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Optimal Decoupling Controllers for Singular Systems

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***Abstract*—The problem of decoupling a singular, linear, and time-invariant system by dynamic compensation into multi-input multi-output subsystems is studied. The set of all controllers that decouple the system and render it regular, proper and stable is determined in parametric form. Optimal and suboptimal decoupling controllers are then obtained by an appropriate selection of the parameters.**

I. Introduction

DECOUPLING is a way to decompose a complex system into non-interacting subsystems. In fact, certain applications necessitate controlling independently different parts of the system. Even if this is not required, the absence of interaction can significantly simplify the synthesis of the desired control laws.

The decoupling problem has received much attention in the literature. For linear time-invariant systems, different approaches have been used and control laws of various structure and complexity applied.

A. State Space Systems

The basic form of decoupling into single-input single-output subsystems is often referred to as the diagonal decoupling. This problem was posed by Voznesenskij [1] and studied by Kavanagh [2], Strejc [3], and Mejerov [4]. The studies were related to the inversion problem of rational matrices. Attention was paid to the existence of proper rational transfer matrices. The issue of stability, however, was not properly addressed.

A deeper insight was provided by the state-space approach. The pioneering work is due to Morgan [5], who posed the problem of decoupling by static state feedback. A solvability condition was given by Falb and Wolovich [6].

The use of a restricted static state feedback, namely the static output feedback, in decoupling was studied by Howze and Pearson [7], Howze [8], and Descusse et al. [9]. This is a very restricted problem, whose solution is hard to obtain, but it is very useful in applications.

A more general form of decoupling into multi-input multi-output subsystems is referred to as the block decoupling. This problem was introduced by Wonham and Morse [10] and Basile and Marro [11]. Using a geometric approach, they determined the solvability of the problem by static state feedback in several special cases.

The decoupling by dynamic state feedback was studied via the geometric approach by Morse and Wonham [12], who obtained a deep insight into the internal structure of the decoupled system. By this time, the problem of decoupling a state space system by dynamic state feedback was solved, including stability or pole distributions that may be achieved while preserving a decoupled structure. The status of non-interacting control was reviewed by Morse and Wonhan [13].

A comeback of the transfer function methods in the study of block decoupling is witnessed through the works of Koussiouris [14] and Hautus and Heymann [15]. A dynamic state feedback was shown to be equivalent with combined dynamic output feedback and feedforward reference compensation, often referred to as a two-degree-of-freedom controller. To address stability issues, the Youla-Kučera parameterization of all stabilizing controllers was invoked, see Kučera [16]. The class of all decoupled transfer matrices that can be achieved by a stabilizing controller was parameterized by Desoer and Gündeş [17]. This result has made it possible to derive the H_2-optimal decoupling controller, see Lee and Bongiorno [18], which minimizes the performance deterioration due to decoupling.

The two-degree-of-freedom controller structure is ideally suited to decoupling since only one of the degrees of freedom is affected by the decoupling requirement. This is not true for a pure feedback, or a one-degree-of-freedom controller. This case is more difficult to solve, as shown by Lin [19], Youla and Bongiorno [20], and Park [21].

B. Singular Systems

The decoupling problem has received due attention also for singular (or descriptor, or implicit) linear time-invariant systems.

The diagonal decoupling by proportional state feedback was studied by Dai [22], who recognized the importance of avoiding impulsive modes. Necessary and sufficient conditions for this type of decoupling were presented by Ailon [23] and Paraskevoupoulos and Komboulis [24]. A polynomial equation approach to obtain a decoupling controller was considered by Gören and Güzelkaya [25] and a matrix fraction description approach was applied by

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Vafiadis and Karcanias [26].

A particular attention was focussed on decoupling by proportional-plus-derivative state feedback, which is a natural type of feedback to use in singular systems. Christodoulou and Paraskevopoulos [27] derived necessary and sufficient solvability conditions and Christodoulou [28] characterized all feedback matrices that decouple the system. Koumboulis and Mertzios [29] then established conditions for decoupling with simultaneous stabilizability or arbitrary pole assignment. As a point of interest, it was shown in [24] that the diagonal decoupling problem of a singular system by static state feedback can be recast as the decoupling problem of a state space system via pure derivative state feedback.

The use of a restricted proportional state feedback, in particular of the static output feedback, to diagonally decouple the system was considered by Paraskevopoulos and Koumboulis [30].

Diagonal decoupling by dynamic state feedback was studied by Mertzios and Christodoulou [31], who gave sufficient solvability conditions and, in case these conditions are satisfied, they determined the class of controller matrices that decouple the system.

The more general form of decoupling into multi-input multi-output subsystems was addressed by Vafiadis and Karcanias [32] using proportional state feedback. The dynamic state feedback or, equivalently, the output feedback combined with dynamic feedforward, has not been studied in the context of the block decoupling of singular systems.

This paper considers a class of singular systems that give rise to a transfer function. This assumption makes it possible to pose the decoupling problem in the most general yet meaningful setting: a singular system in which the measurement output may be different from the output to be decoupled and a singular dynamic controller that features both feedback and feedforward parts. The class of all such controllers that decouple the system and render it useful in applications (making it proper and stable) is determined in parametric form and the parameter is used to obtain the H_2-optimal controller. The solution follows Kučera [33], [34] and it is simple and direct. The controller configuration implies that decoupling and properness/ stability requirements are two independent issues.

II. Problem Formulation

Consider a linear and time-invariant system described by the equations

$$\overline{E}\dot{x}=\overline{A}x+\overline{B}u,\ y=\overline{C}_y x+\overline{D}_y u, \qquad (1)$$

where $x\in R^n$ is a descriptor variable, $u\in R^q$ is the input, $y\in R^p$ is the output, and $\overline{A},\overline{B},\overline{C}_y,\overline{D}_y$ and $\overline{E}$ are real matrices of appropriate dimensions, with E being singular. Such a system is called *singular*, or descriptor, or implicit system and it features impulsive as well as exponential modes. A system defined by (1) is said to be *regular* if the pencil $s\overline{E}-\overline{A}$ is non-singular. Thus, a regular system has a transfer function. Following Kučera [35], a regular system (1) is said to be *proper* if $(s\overline{E}-\overline{A})^{-1}$ is a proper rational matrix (i.e., having no poles at $s=\infty$) and it is said to be *stable* if $(s\overline{E}-\overline{A})^{-1}$ is a stable rational matrix (i.e., having no poles in $\operatorname{Re} s\geq 0$). Thus, a proper system is devoid of impulsive modes while a stable system is devoid of non-decaying exponential modes.

The transfer function of a regular system of the form (1) is

$$S_y(s)=\overline{C}_y(s\overline{E}-\overline{A})^{-1}\overline{B}+\overline{D}_y$$

and it is a matrix over R(s), the field of real rational functions.

Let $p_1, ..., p_k$ be given positive integers that satisfy

$$\textstyle\sum_{i=1}^{k} p_i=p\ .$$

System (1) is said to be *decoupled*, or more specifically ($p_1, ..., p_k$)-decoupled, if there exist positive integers $q_1, ..., q_k$ satisfying

$$\textstyle\sum_{i=1}^{k} q_i=q$$

such that S_y has the block diagonal form

$$S_y:=\begin{bmatrix} S_1 & & \\ & \ddots & \\ & & S_k \end{bmatrix},$$

where S_i is $p_i\times q_i$.

This is not a generic property of the system, but it can be achieved by a suitable compensation [17]. To this effect, let

$$z=\overline{C}_z x+\overline{D}_z u, \qquad (2)$$

with $z\in R^m$ and $\overline{C}_z$, $\overline{D}_z$ real matrices of appropriate dimensions, denote the output that is available for measurement. The corresponding transfer function of a regular system defined by (1) – (2) is

$$S_z(s)=\overline{C}_z(s\overline{E}-\overline{A})^{-1}\overline{B}+\overline{D}_z$$

and it is a matrix over R(s), the field of real rational functions.

The most suitable singular, linear and time-invariant controller can then be described by the equation

$$\widetilde{E}\dot{w}=\widetilde{A}w+\widetilde{B}_v v+\widetilde{B}_z z,\ u=\widetilde{C}w+\widetilde{D}_v v+\widetilde{D}_z z, \qquad (3)$$

where $v\in R^r$ is an external reference input. A regular controller of the form (3) has the transfer function between v and u given by

$$K_v(s)=\widetilde{C}(s\widetilde{E}-\widetilde{A})^{-1}\widetilde{B}_v+\widetilde{D}_v$$

and the transfer function between z and u given by

$$K_z(s)=\widetilde{C}(s\widetilde{E}-\widetilde{A})^{-1}\widetilde{B}_z+\widetilde{D}_z\ .$$

These transfer functions are defined over R(s), the field of real rational functions.

Note that the system and controller transfer functions are rational, not necessarily proper and stable matrices.

The controller (3) is a two-degree-of-freedom controller. The resulting closed-loop control system is shown in Fig.1.

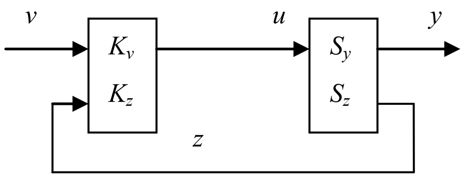

Fig. 1. Control system

The *decoupling problem* is then to find matrices K_v and K_z such that

(i) the control system is regular;

(ii) the transfer matrix

$$T = S_y(I - K_z S_z)^{-1} K_v$$

from v to y is suitably block diagonal.

Obviously, unless additional provisions are made, the decoupling problem is trivial as it could be solved by $K_v = 0$. Thus it is necessary to impose certain admissibility condition on the decoupling controller in order to make the problem meaningful, for example

$$\text{rank}\, T = \text{rank}\, S_y$$

over R(s), the field of rational functions.

Another requirement, frequently imposed on the decoupled system in practice, is that of properness and stability. This requirement means that the system is devoid of impulsive modes and all its exponential modes converge to zero from any initial values.

III. PRELIMINARIES

A proper and stable singular system gives rise to a proper and stable transfer function. In order to study properness and stability of the decoupled system it is convenient to express the transfer matrices S_z, S_y and K_z, K_v in the following fractional form

$$\begin{bmatrix} S_z \\ S_y \end{bmatrix} := \begin{bmatrix} B \\ C \end{bmatrix} A^{-1} \tag{4}$$

$$\begin{bmatrix} K_z & K_v \end{bmatrix} := P^{-1} \begin{bmatrix} -Q & R \end{bmatrix}, \tag{5}$$

where

$$A, \begin{bmatrix} B \\ C \end{bmatrix}$$

are proper and stable rational matrices that are right coprime and

$$P, \begin{bmatrix} -Q & R \end{bmatrix}$$

are proper and stable rational matrices that are left coprime.

These proper and stable fractional representations exist and are unique up to right and left multiplication, respectively, by a unimodular matrix. Recall that a proper and stable rational matrix is said to be *unimodular* if its inverse exists and is proper and stable.

The overall system transfer function then reads

$$T = C(PA + QB)^{-1} R\,. \tag{6}$$

The fundamental assumption we make here is that the part of the given system that is not controllable from u is proper and stable and the part of the given system that is not jointly observable from y, z is proper and stable. Similarly, we assume that the controller is realized in such a manner that its part that is not jointly controllable from v, z is proper and stable and its part that is not observable from u is proper and stable. Thus, the uncontrollable/ unobservable parts of the system are devoid of impulsive and non-decaying exponential modes.

The issue of regularity, properness, and stability of the overall system is then solved as follows.

Lemma1. The overall system described by (4) and (5) is regular, proper, and stable if and only if the matrix $PA + QB$ is unimodular.

Proof. In the overall system, inject inputs f and g as shown in Fig. 2.

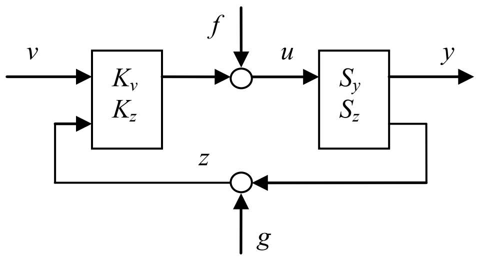

Fig. 2. Control system with the complete set of independent inputs and outputs

It follows from (6) that the overall system is regular if and only if the matrix $PA + QB$ is non-singular. Furthermore, the overall system is proper and stable if and only if the nine transfer matrices between the inputs f, g, v and the outputs u, y, z given by

$$\begin{bmatrix} u \\ z \\ y \end{bmatrix} = \begin{bmatrix} A \\ B \\ C \end{bmatrix} (PA + QB)^{-1} \begin{bmatrix} P & -Q & R \end{bmatrix} \begin{bmatrix} f \\ g \\ v \end{bmatrix}$$

are all proper and stable rational. This statement follows from the assumption of properness and stability of the uncontrollable and unobservable parts of the system.

Now, in view of the coprimeness assumptions on A, B, C and P, Q, R these transfer matrices are proper and stable if and only if $PA + QB$ is a unimodular matrix. □

IV. PROBLEM SOLVABILITY

A simple necessary and sufficient condition will now be established for a system to be decoupled and proper and stable.

Based on the partition $(p_1, ..., p_k)$, write

$$C := \begin{bmatrix} C_1 \\ \vdots \\ C_k \end{bmatrix}, \quad (7)$$

where C_i is a $p_i \times q$ submatrix.

Theorem 1. Given a regular system (1) – (2) in fractional form (4) and partition (7), there exists an admissible regular controller (3) in fractional form (5) such that the overall system is

(i) proper and stable if and only if

A and B are right coprime,

(ii) decoupled if and only if

$$\sum_{i=1}^{k} \text{rank } C_i = \text{rank } C . \quad (8)$$

Proof. (i) Let the overall system be regular, proper, and stable. By Lemma 1, the matrix $PA + QB$ is unimodular whence A and B must be right coprime.

Conversely, let the matrices A and B in (4) be right coprime. Then there exist proper and stable rational matrices P and Q such that

$$PA + QB = I \quad (9)$$

with P invertible and the inverse of P proper.

Then controller (3) in fractional form (5) that is defined by the matrices P and Q from (9) and by an arbitrary proper and stable rational matrix R satisfying rank CR = rank C is admissible since, by (6),

$$\text{rank } T = \text{rank } CR = \text{rank } C = \text{rank } S_y .$$

The resulting system (1), (2), and (3) is regular, proper, and stable in view of Lemma 1 and identity (9).

(ii) Let (5) define an admissible decoupling controller for a system described by (4). Denote

$$K := (PA + QB)^{-1} R .$$

The block diagonal property of the matrix T then implies

$$\text{rank } CK = \sum_{i=1}^{k} \text{rank } C_i K$$

and the admissibility of the controller gives

$$\text{rank } C_i K = \text{rank } C_i ,\ i = 1,...,k.$$

Therefore (8) holds.

The sufficiency will be proved by constructing a suitable R. Denote

$$r_i := \text{rank } C_i ,\ \ i = 1,...,k .$$

Then there exists a $p_i \times p_i$ unimodular proper and stable rational matrix U_i such that

$$C_i = U_i \begin{bmatrix} C_i' \\ 0 \end{bmatrix},$$

where the rows of C_i' are linearly independent over $R(s)$ and where the zero matrix has $p_i - r_i$ rows and may be empty. If (8) holds, then

$$C' := \begin{bmatrix} C_1' \\ \vdots \\ C_k' \end{bmatrix}$$

has linearly independent rows over $R(s)$. Hence there exists a $q \times q$ unimodular, proper and stable rational matrix U' such that

$$C'U' := \begin{bmatrix} D_1 & & & 0 \\ & \ddots & & \vdots \\ & & D_k & 0 \end{bmatrix}, \quad (10)$$

where D_i is an $r_i \times r_i$ diagonal proper and stable rational matrix and where the zero matrices have $q - r$ columns with r defined by

$$r := \sum_{i=1}^{k} r_i .$$

Define an admissible controller (5) by the matrices P and Q from (9) and by the matrix R formed by the first r columns of U'. The transfer matrix (6)

$$T = CR = \begin{bmatrix} U_1 & & \\ & \ddots & \\ & & U_k \end{bmatrix} \begin{bmatrix} \begin{bmatrix} D_1 \\ 0 \end{bmatrix} & & \\ & \ddots & \\ & & \begin{bmatrix} D_k \\ 0 \end{bmatrix} \end{bmatrix}$$

is block diagonal. The resulting system is therefore decoupled and the reference input v has dimension r. □

The interpretation of these solvability conditions is as follows. Condition (i) means that the part of the given system that is not observable from the measured output z is free of impulsive and unstable exponential modes. Condition (ii) calls for the linear independence of any two outputs of the given system that belong to different blocks. The solvability of the decoupling problem thus strongly depends on the partition $(p_1, ..., p_k)$, that is to say, upon the allocation of the outputs into the blocks.

V. CONTROLLER PARAMETERIZATION

When a decoupling controller exists that renders the system regular, proper, and stable, we shall parameterize the class of all such controllers.

The control system given by (4), (5) is regular, proper, and stable if and only if $PA + QB$ is a unimodular matrix by Lemma 1. Thus, these requirements involve only the feedback part K_z of the controller, which surrounds the measurement subsystem S_z. As a result, the parameterization of K_z amounts to the well-known Youla-Kučera parameterization of feedback stabilizing controllers For details, see Kučera [36], [37] and Youla et al. [38].

Let $\overline{P}, \overline{Q}$ be any solution pair of equation (9). Then the solution class of (9) is given by

$$P = \overline{P} + W\overline{B},\, Q = \overline{Q} - W\overline{A}\,, \quad (12)$$

where $\overline{A}$ and $\overline{B}$ are left coprime, proper and stable rational matrices such that

$$\overline{A}^{-1}\overline{B} = BA^{-1}$$

and W is an arbitrary proper and stable rational matrix parameter.

The class of all rational K_z that render the system regular, proper, and stable is then obtained in the form

$$K_z = -P^{-1}Q = -(\overline{P} + W\overline{B})^{-1}(\overline{Q} - W\overline{A})\,,$$

where the parameter W is constrained so that the inverse of $\overline{P} + W\overline{B}$ exists and is proper rational.

Once the control system given by (4) and (5) is regular, proper, and stable, it is decoupled if and only if $T = CR$ by (6). Thus decoupling involves only the feedforward part K_v of the controller.

Partition the $q \times q$ unimodular matrix U' defined in (10) as

$$U' = \begin{bmatrix} U'_r & U'_{q-r} \end{bmatrix},$$

where U'_r has r columns and U'_{q-r} has $q - r$ columns and may be empty. The class of all decoupling K_v is then given by $K_v = P^{-1}R$ with P determined in (12) and

$$R = U'_r \begin{bmatrix} V_1 & & \\ & \ddots & \\ & & V_k \end{bmatrix}, \quad (13)$$

where V_i is an arbitrary $r_i \times r_i$ proper and stable rational matrix parameter. The matrices V_1, ..., V_k in turn parameterize the class of achievable block-diagonal transfer matrices (6) as follows

$$T = \begin{bmatrix} U_1 & & \\ & \ddots & \\ & & U_k \end{bmatrix} \begin{bmatrix} \begin{bmatrix} D_1 \\ 0 \end{bmatrix} & & \\ & \ddots & \\ & & \begin{bmatrix} D_k \\ 0 \end{bmatrix} \end{bmatrix} \begin{bmatrix} V_1 & & \\ & \ddots & \\ & & V_k \end{bmatrix}. \quad (14)$$

The parameterization (12) – (13) reveals that decoupling and properness/stabilization are two independent issues.

VI. ASYMPTOTIC TRACKING

The decoupling constraint can deteriorate system's performance. The bonus of having a parameterized solution set is that the lost performance can easily be controlled by an appropriate choice of the parameters V_1, ..., V_k and W.

Suppose that the control objective is for each block of outputs y_i to asymptotically track the corresponding block of reference inputs v_i. Thus suppose that $p_i = r_i$ for $i = 1$, ..., k, i.e., there are as many reference inputs as controlled outputs in each block. The tracking error for each block is

$$e_i := v_i - y_i = H_i v_i\,.$$

Suppose that the reference input is given by

$$v_i = G_i^{-1} g_i\,, \quad (15)$$

where G_i is a fixed proper and stable rational matrix and g_i is an unspecified proper and stable rational vector that captures the effect of initial conditions. Thus (15) defines a class of references with a specified dynamics.

Asymptotic tracking means that

$$e_i = H_i G_i^{-1} g_i$$

is a proper and stable rational vector. Thus G_i must be absorbed in H_i . In view of (17), H_i has the generic form

$$H_i = I - F_i V_i\,,$$

where $F_i := U_i D_i$ and V_i are proper and stable rational matrices with F_i fixed and V_i an arbitrary parameter to be specified. Therefore, asymptotic tracking is possible if and only if there exists a proper and stable rational matrix Z_i satisfying

$$F_i V_i + Z_i G_i = I\,. \quad (16)$$

Let $\overline{V}_i, \overline{Z}_i$ be any solution pair of equation (16). Then the solution class of (16) is given by

$$V_i = \overline{V}_i + N_i G_i,\, Z_i = \overline{Z}_i - F_i N_i\,,$$

where N_i is an arbitrary proper and stable rational matrix parameter. Thus, the set of reference-to-error transfer functions that achieve asymptotic reference tracking in a decoupled system is

$$H_i = \overline{Z}_i G_i - F_i N_i G_i\,. \quad (17)$$

VII. OPTIMAL CONTROLLERS

The benefits of controller parameterization will now be demonstrated in the case of H_2 control design [34].

Suppose that for each block, the reference-to-error transfer function H_i parameterized in (17) is to have least H_2 norm with respect to N_i. So as to achieve this task, determine the inner-outer factorization of F_i ,

$$F_i := F_{iI} F_{iO}\,,$$

where F_{iI} is inner and F_{iO} is outer. Note that G_i is outer for typical references such as steps, ramps, or harmonic signals.

As F_{iI} is inner, premultiplication by F_{iI}^{-1} preserves the H_2 norm,

$$\left\| H_i \right\| = \left\| F_{iI}^{-1} H_i \right\| = \left\| F_{iI}^{-1} \overline{Z}_i G_i - F_{iO} N_i G_i \right\|.$$

Write

$$F_{iI}^{-1} \overline{Z}_i G_i = F_{iI}^{-1} K_i + L_i\,,$$

where K_i, L_i are proper and stable rational matrices with K_i strictly proper. Note that F_{iI}^{-1} has poles only in Re$s > 0$. Then

$$\|H_i\|^2 = \|F_{iI}^{-1}K_i + (L_i - F_{iO}N_iG_i)\|^2 = \|F_{iI}^{-1}K_i\|^2 + \|L_i - F_{iO}N_iG_i\|^2 \quad (18)$$

because the cross terms contribute nothing to the norm. This is a complete square in which only the second term depends on N_i. Therefore, a unique N_i that attains the minimum of the norm for subsystem i is

$$N_i = F_{iO}^{-1}L_iG_i^{-1} \quad (19)$$

provided N_i is proper and stable rational matrix.

VIII. SUBOPTIMAL CONTROLLERS

Unfortunately, matrix (18) is generically unstable for typical references due to the presence of $j\omega$-zeros in G_i. This impasse can be obviated by sacrificing the optimality and focusing on suboptimal controllers.

Select proper and stable rational matrices M_i, N_i so that

$$L_i = M_i + F_{iO}N_iG_i$$

holds with M_i strictly proper and having a small H_2 norm; in fact, as small as desired. Then, using (21),

$$\|H_i\|^2 = \|F_{iI}^{-1}K_i\|^2 + \|M_i\|^2$$

and the parameter M_i defines a suboptimal controller, for which the resulting H_2 norm of H_i is only an incremental addition to the unattainable infimum.

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