

Driving microalgal production in raceway systems to near optimal productivities

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Abstract—We propose a simple operational criterion for raceway systems that when integrated in a closed-loop controller allows to attain microalgal productivities very near to the maximal theoretical productivities. The strategy developed was tested numerically by using a mathematical model of microalgal growth in raceways.

I. INTRODUCTION

Microalgae are promising feedstocks for the production of high value compounds including biodiesel [1]. However, despite this favorable potential, microalgae production in large scale basis is probably carried out far from an optimal working mode. Here, we refer to the technology of raceways, which are the systems generally used for large microalgae production.

The difficulty of achieving optimal productivities of microalgae in outdoor systems results from the high interaction of phenomena that take place during growth and the low level of control that we have on them. This factor makes the whole process inefficient under an environment that is fluctuating by nature.

The challenge of optimizing microalgal culture systems is a broad endeavour that includes reactor design and strain selection. Furthermore, optimal performances can be achieved by acting on operational variables, such as nutrient feeding rate. In this aspect, mathematical models are powerful tools, because optimization can be model driven. Since microalgal metabolism is mainly influenced by nutrient availability, light intensity and temperature, several models have been developed to account for these factors [2], [3]. A work of synthesis has been done to provide a representative model of microalgal dynamics by keeping a relative simple structure [4] that might be suitable for control processes.

The task of bringing a process to optimality by acting on the system inputs is the goal of optimal

control. The optimal control problem, here considered, consists in finding the trajectories of the manipulated variables maximizing a given criterion on a finite time horizon. This problem can be solved by using the Pontryagin's maximum principle or by numerical optimization. By applying the Pontryagin's principle, a theoretical study on a simplified model of microalgae growth provided guidelines on the form of an optimal controller of productivity [5]. A series of simplifications were needed to provide a simple model suitable for applying the Pontryagin's maximum principle. The distance between the resulting control and the optimal strategy for a more realistic model is therefore difficult to assess.

For complex models, methods based on numerical optimization are, in practice, the most used. The numerical approach often takes place in open loop (OL) fashion, without taking into account the real state of the system. For a real implementation, however, available measurements (either online or offline) must be used to compensate for disturbances and to correct model mismatches [6]. An example of this type of strategy was developed for the optimization of biomass productivities for the heterotrophic microalgae *Auxenochlorella protothecoides* [7]. This approach, however, is computationally expensive and requires sophisticated algorithms of adaptation and proof of process stability is lacking.

A practical alternative for attaining optimal performance consists in developing a closed loop (CL) control strategy that will approach the performances of the optimal law [8]. For photobioreactors, the phenomenon of light transfer to the culture governs the performance of the system [9]. Based on this principle, we propose in this work a simple operational criterion that implies near optimal performance. The proposed strategy has the advantage that it can be implemented in a straightforward manner in a classical CL control.

As a basis, we integrated the models developed by [4], [10], [11] for a planar culturing device. These models have shown to reproduce experimental data of lab scale systems. Here, we extend such models to account for characteristics of raceway systems. Our *in silico* case study takes the con-

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figuration of a pilot-scale open raceway (Algotron) located at INRA-LBE, Narbonne (France).

II. MODELING

Under the assumption that nitrogen and light are the limiting factors for the growth of microalgae, we combined the biomass model from [4] to the lipid production model proposed in [10], [11]. It results in the following mass balance equations for a completely mixed reactor at constant volume V

$$\dot{s} = f_i s_{in}/V - f_i s/V - \rho(\cdot)x, \quad (1)$$

$$\dot{q}_n = \rho(\cdot) - (\mu(\cdot) - R(\cdot))q_n, \quad (2)$$

$$\dot{x} = (\mu(\cdot) - f_i/V - R(\cdot))x, \quad (3)$$

$$\dot{x}_l = \beta q_n \mu(\cdot)x - \gamma \rho(\cdot)x - r_0 \phi_T x_l - f_i x_l/V, \quad (4)$$

where s (g N m⁻³) is the extracellular nitrogen concentration and q_n (g N (g C)⁻¹) is the internal nitrogen quota. The model includes the concentration of the total carbon biomass x (g C m⁻³) and the carbon biomass concentration of storage lipids x_l (g C m⁻³). The influent nitrogen concentration is s_{in} (g N m⁻³). The influent flow rate is f_i (m³ d⁻¹). The functions $\mu(\cdot)$ and $\rho(\cdot)$ represent the kinetics of growth rate and nitrogen uptake. The respiration rate is defined by $R(\cdot)$. The definition of the model parameters and their values are given in Table I.

The temperature exerts a strong influence on microalgae growth. This effect is included in the model in two manners. Firstly, it is assumed, in line with [3], that temperature has an homogeneous effect on uptake, growth and respiration rates. Secondly, following the work of [2], the chlorophyll:nitrogen ratio (Chl:N) was set to be dependent on the temperature and light. The equations are detailed later on.

To model the growth rate, the following is assumed:

(i) Microalgal growth is uncoupled dynamically to nutrient uptake. Growth kinetics follows the cell quota model of Droop [12].

(ii) The absorption of light in the raceway follows the Lambert-Beer law. Thus, for a given depth z , the corresponding light intensity I_z ($\mu\text{E m}^{-2}\text{s}^{-1}$) satisfies

$$I_z = I_0 \exp(-\xi z), \quad (5)$$

where I_0 ($\mu\text{E m}^{-2}\text{s}^{-1}$) is the incident light and ξ is the light attenuation factor, which depends on the chlorophyll concentration Chl

$$\xi = a\text{Chl} + b. \quad (6)$$

At the bottom of the reactor $z = L$. The term ξL is known as optical depth (λ). It should be noted that I_0 varies in time in an oscillatory fashion. For a given day, I_0 follows an increasing behaviour until

noon, then decreases until midnight.

(iii) Light intensity affects the growth rate. This effect is described by a Monod type kinetics. For a given depth z ($0 \leq z \leq L$) with intensity I_z , the growth rate at hypothetical infinite nitrogen quota is

$$\mu_z = \bar{\mu} \frac{I_z}{I_z + K_{SI}}. \quad (7)$$

Finally, the growth rate is represented by an average growth rate obtained by integration of (7) along the raceway depth. The resulting equation for the growth rate reads

$$\mu(\cdot) = \bar{\mu} \phi_T \left(1 - \frac{Q_0}{q_n}\right), \quad (8)$$

with

$$\bar{\mu} = \frac{\bar{\mu}}{\xi L} \ln \frac{I_0 + K_{SI}}{I_0 e^{-\xi L} + K_{SI}},$$

$\phi_T =$

$$\frac{(T - T_{\max})(T - T_{\min})^2}{(T_{\text{opt}} - T_{\min}) [(T_{\text{opt}} - T_{\min})(T - T_{\text{opt}}) - (T_{\text{opt}} - T_{\max})(T_{\text{opt}} + T_{\min} - 2T)]}.$$

The term ϕ_T represents the effect of the temperature (T , °C). It is described by the model developed for bacteria by [13] and validated for microalgae by [14].

Nitrogen uptake rate ($\rho(\cdot)$) is modeled by a modified Michaelis-Menten kinetics [10].

$$\rho(\cdot) = \bar{\rho} \phi_T \frac{s}{s + K_s} \left(v + (1 - v) \frac{\bar{I}^m}{\bar{I}^m + \varepsilon_I^m} \right) \left(1 - \frac{q_n}{Q_l} \right). \quad (9)$$

The nitrogen uptake rate is a function of the average irradiance in the raceway \bar{I} . Additionally, nutrient uptake is regulated by the internal nitrogen quota, *i.e.*, when the cells are nutrient saturated, uptake rate stops. The equation includes a light regulating factor (in the form of a Hill-type function). Therefore, when the cells enter to the dark period, the nutrient uptake rate exhibits a slowdown.

The model includes an overall respiration rate $R(\cdot)$, that gathers maintenance respiration and biosynthesis cost:

$$R(\cdot) = r_0 \phi_T + \varphi \rho(\cdot), \quad (10)$$

with r_0 the maintenance respiration and φ the biosynthesis cost coefficient.

Additionally, it is assumed that chlorophyll concentration (Chl) is correlated to particulate nitrogen (xq_n) [4]. The Chl:N ratio (θ_N) is influenced by light and temperature following [2]

$$\theta_N^{-1} = (g_1 - g_2 T) + g_3 \bar{I} \exp(-g_4 T). \quad (11)$$

Here, it is implicitly assumed that the cells are photoacclimated at the average light intensity \bar{I} .

The environmental variables light intensity and temperature govern reactor performance. They can be accessible from online sensors and mathematical models [15]. In the present study, mathematical modeling supported by meteorological data was used for the location of Narbonne, France. The results presented here correspond to typical environmental conditions for the month of June.

Model parameters were taken from studies on the microalgae *Isochrysis* aff. *galbana*, when available. The parameters describing ϕ_T are those obtained for *Nannochloropsis oceanica* [14].

III. DRIVING RACEWAY OPERATION TO NEAR OPTIMAL PERFORMANCE.

A. Optimal problem statement

We are interested in designing a control law on the input flow rate (f_i) that allows to bring either the biomass productivity (P_x) or the lipid productivity (P_l) very close to their maximal values. For the time horizon t_f , the maximal productivities can be obtained by solving the following optimal control

$$\begin{aligned} \max_{f_i(t)} \int_{t_0}^{t_f} \psi(t, \mathbf{x}(t), f_i(t)) dt. \\ \text{s.t.} \\ 0 \leq f_i(t) \leq f_{\max} \\ \dot{\mathbf{x}} = \mathbf{g}(\mathbf{x}, f_i, t), \mathbf{x}(0) = \mathbf{x}_0. \end{aligned} \quad (12)$$

With \mathbf{x} the state vector and f_{\max} the upper bound of the input flow rate. For optimizing biomass productivity P_x ,

$$\psi(t, \mathbf{x}(t), f_i(t)) = f_i(t) x(t). \quad (13)$$

For optimizing lipid productivity P_l ,

$$\psi(t, \mathbf{x}(t), f_i(t)) = f_i(t) x_l(t). \quad (14)$$

For the sake of clarity, we will call CP_x the optimal controller that maximizes biomass productivity and CP_l the optimal controller that maximizes lipid productivity.

The Matlab toolbox DOTcvsSB [16] was used for solving the optimal control problems numerically.

B. Quasi optimal CL control

Solving the optimal control problem (12) might be computationally expensive and difficult to implement in practice. For a real implementation, it will be desirable to identify a controlled variable that when regulated towards a set point will ensure that the system operates close to optimality. In this respect and since light transfer is a crucial phenomenon for microalgal growth, we propose

TABLE I
MODEL PARAMETERS

Parameters	Definition	Value
β	Fatty acid synthesis coefficient	3.80 g C (g N) ⁻¹
ϵ_l	Dissociation light constant	50 $\mu\text{E m}^{-2}\text{s}^{-1}$
ϕ	Biosynthesis cost coefficient	1.30 g C (g N) ⁻¹
γ	Fatty acid mobilization coefficient	2.90 g C (g N) ⁻¹
ν	Reduction factor of nitrogen uptake during night	0.19
$\bar{\mu}$	Theoretical maximum specific growth rate	2.11 d ⁻¹
\bar{p}	Maximum uptake rate	0.10 g N (g C d) ⁻¹
a	Light attenuation due to chlorophyll	2.0 m ² (g Chl) ⁻¹
b	Light attenuation due to background turbidity	0.087 m ⁻¹
g_1	Coefficient Eq. (11)	16.74 g N (g Chl) ⁻¹
g_2	Coefficient Eq. (11)	0.39 g N (g Chl °C) ⁻¹
g_3	Coefficient Eq. (11)	0.0014
g_4	Coefficient Eq. (11)	g N (g Chl $\mu\text{E m}^{-2}\text{s}^{-1}$) ⁻¹
K_s	Nitrogen saturation constant	0.0015 (°C) ⁻¹
K_{sl}	Light saturation constant	0.018 g N m ⁻³
L	Pond depth	150 $\mu\text{E m}^{-2}\text{s}^{-1}$
m	Hill coefficient	0.30 m
Q_l	Saturation cell quota	3.0
Q_0	Minimal nitrogen cell quota	0.20 g N (g C) ⁻¹
r_0	Maintenance respiration rate	0.05 g N (g C) ⁻¹
s_{in}	Influent nitrogen concentration	0.01 d ⁻¹
S	Pond surface	50 g N m ⁻³
T_{\min}	Lower temperature for microalgae growth	57.0 m ²
T_{\max}	Upper temperature for microalgae growth	-0.20 °C
T_{opt}	Temperature at which growth rate is maximal	33.30 °C
V	Raceway volume	26.70 °C
		17.10 m ³

the efficiency of light absorption (η_L) to be such a controlled variable.

$$\eta_L = \frac{I_0 - I_L}{I_0} = 1 - \exp(-\xi L). \quad (15)$$

A CL controller to regulate η_L is therefore designed. As it will be shown hereafter, this simple controller has a very good ability to maintain the system close optimality.

For a given microalgae, there exists a set point η_L^* that maintains the system near to optimal productivities. In this study a set point $\eta_L^* = 0.95$ was selected. Note that regulating η_L implies the regulation of the optical depth λ . Given the form of the attenuation factor (6), regulating λ is equivalent to regulating the Chlorophyll concentration. This result is very convenient because during darkness we can still regulate the Chlorophyll concentration in such a way that when $I_0 > 0$ the efficiency of light absorption will be close to η_L^* . Note that the measurements of I_0 , I_L , Chl are easily accessible from online sensors.

In the next, we show by means of numerical

simulation, the performance of the raceway by regulating η_L to η_L^* . We use a standard PI controller. Since our premise is that this controller brings the system close to optimality, we call it a quasi optimal (QO) controller.

IV. RESULTS

A. Comparison of the QO controller to optimal strategy

Figure 1 shows the responses of the state variables and the lipid and biomass productivities (P_L , P_x) when applying the QO and the optimal CP_x controllers for a time period of 30 days. To calculate the productivities, it was assumed that carbon contributes to the 56% of ash-free dry weight [17]. The productivities are given by the cumulative production divided by the raceway surface and the elapsed time. The QO controller brings the efficiency of light absorption very close to the defined set point. At $t = 6.5$ d, η_L is 95% of η_L^* . The maximal biomass productivity obtained with the CP_x controller is 168 tons dry weight $ha^{-1} a^{-1}$. This value is consistent with productivities reported in the literature [1], [17]. Importantly, the biomass and lipid productivity provided by the QO controller were both 98% of those given by CP_x . After 25 d, the final lipid quota ($q_l = x_l/x$) oscillates with a maximal value of 16%. This relative low level of lipids is due to the fact that *I. galbana* is known to have a low lipid content.

To assess the maximal lipid productivity that can be attained, the optimal controller CP_l was calculated. For the model parameters used here, it resulted that optimizing lipid productivity was equivalent to optimizing biomass productivity. Thus, the response of the system behaviour when applying the CP_l controller was very similar to the response obtained when applying the CP_x controller. This result is interesting because it is often claimed the conflict between optimizing lipid productivity and optimizing biomass productivity. Indeed, such a conflict occurs when light is constant. For a diurnal light cycle, however, our results suggest that there is not discrepancy between optimizing lipid productivity and optimizing biomass productivity. For both performance indexes, the cells should grow as much as they can in the light period to accumulate enough carbon. The higher the biomass concentration, the higher the available carbon source that can be potentially directed to the lipid pool.

The results presented here are very promising. We show that the QO controller performs as well as the optimal controllers, confirming our hypothesis that regulating η_L makes it possible to attain high productivities both in lipid and biomass. The response of the QO controller suggests that an

optimal strategy consists in driving the biomass concentration to a certain value and to allow it oscillate around this point. This result is consistent with the theoretical results presented by [5], where an optimal controller was developed by forcing the biomass concentration to fulfill a periodicity condition.

B. Is the strict compensation condition relevant for diurnal light cycles?

In closed photobioreactors under light constant regime, maximal productivities are attained when the light intensity at the bottom of the reactor equals the light of compensation (G_c), defined as the minimum value of light for a positive net growth rate [18], that is when the growth rate at the bottom of the photobioreactor (μ_L) equals the respiration rate (R). This condition is called the strict compensation condition ($\mu_L = R$). The light of compensation is often expressed as a constant value. This may be the case when the incident light is constant. However, we might notice that for a varying light system, G_c depends on the actual state of the system and thus there is not a fixed value that will bring the system to operate under the strict compensation condition.

For outdoor raceways, where microalgae are exposed to long periods of darkness, respiration affects negatively growth. It is clear that in the dark period, the compensation condition does not play any role on the reactor performance. When the incident light is higher than zero, G_c is thus a dynamic operational variable that depends on the state of the system. To attain the compensation condition, the optical depth must then be adjusted accordingly. At constant volume, the regulation acts on the attenuation factor ξ . This strategy, however, may suffer of reachability problems, as experienced in the study of [19], where the light at the bottom of the photobioreactor could not be maintained at the defined set point due to the dynamic boundary imposed by the growth rate. To enlarge the discussion in this point, we assess by means of simulations if the strict compensation condition could be fulfilled in a diurnal light cycle and if it is relevant to attain this condition to reach maximal productivities. The following optimal control problem was defined

$$\min_{f_i(t)} \int_{t_0}^{t_f} (\mu_L - R)^2 dt. \quad (16)$$

The controller optimizing (16) is called CP_c .

Figure 2 shows the ratio μ_L/R . The results are given for the optimal controller CP_c . It is observed that, for the light period, $\mu_L > R$ and that the compensation condition is not strictly fulfilled all

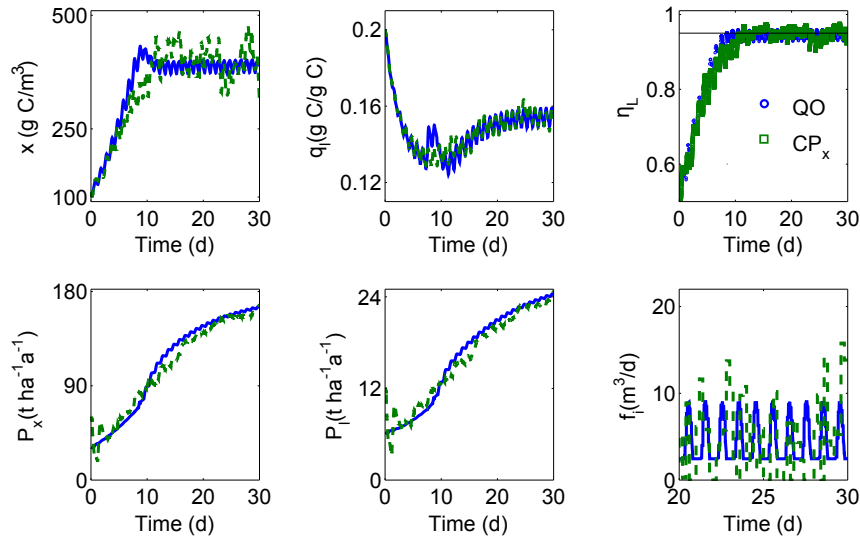


Fig. 1. Trajectories of state variables and productivities given by the QO controller (solid blue line) and the CP_x controller (dashed green line). The top right plot shows the evolution η_L during the light period. The horizontal continuous line is the set point $\eta_L^* = 0.95$.

the time. The results indicate that attaining the strict compensation throughout the day might not be physically possible due to the dynamic bound imposed by the growth rate.

The optimal controller CP_c resulted in biomass and lipid productivities that were, 100% of those obtained with the optimal controllers CP_x and CP_l . Our results suggest that the closest the system is to the compensation condition the closest the system operates optimally. However, the results also indicate that for a photobioreactor subjected to the diurnal light cycle, the strict compensation condition is not a necessary condition to be fulfilled for achieving maximal productivities. We have also verified that imposing a compensation condition valid around the midday light peak could be inefficient resulting in low productivities.

We must note that when the respiration is negligible, the compensation condition implies almost full absorption of light which is rather difficult to maintain throughout the day. Due to the limitation of reachability of the strict compensation condition and the difficulty associated to the online determination of μ_L and R , we suggest that the strict compensation condition is not a practical criterion for the design of control strategies. By contrary, the strategy that we proposed of controlling η_L is technically feasible for real implementation and provides almost optimal productivities.

C. Comparison of open loop configurations to optimal strategy

To assess the performance of the raceway in OL configuration, the model was simulated initially

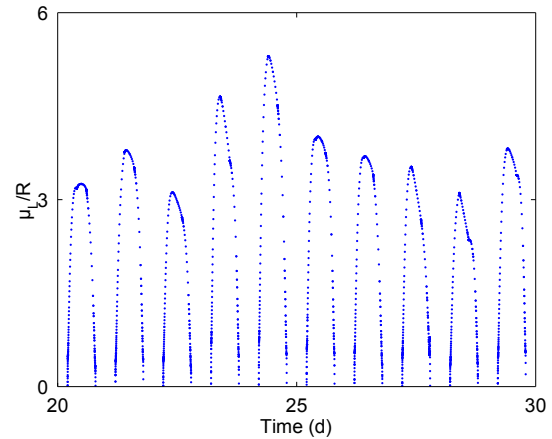


Fig. 2. The strict compensation condition ($\mu_L/R=1$) is not attained along the day. Response obtained for the optimal controller CP_c .

with an input flow rate $f_i = 5.13 \text{ m}^3 \text{ d}^{-1}$. The lipid and biomass productivities were, respectively, 54% and 59% of those obtained with CP_x and CP_l . This means that for an OL configuration, a wrong choice of the input flow rate will imply an unsatisfactory performance. To perform a fair comparison, the optimal control problem (12) was solved by setting a constant flow rate. The optimal flow rate was found to be $3.22 \text{ m}^3 \text{ d}^{-1}$ and the resulting biomass productivity was 93% of that obtained with CP_x . Table II summarizes the comparison of the performances of different controllers and configurations evaluated (including the QO controller) relative to the optimal productivities. The high productivity obtained with the optimal

TABLE II

COMPARISON OF PRODUCTIVITY PERFORMANCE.		
	$100P_x/P_x^*$	$100P_l/P_l^*$
QO	98%	98%
CP _c	100%	100%
OL: $f_i^* = 3.22 \text{ m}^3 \text{ d}^{-1}$	93%	91%
OL: $f_i = 5.13 \text{ m}^3 \text{ d}^{-1}$	54%	59%

* productivities obtained by the optimal controllers CP_x and CP_l.
 * optimal constant flow rate.

constant flow rate is not surprising. Indeed, it has been shown experimentally that with an adequate constant flow rate it is possible to attain high productivities [19]. This result may suggest that, when the microalgae are not nutrient limited, light and temperature exert such a strong influence on the system behaviour that the improvement of the performance that can be reached by manipulating the dilution rate is only marginal. This finding might, at first sight, discourages the development of control strategies since it appears that even with a constant flow rate a satisfactory performance can be attained. This result, however, must be taken with caution. Indeed, we argue in favor of the QO controller over the other control strategies analyzed. The QO controller has the advantage to operate in CL fashion. Hence, it can be easily tuned for a real scenario subjected to disturbances and technical failures. The optimal controllers can also be in CL fashion. However, its implementation is more demanding than that of the QO controller. If the optimal controllers are used in OL, they are not adapted to account for model uncertainty and potential disturbances, which can lead to suboptimal operation. Indeed, to assess the robustness of the controllers under model uncertainty, the value of $\bar{\mu}$ was decreased 30% of the nominal value shown in Table I. When applying the input obtained with the CP_x controller, the biomass productivity was $87.5 \text{ ha}^{-1} \text{ a}^{-1}$. When applying the QO controller, the biomass productivity was $102 \text{ ha}^{-1} \text{ a}^{-1}$. This value is 17% higher than the productivity obtained with CP_x. The performances of the QO controller might be even better with a nonlinear control design. The design of such a controller is currently in process.

V. CONCLUSIONS

Controlling the efficiency of light absorption makes it possible to attain maximal productivities. The overall performance of the QO controller developed here and its practical advantages for real implementation makes it a suitable control strategy for optimizing microalgae production in raceways.

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