

P·SPR·D Control Design via LMI for Linear MIMO Systems and its Extension to Adaptive Control

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Abstract

This paper concerns with P·SPR·D control for set-point servo problem of linear MIMO systems and its extension to adaptive control. The P·SPR·D control is a structured controller introducing a SPR (strict positive real) element instead of I element for PID control. The purpose is to design a P·SPR·D controller for asymptotical stabilization and to adjust P, SPR, D gain matrices for the improvement of control performances. We make stability analysis of the P·SPR·D control via LMI, based on Lyapunov's direct method. We extend the P·SPR·D control to adaptive P·SPR·D control in the case where plant parameters change. Namely, we make an effort to stabilize the closed-loop system and to improve convergence speed to the desired equilibrium by adding an adaptive control mode with a time-varying gain matrix to the usual P·SPR·D controller. Consequently the closed-loop system keeps asymptotical stability against unknown plant-parameter changes. For the adaptive P·SPR·D control we propose, it is not necessary to assume the minimum phase property of the plant. The P·SPR·D control can be modified to P·SPR·D+I control for improving the steady state performance (an off-set).

Keywords : P·SPR·D control, linear MIMO systems, stabilization, Lyapunov's direct method, LMI, adaptive control

I. INTRODUCTION

This paper is concerned with design of P·SPR·D control for linear MIMO systems and its extension to adaptive control. The P·SPR·D control consisting of P (proportional) and D (derivative) modes plus SPR (strict positive real) element has been acknowledged its effectiveness by applications to both linear MIMO systems and affine nonlinear systems.

In the past we derived the P·SPR·D control [24], [15] by applying high gain output feedback theorem [12], [4]. Refs.[17], [18], [19] represent applying the P·SPR·D control for affine nonlinear systems. The L_2 -gain disturbance attenuation problem was also studied in Ref.[16].

In this paper, based on the Lyapunov stability theorem, we make stability analysis of the P·SPR·D control by the manner different from Ref.[15], in which we applied the high gain output feedback theorem for the proof. And by applying LMI(Linear Matrix Inequality) technique, we investigate the

existence of P, SPR, D gain matrices that stabilize the closed-loop system and how to determine them.

We also extend the P·SPR·D control to adaptive P·SPR·D control in the case where plant parameters change. In other words, by adding an adaptive control mode with a time-varying gain matrix to the usual P·SPR·D controller, we make an effort such that the closed-loop system maintains to be stable against unknown plant-parameter variations and such that its convergence speed to the desired equilibrium may be improved. It is also noted that by introducing the SPR element one can relax assumptions with respect to conditions such as minimum phase and/or relative degree, which were assumed in many adaptive control theories.

Most of adaptive control laws are based on a gradient descent method essentially. MRACS has been extensively investigated for adaptive control of servo problems. As a design method of adaptive control systems based on not the augmented error signal [6] but the tracking error, there exist adaptive back-stepping methods [5], [7] and the high order tuning [9] by the dynamic certainty equivalence principle. Stability analysis of MRACS was made in Ref.[8]. Note that stability analysis of adaptive control systems is fundamentally based on the strict positive realness and the passivity. The readers may refer to review papers [10], [21] in regard to classical adaptive control theory.

A simple adaptive control law [3] can be obtained under the certain condition, that is, one can obtain a very practical adaptive controller (time-varying PI control), if the closed-loop transfer function by constant gain output feedback is strictly positive real. Adaptive PID control is also studied in Ref.[25], where P, D gain matrices are modified as time-varying ones.

In most of past researches, to analyze stability of adaptive control systems, they necessitated such assumptions that a plant was minimum phase and/or relative degree of the plant was known. For the adaptive P·SPR·D control law we propose, it is not necessary to assume about the relative degree and/or the minimum phase property of the plant.

II. P·SPR·D CONTROL FOR SET-POINT SERVO PROBLEM

Consider the following MIMO system:

$$\dot{x}(t) = Ax(t) + Bu(t), \quad x(0) = x_0 \quad (1)$$

$$\mathbf{y}(t) = C\mathbf{x}(t) \quad (2)$$

where $\mathbf{x}(t) \in R^n$, $\mathbf{u}(t) \in R^r$, $\mathbf{y}(t) \in R^m$ are the state vector, the control vector and the output vector, respectively. The system $\{A, B, C\}$ is assumed to be controllable.

PID control is usually given as

$$\mathbf{u}(t) = K_P \mathbf{e}(t) + K_I \int_0^t \mathbf{e}(\tau) d\tau + K_D \dot{\mathbf{e}}(t) + \mathbf{m}_0 \quad (3)$$

where $\mathbf{e}(t) = \mathbf{r}(t) - \mathbf{y}(t)$ denotes the error of output from the desired value $\mathbf{r}(t)$, and $K_P, K_I, K_D \in R^{r \times m}$ are P, I, D gain matrices called Proportional, Integral, and Derivative, respectively, and \mathbf{m}_0 denotes the manual reset quantity.

For system (1),(2) let us consider a set-point servo problem with the desired output $\mathbf{r}(t) = \mathbf{y}^*$.

We assume :

$$\text{[Assumption 1]} \text{ rank} \begin{bmatrix} A & B \\ C & 0 \end{bmatrix} = n + m$$

An equilibrium state \mathbf{x}_e holding the output at \mathbf{y}^* must satisfy the following relation:

$$\mathbf{0} = A\mathbf{x}_e + B\bar{\mathbf{u}}, \quad \mathbf{y}^* = C\mathbf{x}_e$$

Since this relation consists of $(n+m)$ equations and $(n+r)$ variables, when $r \geq m$, $(r-m)$ state variables \mathbf{x}_{eN} can be set arbitrary value \mathbf{x}_{eN}^* , but the remained state variables \mathbf{x}_{eB} and $\bar{\mathbf{u}}$ are determined dependently. Putting such an equilibrium as $\mathbf{x}^* = \begin{bmatrix} \mathbf{x}_{eN}^* \\ \mathbf{x}_{eB}(\mathbf{x}_{eN}^*, \mathbf{y}^*) \end{bmatrix}$ and $\mathbf{u}^* = \bar{\mathbf{u}}(\mathbf{x}_{eN}^*, \mathbf{y}^*)$, we have

$$\mathbf{0} = A\mathbf{x}^* + B\mathbf{u}^* \quad (4)$$

$$\mathbf{y}^* = C\mathbf{x}^* \quad (5)$$

Next, let the state error from the equilibrium \mathbf{x}^* be

$$\mathbf{e}_x(t) = \mathbf{x}(t) - \mathbf{x}^* \quad (6)$$

We also obtain the output error from (2),(5),(6) as follows.

$$\mathbf{e}(t) = \mathbf{y}^* - \mathbf{y}(t) = C(\mathbf{x}^* - \mathbf{x}(t)) = -C\mathbf{e}_x(t) \quad (7)$$

Differentiate (6) with the use of (1),(4) to obtain the state error system

$$\begin{aligned} \dot{\mathbf{e}}_x(t) &= A\mathbf{x}(t) + B\mathbf{u}(t) - (A\mathbf{x}^* + B\mathbf{u}^*) \\ &= A\mathbf{e}_x(t) + B(\mathbf{u}(t) - \mathbf{u}^*) \end{aligned} \quad (8)$$

Accordingly, if we can asymptotically stabilize the error system (8) and can make $\mathbf{e}_x(t) \rightarrow \mathbf{0}$ as $t \rightarrow \infty$, we have $\mathbf{e}(t) \rightarrow \mathbf{0}$, that is, $\mathbf{y}(t) \rightarrow \mathbf{y}^*$. So the set-point servo problem can be solved.

In this paper, by substituting a SPR (strict positive real) element instead of the I element of PID control, we propose the following P-SPR-D control

$$\dot{\boldsymbol{\zeta}}(t) = D\boldsymbol{\zeta}(t) + \mathbf{e}(t), \quad \boldsymbol{\zeta}(0) = \mathbf{0} \quad (9)$$

$$\mathbf{u}(t) = K_P \mathbf{e}(t) + K_S \boldsymbol{\zeta}(t) + K_D \dot{\mathbf{e}}(t) + \mathbf{m}_0 \quad (10)$$

where $\boldsymbol{\zeta}(t) \in R^m$ and (9) represents the SPR element with negative definite $D \in R^{m \times m}$, and $K_S \in R^{r \times m}$ denotes the SPR gain matrix.

Since it holds from (7) and (8) that

$$\dot{\mathbf{e}}(t) = -C(A\mathbf{e}_x(t) + B(\mathbf{u}(t) - \mathbf{u}^*)) \quad (11)$$

we have

$$\begin{aligned} \mathbf{u}(t) &= -K_P C\mathbf{e}_x(t) + K_S \boldsymbol{\zeta}(t) \\ &\quad - K_D C(A\mathbf{e}_x(t) + B(\mathbf{u}(t) - \mathbf{u}^*)) + \mathbf{m}_0 \end{aligned}$$

by substituting (7),(11) into (10).

Furthermore, arranging this equation, we obtain

$$\begin{aligned} \mathbf{u}(t) &= -(I_r + K_D C B)^{-1} K_P C\mathbf{e}_x(t) \\ &\quad + (I_r + K_D C B)^{-1} K_S \boldsymbol{\zeta}(t) \\ &\quad - (I_r + K_D C B)^{-1} K_D C(A\mathbf{e}_x(t) - B\mathbf{u}^*) \\ &\quad + (I_r + K_D C B)^{-1} \mathbf{m}_0 \\ &= -(I_r + K_D C B)^{-1} (K_P C + K_D C A)\mathbf{e}_x(t) \\ &\quad + (I_r + K_D C B)^{-1} K_S \boldsymbol{\zeta}(t) \\ &\quad + (I_r + K_D C B)^{-1} K_D C B \mathbf{u}^* \\ &\quad + (I_r + K_D C B)^{-1} \mathbf{m}_0 \\ &= -K_E \mathbf{e}_x(t) + K_{\Xi} \boldsymbol{\zeta}(t) + K_U (K_D C B \mathbf{u}^* + \mathbf{m}_0) \end{aligned} \quad (12)$$

where $K_E \triangleq (I_r + K_D C B)^{-1} (K_P C + K_D C A)$

$$K_{\Xi} \triangleq (I_r + K_D C B)^{-1} K_S$$

$$K_U \triangleq (I_r + K_D C B)^{-1}$$

By substituting (12) into (8), we obtain the closed-loop error system

$$\begin{aligned} \dot{\mathbf{e}}_x(t) &= A\mathbf{e}_x(t) + B(\mathbf{u}(t) - \mathbf{u}^*) \\ &= A\mathbf{e}_x(t) + B\{-K_E \mathbf{e}_x(t) + K_{\Xi} \boldsymbol{\zeta}(t) \\ &\quad + K_U (K_D C B \mathbf{u}^* + \mathbf{m}_0) - \mathbf{u}^*\} \\ &= (A - BK_E)\mathbf{e}_x(t) + BK_{\Xi} \boldsymbol{\zeta}(t) - BK_U \mathbf{u}^* + BK_U \mathbf{m}_0 \\ &= (A - BK_E)\mathbf{e}_x(t) + BK_{\Xi} \boldsymbol{\zeta}(t) + BK_U (\mathbf{m}_0 - \mathbf{u}^*) \end{aligned} \quad (13)$$

Accordingly, by combining (13) and (9), the closed-loop error system with the P-SPR-D control becomes

$$\begin{bmatrix} \dot{\mathbf{e}}_x(t) \\ \dot{\boldsymbol{\zeta}}(t) \end{bmatrix} = \begin{bmatrix} A - BK_E & BK_{\Xi} \\ -C & D \end{bmatrix} \begin{bmatrix} \mathbf{e}_x(t) \\ \boldsymbol{\zeta}(t) \end{bmatrix} + \begin{bmatrix} BK_U (\mathbf{m}_0 - \mathbf{u}^*) \\ \mathbf{0} \end{bmatrix} \quad (14)$$

Now let us set the manual reset quantity as $\mathbf{m}_0 = \mathbf{u}^*$. Then the closed-loop error system becomes

$$\begin{bmatrix} \dot{\mathbf{e}}_x(t) \\ \dot{\boldsymbol{\zeta}}(t) \end{bmatrix} = \begin{bmatrix} A - BK_E & BK_{\Xi} \\ -C & D \end{bmatrix} \begin{bmatrix} \mathbf{e}_x(t) \\ \boldsymbol{\zeta}(t) \end{bmatrix} \quad (15)$$

At the same time, P-SPR-D control (10) is given as

$$\mathbf{u}(t) = K_P \mathbf{e}(t) + K_S \boldsymbol{\zeta}(t) + K_D \dot{\mathbf{e}}(t) + \mathbf{u}^* \quad (16)$$

Now define a Lyapunov function with $P = P^T > 0, S = S^T > 0$

$$V(\mathbf{e}_x(t), \boldsymbol{\zeta}(t)) = \frac{1}{2} \begin{bmatrix} \mathbf{e}_x(t) \\ \boldsymbol{\zeta}(t) \end{bmatrix}^T \begin{bmatrix} P & O \\ O & S \end{bmatrix} \begin{bmatrix} \mathbf{e}_x(t) \\ \boldsymbol{\zeta}(t) \end{bmatrix} \quad (17)$$

and take its time derivative along (15) to obtain

$$\begin{aligned}\dot{V}(\mathbf{e}_x(t), \zeta(t)) &= \begin{bmatrix} \mathbf{e}_x(t) \\ \zeta(t) \end{bmatrix}^T \begin{bmatrix} P & O \\ O & S \end{bmatrix} \begin{bmatrix} \dot{\mathbf{e}}_x(t) \\ \dot{\zeta}(t) \end{bmatrix} \\ &= \begin{bmatrix} \mathbf{e}_x(t) \\ \zeta(t) \end{bmatrix}^T M \begin{bmatrix} \mathbf{e}_x(t) \\ \zeta(t) \end{bmatrix}\end{aligned}\quad (18)$$

where

$$M \triangleq \begin{bmatrix} P(A - BK_E) & BK_{\Xi} \\ -SC & SD \end{bmatrix}$$

Using a relation $\mathbf{x}^T M \mathbf{x} = \frac{1}{2} \mathbf{x}^T (M + M^T) \mathbf{x}$, we have

$$\dot{V}(\mathbf{e}_x(t), \zeta(t)) = \frac{1}{2} \begin{bmatrix} \mathbf{e}_x(t) \\ \zeta(t) \end{bmatrix}^T (M + M^T) \begin{bmatrix} \mathbf{e}_x(t) \\ \zeta(t) \end{bmatrix}\quad (19)$$

where

$$(M + M^T) = \begin{bmatrix} P(A - BK_E) + (A^T - K_E^T B^T)P & PBK_{\Xi} - C^T S \\ -SC + K_{\Xi}^T B^T P & SD + D^T S \end{bmatrix}$$

In order that the closed-loop system be asymptotically stable, $\dot{V}(\mathbf{e}_x, \zeta) < 0$ is sufficient. For that it is sufficient that there exist $P = P^T > 0, S = S^T > 0, K_E, K_{\Xi}$ which satisfy

$$\begin{bmatrix} P(A - BK_E) + (A^T - K_E^T B^T)P & PBK_{\Xi} - C^T S \\ -SC + K_{\Xi}^T B^T P & SD + D^T S \end{bmatrix} < 0\quad (20)$$

Note that (20) is equivalent to the Lyapunov inequality

$$\begin{bmatrix} P & O \\ O & S \end{bmatrix} \begin{bmatrix} A - BK_E & BK_{\Xi} \\ -C & D \end{bmatrix} + \begin{bmatrix} A - BK_E & BK_{\Xi} \\ -C & D \end{bmatrix}^T \begin{bmatrix} P & O \\ O & S \end{bmatrix} < 0$$

which gives a necessary and sufficient condition that system (15) be asymptotically stable.

Furthermore, we have from (20)

$$\begin{bmatrix} P(A - BK_E) + (A^T - K_E^T B^T)P & PBK_{\Xi} \\ K_{\Xi}^T B^T P & O \end{bmatrix} - \begin{bmatrix} O & C^T \\ C & -D - D^T \end{bmatrix} < 0$$

This implies that system $\{A - BK_E, BK_{\Xi}, C, D\}$ is positive real.

Since the matrix of (19), $M + M^T$ is symmetric one, it is necessary and sufficient for this to be negative definite that

$$\begin{aligned}SD + D^T S &< 0 \\ P(A - BK_E) + (A^T - K_E^T B^T)P - (PBK_{\Xi} - C^T S) \\ &\times (SD + D^T S)^{-1} (-SC + K_{\Xi}^T B^T P)^T < 0\end{aligned}\quad (21)$$

from Schur complement¹.

¹ [Lemma 1] (Schur complement) In order that symmetric real matrix $\Theta = \begin{bmatrix} \Theta_{11} & \theta_{12} \\ \theta_{12}^T & \Theta_{22} \end{bmatrix}$ be positive definite, it is necessary and sufficient that

(i) $\Theta_{11} > 0$ and $\Theta_{22} - \theta_{12}^T \Theta_{11}^{-1} \theta_{12} > 0$
or (ii) $\Theta_{22} > 0$ and $\Theta_{11} - \theta_{12} \Theta_{22}^{-1} \theta_{12}^T > 0$

To solve the matrix inequality (20), let it result in LMI solving problem by performing congruent transformation and choosing new variables appropriately. Make the congruent transformation with $T = \begin{bmatrix} P^{-1} & O \\ O & S^{-1} \end{bmatrix}$ to obtain

$$\begin{aligned}&\begin{bmatrix} P^{-1} & O \\ O & S^{-1} \end{bmatrix} \times \\ &\begin{bmatrix} P(A - BK_E) + (A^T - K_E^T B^T)P & PBK_{\Xi} - C^T S \\ -SC + K_{\Xi}^T B^T P & SD + D^T S \end{bmatrix} \\ &\times \begin{bmatrix} P^{-1} & O \\ O & S^{-1} \end{bmatrix}^T \\ &= \begin{bmatrix} (A - BK_E) + P^{-1}(A^T - K_E^T B^T)P & BK_{\Xi} - P^{-1}C^T S \\ -C + S^{-1}K_{\Xi}^T B^T P & D + S^{-1}D^T S \end{bmatrix} \\ &\times \begin{bmatrix} P^{-1} & O \\ O & S^{-1} \end{bmatrix}^T \\ &= \begin{bmatrix} (A - BK_E)P^{-1} + P^{-1}(A^T - K_E^T B^T) & BK_{\Xi}S^{-1} - P^{-1}C^T \\ -CP^{-1} + S^{-1}K_{\Xi}^T B^T & DS^{-1} + S^{-1}D^T \end{bmatrix} \\ &< 0\end{aligned}\quad (22)$$

By introducing here new variables $Y = P^{-1}, Z = S^{-1}, G_1 = K_E P^{-1}, G_2 = K_{\Xi} S^{-1}$, (22) becomes as

$$\begin{bmatrix} AY - BG_1 + YA^T - G_1^T B^T & BG_2 - YC^T \\ -CY + G_2^T B^T & DZ + ZD^T \end{bmatrix} < 0\quad (23)$$

Since (23) is a LMI, we can confirm the existence of $Y > 0, Z > 0, G_1, G_2$ by using semi-definite programming. If we can obtain $Y > 0, Z > 0, G_1, G_2$ satisfying LMI (23), since Y and Z become nonsingular, $P = Y^{-1} > 0, S = Z^{-1} > 0, K_E = G_1 P = GY^{-1}$ and $K_{\Xi} = G_2 S = G_2 Z^{-1}$ satisfy (20). Consequently, stable P-SPR-D controller gains are obtained from the solution of LMI (23). That is, we have only to calculate K_P, K_D, K_S from

$$\begin{aligned}K_E &= (I_r + K_D C B)^{-1} (K_P C + K_D C A) \\ K_{\Xi} &= (I_r + K_D C B)^{-1} K_S\end{aligned}$$

The reader may refer to [1], [2], [22] for details of LMI technique.

A method of determining K_P, K_D, K_S from the above relations is given in Refs.[15], [24]. These references also suggest a tuning method of P-SPR-D controller parameters based on the eigen-value assignment method [23].

Meanwhile, if $\mathbf{m}_0 \neq \mathbf{u}^*$, or when one sets $\mathbf{m}_0 = \mathbf{0}$, an off-set, $\mathbf{e}(\infty) = \mathbf{y}^* - \mathbf{y}(\infty)$, occurs caused by the second term in the right-hand side of (14), even when a transition matrix of (14) is asymptotically stable. An amount of off-set depends on K_P, K_S, K_D and D , and it is determined from the steady state error

$$\begin{aligned}\begin{bmatrix} \mathbf{e}_x(\infty) \\ \zeta(\infty) \end{bmatrix} &= - \begin{bmatrix} A - BK_E & BK_{\Xi} \\ -C & D \end{bmatrix}^{-1} \\ &\times \begin{bmatrix} BK_U(\mathbf{m}_0 - \mathbf{u}^*) \\ \mathbf{0} \end{bmatrix}\end{aligned}\quad (24)$$

For the regulation problem (i.e. $\mathbf{x}^* = \mathbf{0}$), however, the state $\mathbf{x}(t)$ can be converged to the origin by the P-SPR-D control with $\mathbf{m}_0 = \mathbf{0}$ (no off-set occurs).

Finally let us consider a counterplan in case that the manual reset quantity $\mathbf{m}_0 = \mathbf{u}^*$ cannot be calculated or is not available for the P-SPR-D control. Since the stability of transient state is guaranteed sufficiently by the high gain output feedback, we devise only a countermove for a steady state error (i.e. off-set). We suggest the following two methods in order to compensate $\mathbf{m}_0 = \mathbf{u}^*$.

The first one is to use I control such that the following equation is adopted instead of (10) or (16)

$$\mathbf{u}(t) = K_P \mathbf{e}(t) + K_S \zeta(t) + K_D \dot{\mathbf{e}}(t) + K_I \int_0^t \mathbf{e}(\tau) d\tau, \quad (25)$$

which is called the P-SPR-D+I control.

The second one is to use feedforward mode as follows.

$$\mathbf{u}(t) = K_P \mathbf{e}(t) + K_S \zeta(t) + K_D \dot{\mathbf{e}}(t) + K_F(t) \mathbf{y}^* \quad (26)$$

$$\dot{K}_F(t) = S^{-1} \mathbf{e}(t) \mathbf{y}^{*T}, \quad S > 0, \quad (27)$$

which is called the P-SPR-D+Feedforward control. Here $K_F(t)$ denotes the time-variant feedforward gain matrix and S is a weighting coefficient. Note that this idea originates in the direct adaptive control algorithm [3].

III. ADAPTIVE P-SPR-D CONTROL

In this chapter we consider the following adaptive P-SPR-D control, expecting further performance improvement than the usual P-SPR-D control ;

$$\begin{aligned} \mathbf{u}(t) = & K_P \mathbf{e}(t) + K_S \zeta(t) + K_D \dot{\mathbf{e}}(t) \\ & + K_A(t)(W_1 \mathbf{e}(t) + W_2 \dot{\mathbf{e}}(t)) + \mathbf{m}_0 \end{aligned} \quad (28)$$

where $W_1, W_2 \in R^{m \times m}$ denote the weighting positive-definite diagonal matrices. Note that $K_A(t) \in R^{r \times m}$ is a time-varying adaptive control gain matrix.

Substitute (7),(11) into (28) to obtain

$$\begin{aligned} \mathbf{u}(t) = & -K_P C \mathbf{e}_x(t) + K_S \zeta(t) \\ & -K_D C (A \mathbf{e}_x(t) + B(\mathbf{u}(t) - \mathbf{u}^*)) \\ & + K_A(t)(W_1 \mathbf{e}(t) + W_2 \dot{\mathbf{e}}(t)) + \mathbf{m}_0 \end{aligned}$$

Furthermore, arranging this equation, we obtain

$$\begin{aligned} \mathbf{u}(t) = & -(I_r + K_D C B)^{-1} K_P C \mathbf{e}_x(t) \\ & + (I_r + K_D C B)^{-1} K_S \zeta(t) \\ & - (I_r + K_D C B)^{-1} K_D C (A \mathbf{e}_x(t) - B \mathbf{u}^*) \\ & + (I_r + K_D C B)^{-1} K_A(t)(W_1 \mathbf{e}(t) + W_2 \dot{\mathbf{e}}(t)) \\ & + (I_r + K_D C B)^{-1} \mathbf{m}_0 \\ = & -K_E \mathbf{e}_x(t) + K_\Xi \zeta(t) \\ & + K_U K_A(t)(W_1 \mathbf{e}(t) + W_2 \dot{\mathbf{e}}(t)) \\ & + K_U (K_D C B \mathbf{u}^* + \mathbf{m}_0) \end{aligned} \quad (29)$$

where $K_E \triangleq (I_r + K_D C B)^{-1} (K_P C + K_D C A)$

$$K_\Xi \triangleq (I_r + K_D C B)^{-1} K_S$$

$$K_U \triangleq (I_r + K_D C B)^{-1}$$

Substitute this into (8), we obtain the closed-loop error system

$$\begin{aligned} \dot{\mathbf{e}}_x(t) = & A \mathbf{e}_x(t) + B \{-K_E \mathbf{e}_x(t) + K_\Xi \zeta(t) \\ & + K_U K_A(t)(W_1 \mathbf{e}(t) + W_2 \dot{\mathbf{e}}(t)) \\ & + K_U (K_D C B \mathbf{u}^* + \mathbf{m}_0) - \mathbf{u}^*\} \\ = & (A - B K_E) \mathbf{e}_x(t) + B K_\Xi \zeta(t) \\ & + B K_U K_A(t)(W_1 \mathbf{e}(t) + W_2 \dot{\mathbf{e}}(t)) \\ & - B K_U \mathbf{u}^* + B K_U \mathbf{m}_0 \\ = & (A - B K_E) \mathbf{e}_x(t) + B K_\Xi \zeta(t) \\ & + B K_U K_A(t)(W_1 \mathbf{e}(t) + W_2 \dot{\mathbf{e}}(t)) \\ & + B K_U (\mathbf{m}_0 - \mathbf{u}^*) \end{aligned} \quad (30)$$

Accordingly, by combining (9) and (30), the closed-loop error system with the adaptive P-SPR-D control becomes

$$\begin{aligned} \begin{bmatrix} \dot{\mathbf{e}}_x(t) \\ \dot{\zeta}(t) \end{bmatrix} = & \begin{bmatrix} A - B K_E & B K_\Xi \\ -C & D \end{bmatrix} \begin{bmatrix} \mathbf{e}_x(t) \\ \zeta(t) \end{bmatrix} \\ & + \begin{bmatrix} B \\ O \end{bmatrix} K_U K_A(t)(W_1 \mathbf{e}(t) + W_2 \dot{\mathbf{e}}(t)) \\ & + \begin{bmatrix} B K_U (\mathbf{m}_0 - \mathbf{u}^*) \\ \mathbf{0} \end{bmatrix} \end{aligned} \quad (31)$$

Now let us set the manual reset quantity as $\mathbf{m}_0 = \mathbf{u}^*$. Then the closed-loop error system becomes

$$\begin{aligned} \begin{bmatrix} \dot{\mathbf{e}}_x(t) \\ \dot{\zeta}(t) \end{bmatrix} = & \begin{bmatrix} A - B K_E & B K_\Xi \\ -C & D \end{bmatrix} \begin{bmatrix} \mathbf{e}_x(t) \\ \zeta(t) \end{bmatrix} \\ & + \begin{bmatrix} B \\ O \end{bmatrix} K_U K_A(t)(W_1 \mathbf{e}(t) + W_2 \dot{\mathbf{e}}(t)) \end{aligned} \quad (32)$$

At the same time the adaptive P-SPR-D control (28) is given as

$$\begin{aligned} \mathbf{u}(t) = & K_P \mathbf{e}(t) + K_S \zeta(t) + K_D \dot{\mathbf{e}}(t) \\ & + K_A(t)(W_1 \mathbf{e}(t) + W_2 \dot{\mathbf{e}}(t)) + \mathbf{u}^* \end{aligned} \quad (33)$$

Now define the Lyapunov function

$$\begin{aligned} V(\mathbf{e}_x, \zeta, K_A(t)) = & \frac{1}{2} \begin{bmatrix} \mathbf{e}_x \\ \zeta \end{bmatrix}^T \begin{bmatrix} P & O \\ O & S \end{bmatrix} \begin{bmatrix} \mathbf{e}_x \\ \zeta \end{bmatrix} \\ & + \frac{1}{2} \text{tr} K_A(t)^T K_A(t) \end{aligned} \quad (34)$$

with $P = P^T > 0, S = S^T > 0$ and calculate its time derivative along (8),(7),(9),(33) in the same way as (19) to obtain

$$\begin{aligned} \dot{V}(\mathbf{e}_x, \zeta, K_A(t)) = & \begin{bmatrix} \mathbf{e}_x \\ \zeta \end{bmatrix}^T \begin{bmatrix} P & O \\ O & S \end{bmatrix} \begin{bmatrix} \dot{\mathbf{e}}_x \\ \dot{\zeta} \end{bmatrix} + \text{tr} K_A(t)^T \dot{K}_A(t) \\ = & \frac{1}{2} \begin{bmatrix} \mathbf{e}_x \\ \zeta \end{bmatrix}^T (M + M^T) \begin{bmatrix} \mathbf{e}_x \\ \zeta \end{bmatrix} \\ & + \begin{bmatrix} \mathbf{e}_x \\ \zeta \end{bmatrix}^T \begin{bmatrix} P & O \\ O & S \end{bmatrix} \begin{bmatrix} B \\ O \end{bmatrix} K_U K_A(t)(W_1 \mathbf{e} + W_2 \dot{\mathbf{e}}) \\ & + \text{tr} K_A(t)^T \dot{K}_A(t) \\ = & \frac{1}{2} \begin{bmatrix} \mathbf{e}_x \\ \zeta \end{bmatrix}^T (M + M^T) \begin{bmatrix} \mathbf{e}_x \\ \zeta \end{bmatrix} \end{aligned}$$

$$\begin{aligned}
& +e_x^T PBK_U K_A(t)(W_1 e + W_2 \dot{e}) \\
& +\text{tr}K_A(t)^T \dot{K}_A(t)
\end{aligned} \tag{35}$$

where

$$(M + M^T) = \begin{bmatrix} P(A - BK_E) + (A^T - K_E^T B^T)P & PBK_\Xi - C^T S \\ -SC + K_\Xi^T B^T P & SD + D^T S \end{bmatrix}$$

(The argument t is abbreviated except for the case of necessary emphasis.)

Now let

$$\begin{aligned}
\dot{K}_A(t) = & K_U^T B^T P(C^T C)^{-1} C^T (W_1 e(t)e(t)^T + W_2 \dot{e}(t)e(t)^T) \\
& - RK_A(t)
\end{aligned} \tag{36}$$

where $R \in R^{r \times r}$ is a positive definite matrix. Then the third term of (35) becomes as follows².

$$\begin{aligned}
& \text{tr}K_A(t)^T \dot{K}_A(t) \\
& = \text{tr}K_A(t)^T K_U^T B^T P(C^T C)^{-1} C^T (W_1 e e^T + W_2 \dot{e} e^T) \\
& \quad - \text{tr}K_A(t)^T RK_A(t) \\
& = (W_1 e)^T K_A(t)^T K_U^T B^T P(C^T C)^{-1} C^T (-C e_x) \\
& \quad + (W_2 \dot{e})^T K_A(t)^T K_U^T B^T P(C^T C)^{-1} C^T (-C e_x) \\
& \quad - \text{tr}K_A(t)^T RK_A(t) \\
& = -(e^T W_1 + \dot{e}^T W_2) K_A(t)^T K_U^T B^T P e_x \\
& \quad - \text{tr}K_A(t)^T RK_A(t) \\
& = -(e_x^T PBK_U K_A(t)(W_1 e + W_2 \dot{e}) \\
& \quad - \text{tr}K_A(t)^T RK_A(t)
\end{aligned} \tag{37}$$

Hence, by substituting this into (35), we obtain the following.

$$\begin{aligned}
\dot{V}(e_x, \zeta, K_A(t)) = & \frac{1}{2} \begin{bmatrix} e_x \\ \zeta \end{bmatrix}^T (M + M^T) \begin{bmatrix} e_x \\ \zeta \end{bmatrix} \\
& - \text{tr}K_A(t)^T RK_A(t)
\end{aligned} \tag{38}$$

Since $-\text{tr}K_A(t)^T RK_A(t)$ is negative definite, (38) becomes negative definite when $(M + M^T)$ is negative definite. Consequently, an equilibrium of the closed-loop error system (8),(7),(9),(33),(36) is asymptotically stable.

However, we cannot execute adaptive P-SPR-D control (33),(36) for the unknown plants, when $\{A, B, C\}$ are not exactly known. Hence, we first choose K_P, K_S, K_D such that $(M + M^T)$ becomes negative definite in regard to nominal $\{A, B, C\}$. u^* is also calculated with the nominal $\{A, B, C\}$. Then, let us consider the following adaptive control law instead of (36).

$$\dot{K}_A(t) = \Gamma(W_1 e(t)e(t)^T + W_2 \dot{e}(t)e(t)^T) - RK_A(t) \tag{39}$$

where $\Gamma \in R^{r \times m}$ denotes the adaptive gain.

If we can choose $\Gamma = K_U^T B^T P(C^T C)^{-1} C^T$, the closed-loop error system becomes asymptotically stable as mentioned above.

²When $W \in R^{n \times n}$, it holds that $\text{tr}Wxy^T = x^T Wy$.

When (39) is implemented, however, the third term of (35) becomes as follows.

$$\begin{aligned}
& \text{tr}K_A(t)^T \dot{K}_A(t) \\
& = \text{tr}K_A(t)^T \Gamma(W_1 e e^T + W_2 \dot{e} e^T) \\
& \quad - \text{tr}K_A(t)^T RK_A(t) \\
& = (W_1 e)^T K_A(t)^T \Gamma e + (W_2 \dot{e})^T K_A(t)^T \Gamma e \\
& \quad - \text{tr}K_A(t)^T RK_A(t) \\
& = (e^T W_1 + \dot{e}^T W_2) K_A(t)^T \Gamma e \\
& \quad - \text{tr}K_A(t)^T RK_A(t)
\end{aligned} \tag{40}$$

Accordingly, (35) becomes

$$\begin{aligned}
\dot{V}(e_x, \zeta, K_A(t)) = & \frac{1}{2} \begin{bmatrix} e_x \\ \zeta \end{bmatrix}^T (M + M^T) \begin{bmatrix} e_x \\ \zeta \end{bmatrix} \\
& + e_x^T PBK_U K_A(t)(W_1 e + W_2 \dot{e}) \\
& + (e^T W_1 + \dot{e}^T W_2) K_A(t)^T \Gamma (-C e_x) \\
& - \text{tr}K_A(t)^T RK_A(t) \\
& = \frac{1}{2} \begin{bmatrix} e_x \\ \zeta \end{bmatrix}^T (M + M^T) \begin{bmatrix} e_x \\ \zeta \end{bmatrix} \\
& + e_x^T (PBK_U - C^T \Gamma^T) K_A(t)(W_1 e + W_2 \dot{e}) \\
& - \text{tr}K_A(t)^T RK_A(t)
\end{aligned} \tag{41}$$

Consequently, if $\Gamma = K_U^T B^T P(C^T C)^{-1} C^T$ holds and so it becomes $PBK_U - C^T \Gamma^T = O$, then the second term of (41) becomes zero. As a result, we have $\dot{V}(e_x, \zeta, K_A(t)) < 0$ and thus the closed-loop error system (8),(7),(9),(33),(39) is asymptotically stable.

Incidentally, given P, B, K_U, C^T , the adaptive gain Γ satisfying $PBK_U - C^T \Gamma^T = O$ can be calculated as follows.

$$K_U^T B^T P = \Gamma C$$

Accordingly,

$$\Gamma = K_U^T B^T P C^T (C C^T)^{-1} = K_U^T B^T P (C^T C)^{-1} C^T$$

Even though $PBK_U - C^T \Gamma^T \neq O$, however, if a value of $e_x^T (PBK_U - C^T \Gamma^T) K_A(t)(W_1 e + W_2 \dot{e})$ is not too large, we obtain $\dot{V}(e_x, \zeta, K_A(t)) < 0$ when $(M + M^T)$ is sufficiently small negative definite. Consequently, the system is asymptotically stable. At least there exists such $P = P^T > 0$ and $S = S^T > 0$.

In other words, if there exist $P > 0, S > 0, K_P, K_S, K_D$ such that $(M + M^T) < 0$ is satisfied for nominal $\{A, B, C\}$, then there is enough possibility of holding $(M + M^T) < 0$ and $\dot{V}(e_x, \zeta, K_A(t)) < 0$, even when $\{A, B, C\}$ varies a little bit. As a result the closed-loop error system can be expected to maintain stable. Hence, as mentioned above, an equilibrium of the closed-loop error system (8),(7),(9),(33),(39) is asymptotically stable.

Next let us consider a special case when $r = m, CB = O$ and the plant $\{A, B, C\}$ is passive.

Define the Lyapunov function

$$\begin{aligned}
V(e_x, \zeta, K_A(t)) = & \frac{1}{2} \begin{bmatrix} e_x \\ \zeta \end{bmatrix}^T \begin{bmatrix} P & O \\ O & S \end{bmatrix} \begin{bmatrix} e_x \\ \zeta \end{bmatrix} \\
& + \frac{1}{2} \text{tr}K_A(t)^T \Gamma^{-1} K_A(t)
\end{aligned} \tag{42}$$

with $P = P^T > 0, S = S^T > 0$ and a positive definite $\Gamma \in R^{m \times m}$, and calculate its time derivative along (8),(7),(9),(33) in the same way as (19) to obtain

$$\begin{aligned}
\dot{V}(e_x, \zeta, K_A(t)) &= \begin{bmatrix} e_x \\ \zeta \end{bmatrix}^T \begin{bmatrix} P & O \\ O & S \end{bmatrix} \begin{bmatrix} \dot{e}_x \\ \dot{\zeta} \end{bmatrix} \\
&\quad + \text{tr} K_A(t)^T \Gamma^{-1} \dot{K}_A(t) \\
&= \frac{1}{2} \begin{bmatrix} e_x \\ \zeta \end{bmatrix}^T (M + M^T) \begin{bmatrix} e_x \\ \zeta \end{bmatrix} \\
&\quad + \begin{bmatrix} e_x \\ \zeta \end{bmatrix}^T \begin{bmatrix} P & O \\ O & S \end{bmatrix} \begin{bmatrix} B \\ O \end{bmatrix} K_U K_A(t) (W_1 e + W_2 \dot{e}) \\
&\quad + \text{tr} K_A(t)^T \Gamma^{-1} \dot{K}_A(t) \\
&= \frac{1}{2} \begin{bmatrix} e_x \\ \zeta \end{bmatrix}^T (M + M^T) \begin{bmatrix} e_x \\ \zeta \end{bmatrix} \\
&\quad + e_x^T P B K_U K_A(t) (W_1 e + W_2 \dot{e}) \\
&\quad + \text{tr} K_A(t)^T \Gamma^{-1} \dot{K}_A(t) \tag{43}
\end{aligned}$$

where

$$\begin{aligned}
(M + M^T) &= \\
&\begin{bmatrix} P(A - BK_E) + (A^T - K_E^T B^T)P & PBK_\Xi - C^T S \\ -SC + K_\Xi^T B^T P & SD + D^T S \end{bmatrix}
\end{aligned}$$

And consider the following adaptive control law instead of (36) ;

$$\dot{K}_A(t) = \Gamma(W_1 e(t)e(t)^T + W_2 \dot{e}(t)e(t)^T) - RK_A(t) \tag{44}$$

where $\Gamma \in R^{m \times m}$ denotes the adaptive gain.

But, when (44) is implemented, the third term of (43) becomes as follows.

$$\begin{aligned}
&\text{tr} K_A(t)^T \Gamma^{-1} \dot{K}_A(t) \\
&= \text{tr} K_A(t)^T (W_1 e e^T + W_2 \dot{e} \dot{e}^T) \\
&\quad - \text{tr} K_A(t)^T R K_A(t) \\
&= (e^T W_1 + \dot{e}^T W_2) K_A(t)^T e \\
&\quad - \text{tr} K_A(t)^T R K_A(t) \tag{45}
\end{aligned}$$

Accordingly, (43) becomes

$$\begin{aligned}
\dot{V}(e_x, \zeta, K_A(t)) &= \frac{1}{2} \begin{bmatrix} e_x \\ \zeta \end{bmatrix}^T (M + M^T) \begin{bmatrix} e_x \\ \zeta \end{bmatrix} \\
&\quad + e_x^T (P B K_U K_A(t) (W_1 e + W_2 \dot{e}) \\
&\quad + (e^T W_1 + \dot{e}^T W_2) K_A(t)^T (-C e_x) \\
&\quad - \text{tr} K_A(t)^T R K_A(t) \\
&= \frac{1}{2} \begin{bmatrix} e_x \\ \zeta \end{bmatrix}^T (M + M^T) \begin{bmatrix} e_x \\ \zeta \end{bmatrix} \\
&\quad + e_x^T (P B K_U - C^T) K_A(t) (W_1 e + W_2 \dot{e}) \\
&\quad - \text{tr} K_A(t)^T R K_A(t) \tag{46}
\end{aligned}$$

Consequently, if $P B K_U = C^T$ holds and so it becomes $P B K_U - C^T = O$, then the second term of (46) becomes zero. As a result, we have $\dot{V}(e_x, \zeta, K_A(t)) < 0$ and thus the system is asymptotically stable.

It is obvious that $P B K_U = C^T$ equals to $P B = C^T$ when $C B = O$. Therefore, if the plant has the relative degree more

than or equal to 2 and is passive, that is, if $C B = O$ and $P B = C^T$ from the Kalman-Yakubovich-Popov property, then the adaptive P-SPR·D control (33),(44) is asymptotically stable for any positive definite Γ , provided that the matrix inequality (20) holds.

Incidentally, the adaptive control law

$$\dot{K}_A(t) = \Gamma(W_1 e(t)e(t)^T + W_2 \dot{e}(t)e(t)^T) - RK_A(t)$$

implies the steepest descent method of dynamical system essentially for the performance function

$$F(e(t)) = \frac{1}{2} e(t)^T e(t)$$

$$\text{or } F(e(t), \dot{e}(t)) = \frac{1}{2} e(t)^T e(t) + \frac{1}{2} \dot{e}(t)^T \dot{e}(t)$$

and the penalty function

$$P(K_A(t)) = \frac{1}{2} \text{tr} K_A(t)^T R K_A(t), \quad R > 0$$

Hence, a function of adaptive control (39) is locally stable itself, even when $\{A, B, C\}$ changes to some extent.

To derive the gradient function of dynamical systems with respect to $K_A(t)$, theory and methods developed for Direct Gradient Descent Control [11], [13], [14] is useful. PID-type control with time-varying gain matrices $K_P(t), K_I(t), K_D(t)$ was investigated in Ref. [20] also. The adaptive control law can be investigated and justified not only for stability but also for optimality from a view-point of gradient descent method.

How to decide Γ for good control performance is an interesting but not easy future topic. It is also noted that ΓW_1 and ΓW_2 are considered actual adaptive gains, which we must choose adequately in practice. As the adaptive gain Γ exists at least, it is not difficult to choose reasonable Γ in case of SISO systems by trial and error from the nominal $\{A, B, C\}$.

If $m_0 \neq u^*$, however, an off-set, $e(\infty) = y^* - y(\infty)$, occurs even when the closed-loop system by the adaptive P-SPR·D control is asymptotically stable around an equilibrium. To decrease the off-set, we can add I control (Integral control) and carry out adaptive P-SPR·D+I control as well as in the previous chapter II.

IV. NUMERICAL EXAMPLE

Consider an example of 5 dimensional 2-input 2-output unstable system

$$\dot{x}(t) = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 & 2 \\ 0 & -1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & -1 \end{bmatrix} u(t) \tag{47}$$

$$y(t) = \begin{bmatrix} 1 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix} x(t) \tag{48}$$

This system has eigenvalues $\{0, 0, -1, 1 \pm i\}$ which implies that the plant is unstable.

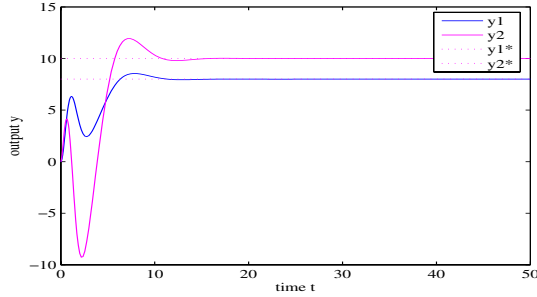


Fig.1 Time histories of output y by P-SPR-D Control

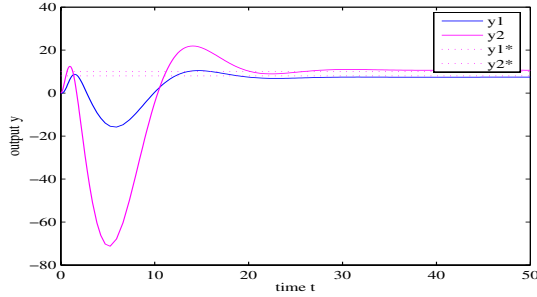


Fig.2 Time histories of output y by P-SPR-D+I Control

By setting P, SPR, D gain matrices as

$$K_P = \begin{bmatrix} 6 & -6 \\ -9 & -2 \end{bmatrix}, K_S = \begin{bmatrix} 10 & 5 \\ 5 & 10 \end{bmatrix}, K_D = \begin{bmatrix} 5 & -1 \\ -3 & -3 \end{bmatrix}$$

the simulation results by the P-SPR-D control was obtained as shown in Fig.1. Here the desired output is given as $y^* = \begin{bmatrix} 8 \\ 10 \end{bmatrix}$. We see that no off-set yields and convergence speed is very quick.

Fig.2 shows the simulation results by the P-SPR-D+I control with $K_I = \begin{bmatrix} 0.1 & 0.05 \\ 0.05 & 0.1 \end{bmatrix}$, when u^* is not available. It is observed that there is almost no off-set and convergence speed is reasonably quick. It is noted that P, SPR, D, I gain matrices can be set with considerable freedom to assure convergence (asymptotical stability).

V. CONCLUDING REMARKS

We studied the P-SPR-D control and the adaptive P-SPR-D control for the set-point servo problem of linear MIMO systems. Stabilization and tuning of the P-SPR-D control implies high gain output feedback essentially, though Lyapunov's direct method is applied to prove the stability of the closed-loop system in this paper.

It is also remarked that the use of SPR element contributes powerfully to stabilizing the closed-loop system. Namely, by adding the SPR element to PD and/or PID modes, we can improve stabilization ability of the structured controller to a great extent.

In order to establish the adaptive controller, we adopt the adaptive control law introducing a time-varying proportional gain matrix $K_A(t)$ multiplied by $W_1 e(t) + W_2 \dot{e}(t)$, expecting

the performance improvement (stability and convergence) against unknown plant-parameter changes.

Implementation of the P-SPR-D control or the adaptive P-SPR-D control is not difficult with the use of a digital processor.

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