Omega \sum_{j=2}^n c_j \right] + \\ &- c_n^{-1} \left\{ c_1 \rho x_1(k) + c_1 a_2 x_2(k) + \sum_{j=3}^n (c_1 a_j + c_{j-1}) x_j(k) \right\} \end{aligned} \quad (15)$$

In order to find parameters of the hyperplane which will ensure that the system error is eliminated in finite (and the smallest possible) number of control steps, we analyse coefficients of the characteristic polynomial of the closed-loop system state matrix $\mathbf{A}_c = [\mathbf{I}_n - \mathbf{b}(\mathbf{c}^T \mathbf{b})^{-1} \mathbf{c}^T] \mathbf{A}$. The polynomial $\det(z\mathbf{I}_n - \mathbf{A}_c)$ can be expressed as follows

$$\begin{aligned} \det(z\mathbf{I}_n - \mathbf{A}_c) &= z^n + \frac{c_1 a_n + c_{n-1} - \rho c_n}{c_n} z^{n-1} + \\ &+ \frac{c_1 a_{n-1} + c_{n-2} - \rho c_{n-1}}{c_n} z^{n-2} + \dots + \frac{c_1 a_2 - \rho c_2}{c_n} z \end{aligned} \quad (16)$$

A discrete-time system is asymptotically stable if and only if all its eigenvalues are located inside the unit circle on the z plane. Additionally, for the dead-beat performance [27], the characteristic polynomial of the closed-loop system should have the following form

$$\det(z\mathbf{I}_n - \mathbf{A}_c) = z^n, \quad (17)$$

which is satisfied when

$$c_1 = c_n \Psi \text{ and } c_j = c_n \Psi \sum_{i=1}^{j-1} \rho^{i-j} a_{i+1} \text{ for } j \geq 2, \quad (18)$$

where

$$\Psi = \rho^{n-1} / \sum_{i=1}^{n-1} \rho^{i-1} a_{i+1}. \quad (19)$$

Using (10), we can rewrite (19) as follows

$$\Psi = 1 / \sum_{i=1}^r \rho^{-n_i} \alpha_i \beta_i. \quad (20)$$

Hence, the vector \mathbf{c} describing the parameters of the sliding hyperplane has the following form

$$\mathbf{c}^T = \begin{bmatrix} \Psi & \Psi \rho^{-1} a_2 & \Psi (\rho^{-2} a_2 + \rho^{-1} a_3) & \dots \\ \dots & \Psi \sum_{i=1}^{n-2} \rho^{i-n+1} a_{i+1} c_n & 1 & c_n \end{bmatrix}, \quad (21)$$

which guarantees that the closed-loop system has all its eigenvalues located at the origin of the z plane.

Substituting (21) into (15) we get

$$c_1 + (1 - \rho) / \Omega \sum_{j=2}^n c_j = c_n / \Omega. \quad (22)$$

The control law can be expressed in the following way

$$u(k) = \frac{y_{ref}}{\Omega} - \Psi \rho x_1(k) - \Psi \sum_{j=2}^n a_j \sum_{i=j}^n \rho^{i-j} x_i(k). \quad (23)$$

According to the state space representation of inventory system (9), the first state variable denotes the on-hand stock level $x_1(k) = y(k)$, and the other state variables are equal to the delayed control signals generated at the previous $n - 1$ review periods $x_j(k) = u(k - n + j - 1)$. Therefore, taking into account that system order is equal to $n = n_r + 1$ we obtain

$$u(k) = \frac{y_{ref}}{\Omega} - \Psi \rho y(k) - \Psi \sum_{i=1}^r \alpha_i \beta_i \sum_{j=k-n_i}^{k-1} \rho^{k-n_i-j} u(j). \quad (24)$$

The obtained control signal incorporates reference stock level, current stock level and open orders modified with respect to transportation losses and goods decay in the warehouse.

B. Properties of the Proposed Controller

One of the fundamental issues in practical implementation of each inventory policy is to ensure that the quantity of goods shipped to the warehouse is always nonnegative and upper bounded. Therefore, now we introduce a lemma and a theorem which show that the proposed policy indeed ensures these two highly desirable properties.

First of all, it can be noticed from (24) that $u(0) = y_{ref} / \Omega$. Furthermore, for any $k \geq 1$ the following lemma holds.

Lemma: If the proposed inventory policy is applied, then for any $k \geq 1$

$$u(k) = (1 - \rho) \frac{y_{ref}}{\Omega} + \Psi \rho h(k - 1). \quad (25)$$

Proof: Substituting (7) into (24), we get

$$\begin{aligned} u(k) &= \frac{y_{ref}}{\Omega} - \Psi \rho \sum_{i=1}^r \alpha_i \beta_i \sum_{j=0}^{k-n_i-1} \rho^{k-n_i-1-j} u(j) + \\ &+ \Psi \rho \sum_{j=0}^{k-1} \rho^{k-1-j} h(j) + \\ &- \Psi \sum_{i=1}^r \alpha_i \beta_i \sum_{j=k-n_i}^{k-1} \rho^{k-n_i-j} u(j) = \frac{y_{ref}}{\Omega} + \\ &- \Psi \sum_{i=1}^r \alpha_i \beta_i \rho^{-n_i} \sum_{j=0}^{k-1} \rho^{k-j} u(j) + \Psi \sum_{j=0}^{k-1} \rho^{k-j} h(j) \end{aligned} \quad (26)$$

Then applying (20) to (26) we obtain

$$u(k) = \frac{y_{ref}}{\Omega} - \sum_{j=0}^{k-1} \rho^{k-j} u(j) + \Psi \sum_{j=0}^{k-1} \rho^{k-j} h(j) \quad (27)$$

For $k = 1$, it follows immediately from (27) that

$$\begin{aligned} u(1) &= \frac{y_{ref}}{\Omega} - \rho u(0) + \Psi \rho h(0) = \frac{y_{ref}}{\Omega} - \rho \frac{y_{ref}}{\Omega} + \\ &+ \Psi \rho h(0) = (1 - \rho) \frac{y_{ref}}{\Omega} + \Psi \rho h(0) \end{aligned} \quad (28)$$

which shows that the lemma is indeed satisfied for $k = 1$. Now let us assume that (25) is true for all integers up to

some $l > 1$. Using this assumption and (27), the order quantity generated at time instant $l + 1$ can be expressed in the following form

$$\begin{aligned}
u(l+1) &= \frac{y_{ref}}{\Omega} - \sum_{j=0}^l \rho^{l+1-j} u(j) + \Psi \sum_{j=0}^l \rho^{l+1-j} h(j) = \\
&= \frac{y_{ref}}{\Omega} + \rho \frac{y_{ref}}{\Omega} - \rho \frac{y_{ref}}{\Omega} - \rho \sum_{j=0}^{l-1} \rho^{l-j} u(j) - \rho u(l) + \\
&+ \Psi \rho \sum_{j=0}^{l-1} \rho^{l-j} h(j) + \Psi \rho h(l) = \frac{y_{ref}}{\Omega} - \rho \frac{y_{ref}}{\Omega} + \quad (29) \\
&+ \rho \left[\frac{y_{ref}}{\Omega} - \sum_{j=0}^{l-1} \rho^{l-j} u(j) + \Psi \sum_{j=0}^{l-1} \rho^{l-j} h(j) \right] - \rho u(l) + \\
&+ \Psi \rho h(l) = (1-\rho) \frac{y_{ref}}{\Omega} + \Psi \rho h(l)
\end{aligned}$$

Since l is an arbitrary positive integer, it follows from the principle of mathematical induction that (25) is true for any integer $k \geq 1$. This concludes the proof of the lemma.

Theorem 1: If the proposed warehouse management policy is applied, then for any $k \geq 0$ the control signal satisfies the following inequalities

$$(1-\rho) \frac{y_{ref}}{\Omega} \leq u(k) \leq \max \left[\frac{y_{ref}}{\Omega}, (1-\rho) \frac{y_{ref}}{\Omega} + \Psi \rho d_{\max} \right]. \quad (30)$$

Proof: It follows directly from (24) that $u(0) = y_{ref}/\Omega$, and this implies that the theorem is satisfied for $k = 0$. Moreover, since the demand is always bounded as stated by inequalities (3), then for any $k > 0$, from the lemma proved above, we obtain

$$(1-\rho) \frac{y_{ref}}{\Omega} \leq u(k) \leq (1-\rho) \frac{y_{ref}}{\Omega} + \Psi \rho d_{\max}, \quad (31)$$

which ends the proof of Theorem 1.

The next theorem states another important property of the proposed policy, namely it shows that the inventory level never exceeds its reference value. This theorem shows that if the warehouse capacity is selected at least equal to y_{ref} , then enough storage space at the distribution centre for all incoming shipments will always be provided.

Theorem 2: If the proposed inventory management policy is applied, then for any $k \geq 0$ the stock level is always upper bounded by y_{ref} , i.e.

$$y(k) \leq y_{ref}. \quad (32)$$

Proof: Due to initial conditions and lead-time delay, the considered warehouse is empty for any $k \leq n_1$. Hence, we need to show that inequality (32) holds for any $k \geq n_1$.

Let us assume that for some integer $l \geq n_1$, $y(l) \leq y_{ref}$. Then, we will demonstrate that this inequality is also satisfied for $l + 1$. The stock level at the time instant $l + 1$, based on the inventory balance equation, can be expressed as

$$y(l+1) = \rho y(l) + \sum_{i=1}^r \alpha_i \beta_i u(l-n_i) - h(l). \quad (33)$$

Substituting (7) and (27) into (33), we obtain

$$\begin{aligned}
y(l+1) &= \rho y(l) + \\
&+ \sum_{i=1}^r \alpha_i \beta_i \frac{y_{ref}}{\Omega} - \sum_{i=1}^r \alpha_i \beta_i \sum_{j=0}^{l-n_i-1} \rho^{l-n_i-j} u(j) + \\
&+ \Psi \sum_{i=1}^r \alpha_i \beta_i \sum_{j=0}^{l-n_i-1} \rho^{l-n_i-j} h(j) - h(l) = \rho y(l) + \quad (34) \\
&+ y_{ref} - \rho y(l) - \Psi \sum_{i=1}^r \alpha_i \beta_i \sum_{j=l-n_i}^{l-1} \rho^{l-n_i-j} h(j) - h(l) = \\
&= y_{ref} - \Psi \sum_{i=1}^r \alpha_i \beta_i \rho^{-n_i} \sum_{j=l-n_i}^{l-1} \rho^{l-j} h(j)
\end{aligned}$$

Since $h(k)$ is always nonnegative, then $y(l+1) \leq y_{ref}$. Using the principle of the mathematical induction we conclude that the theorem is satisfied for any $k \geq 0$.

Now we will formulate and prove the last theorem, which shows how to select the reference stock level, so that full demand satisfaction is guaranteed. In other words, this theorem demonstrates how big warehouse capacity is needed, to ensure that all sales are realized from the readily available resources.

Theorem 3: If the proposed inventory policy is applied, and the target stock level satisfies the following inequality

$$y_{ref} > \Psi d_{\max} \sum_{i=1}^r \alpha_i \beta_i \rho^{-n_i} \sum_{j=0}^{n_i} \rho^j, \quad (35)$$

then for any $k \geq n$ the stock level is strictly positive.

Proof: Assumption (3) implies that the realized demand is always upper bounded. Hence, taking into account (28), (34) and (35), for any $k \geq n$, we obtain

$$\begin{aligned}
y(k) &= y_{ref} - \Psi \sum_{i=1}^r \alpha_i \beta_i \rho^{-n_i} \sum_{j=l-1-n_i}^{l-1} \rho^{l-1-j} h(j) \geq \\
&\geq y_{ref} - \Psi d_{\max} \sum_{i=1}^r \alpha_i \beta_i \rho^{-n_i} \sum_{j=l-1-n_i}^{l-1} \rho^{l-1-j} > 0
\end{aligned} \quad (36)$$

This concludes the proof.

IV. NUMERICAL EXAMPLE

In order to verify the effectiveness of the proposed supply policy we performed a number of simulation tests.

The system parameters considered in the tests are: review period $T=1$ day, inventory deterioration rate $\sigma=0.07$ (which results in $\rho=1-0.07=0.93$), and the maximum daily demand at the distribution centre $d_{max}=50$ items. The warehouse is replenished from 4 supply sources, and the lead-time delay of each source is: $L_1=n_1T=1$ day, $L_2=n_2T=2$ days, $L_3=n_3T=4$ days, $L_4=n_4T=6$ days. Transportation losses in each transportation channel are: $\alpha_1=0.97$, $\alpha_2=0.93$, $\alpha_3=0.9$ and $\alpha_4=0.86$. Moreover, assuming order partitioning $\beta_1=0.2$, $\beta_2=0.25$, $\beta_3=0.25$, $\beta_4=0.3$, one gets elements a_i ($i=2, \dots, 7$) of first row of state matrix equal to $a_2=0.194$, $a_3=0.236$, $a_4=0$, $a_5=0.215$, $a_6=0$, $a_7=0.258$. Thus Ψ is equal to 0.869, and Ω equals 0.9. According to Theorem 3, in order to obtain full demand satisfaction the reference stock level should be greater than 182.08 items. Thus, we set $y_{ref}=190$ items.

The simulations are run for demand pattern depicted in Fig. 2. This pattern is chosen to test the controller response to abrupt changes of demand, and to verify controller performance in the presence of stochastic demand. The stochastic component turns up after 50 review periods.

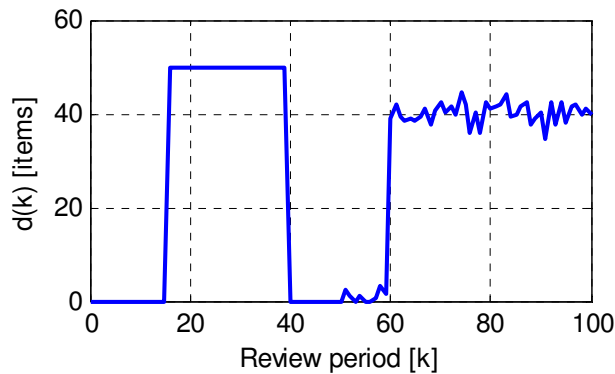


Fig. 2. Imposed demand.

The orders generated by the proposed policy are shown in Fig. 3. It can be easily seen from the figure that the control signal is always nonnegative and bounded. Moreover, the controller rapidly reacts to sudden changes in the demand without oscillations and overshoots.

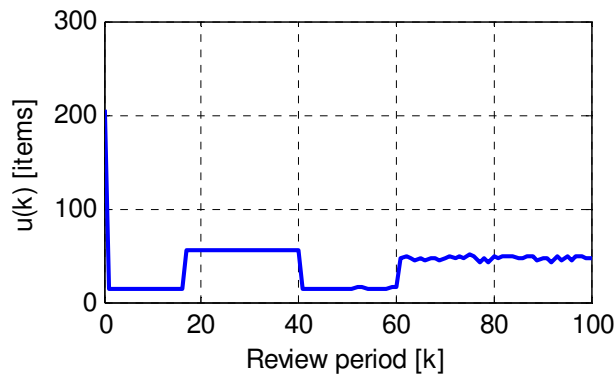


Fig. 3. Order quantities.

Fig. 4 presents the on-hand stock volume. It can be seen from the figure that this volume never drops to zero and does

not increase beyond warehouse capacity y_{ref} . This implies that our supply policy works properly and after the initial phase of the control process it always guarantees full satisfaction of the market demand.

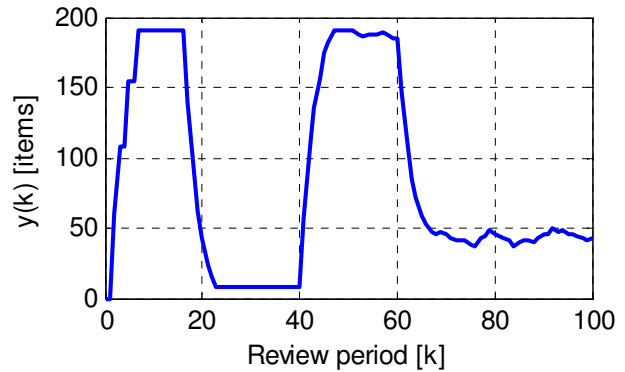


Fig. 4. On-hand stock.

V. CONCLUSIONS

In this paper a new periodic review supply chain management strategy has been proposed. The strategy takes into account perishable inventories with transportation losses, i.e. not only it explicitly concerns goods decay in the warehouse, but it also accounts for the losses which take place during the delivery process. The proposed strategy ensures full demand satisfaction, eliminates the risk of warehouse overflow and always generates non-negative and bounded orders.

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