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Frequency-domain analysis of self-excited oscillations for a class of multivariable relay systems

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Abstract— **This paper investigates certain complex oscillatory phenomena taking place in multivariable relay control systems. A frequency based approach is applied, by making use of such tools of frequency-domain analysis as the Locus of a Perturbed Relay System (LPRS) and transfer properties analysis via the equivalent gain concept. As the novelty, we provide mathematical models and methods of analysis applicable to complex oscillatory modes not dealt with in the existing literature. A thoroughly discussed analysis and simulation example is outlined to validate the presented results.**

Keywords: Relay systems, Multivariable systems, Chattering analysis.

I. INTRODUCTION

The problem of establishing the existence or nonexistence of oscillations, and analyzing the relevant characteristics whenever they exist, arises in many areas of engineering. Control engineers are often involved in the design of feedback systems with pre-specified limit cycle properties in such areas as oscillators and relay control systems (which are widely used in process control applications, see [20]), or even simply to ensure that a system meets performance criteria which exclude the presence of limit cycles. More recently, much interest has also been generated in using the presence of limit cycles, along with the associated mathematical models, to effectively tune industrial PID controllers [1]. The most common method addressing the question of existence and stability of limit cycle oscillations is the describing function (DF) method [2]. The DF method is simple to apply but its main weakness is that it is only an approximate procedure whose accuracy strictly relies on the level of satisfaction of the filtering hypothesis.

An approach to include higher order harmonics thereby enhancing the accuracy of the DF method was proposed by Tsypkin [21]. This result is particularly attractive in that simple graphical techniques were devised to determine the existence of limit cycles and to investigate their characteristics.

Although there is a tremendous amount of work done in the area of relay SISO systems analysis and design, there is very little literature on the extension of these results to multivariable systems with multiple and possibly hysteretic static nonlinearities in the loop.

Due to the coupling between the loops in a multivariable system, different modes of self-excited oscillations are now possible. Each mode differs in frequency and switching patterns in the respective output and input steady state oscillations.

The focus of the present work is on MIMO relay control systems for linear time-invariant (LTI) processes. It is worth noting that to the best of the authors' knowledge all existing literature on MIMO relay systems exclusively dealt with this practically relevant class of processes. In [15], the simplest operating mode, i.e. that with all relay and systems outputs oscillating at the same frequency Ω was investigated for relay MIMO systems in the presence of external harmonic excitation signals having all the same frequency Ω. Conditions for the existence of single frequency oscillations in the autonomous system without external excitation signals were also derived as a particular instance of the general result. The establishment of a single-frequency oscillatory mode is, however, a very specific and rare phenomenon in the unforced MIMO relay systems, while more complex modes of operation are in fact observed in the majority of the actual cases.

In [13] three distinct and more general modes of operation of the MIMO relay system were listed and described.

In the simplest case, referred to as **Mode 1**, all the relay outputs are square waveforms of precisely one fundamental frequency but with a relative phase shift between them. This mode, the same that was studied in [15], can be expected to occur when the coupling in the multivariable plant is **strong**.

In the more complicated **Mode 2**, according to [13], in each loop the relay output has a single fundamental frequency, generally different for each loop. The individual loops thus behave as independent loops, not perturbed by the interaction signals. This mode is expected to occur when the coupling in the multivariable plant is **weak**. We should note, however, that the ideal Mode 2 situation described in [13] can occur only if interaction between the loops is not present

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at all. Even small interaction between the loops results in a more involved mode in which periodic motions do not exist in the exact sense but *almost periodic motions* occur instead. They manifest themselves as oscillations with certain distinct frequency in each loop having a small drift, which requires a different than [13] approach to analyze this phenomenon.

Mode 3, according to [13], features pretty complex and irregular oscillations in one or a few loops when no single distinct frequency of oscillations can be found in any loop. This mode occurs when the coupling between the loops is **moderate**. The resulting signal at the outputs of the system can feature a few different frequencies.

In [13], necessary conditions guaranteeing the appearance of the three modes were derived by means of a some generalization of the Tzypkin method. Unfortunately, the proposed approach involves solution of possibly high number of nonlinear algebraic equations that may be unsuitable for practical application. The approach is also based on the assumption of the existence of a certain fundamental frequency, so that the oscillation frequencies in each individual loop are multiple of the fundamental frequency, which rarely happens in practice. More recently, influence of the periodic excitation on the nonlinear systems behavior has been studied in [16], [17]

It follows from the above considerations that the complex behavior that occurs in the MIMO relay systems has not been sufficiently investigated. Only Mode 1 has been substantially addressed in the literature, while no practically realizable methods allowing one to compute the oscillations parameters in the Modes 2 and 3 have been proposed so far.

A. Plan and contribution of the paper

As a preliminary step, analysis of SISO relay systems perturbed by an external harmonic input is provided. In spite of the simplicity of such a system, many complex types of behaviour can occur in it, depending on the amplitude and frequency of the harmonic input. Analysis of such system is very helpful to the derivation of conditions and properties to be exploited in the framework of the subsequent MIMO analysis. A classification of the possible behaviours occurring in the perturbed relay SISO system has been seminally suggested by Pospelov [19] in terms of "**periodic mode**", "**tracking mode**" and "**complex oscillations mode**".

It is shown, after that, that the MIMO relay system can be transformed into a multiloop structure in which the relay control signals are explicitly applied to the input point of each other loop, so that analysis of each individual loop as having its own self-excited oscillations and an external input (caused by the interaction with the other loops) becomes possible. This is conveniently done by the appropriate use of the LPRS method and the associated transfer properties analysis via equivalent gain (see [5], [6]).

The contribution of the present work is to develop a suitable model of the oscillatory process in a two-input-two-output (TITO) relay system that features the mode of cross-influencing (complex) oscillations, to explain this phenomenon using the frequency-domain approach and to address practically suitable computations of the frequency and the amplitude of the oscillations in each loop.

The paper is structured as follows: Section II deals with the analysis of SISO relay systems perturbed by an external harmonic input. Section III studies the TITO multivariable case and presents the main results, namely the frequency-domain analysis of the cross-influencing oscillatory phenomenon, and the computation of the oscillation parameters. Section IV outlines a thoroughly developed analysis and simulation example, and Section V gives some concluding remarks and perspectives for next developments.

II. SISO RELAY SYSTEMS WITH EXTERNAL HARMONIC EXCITATION

We preliminarily consider the SISO relay control system in Figure 1, with a standard symmetric hysteretic relay controller (with amplitude c and hysteresis b) and with the external harmonic input

$$r(t) = a_r sin(\omega t) \quad (1)$$

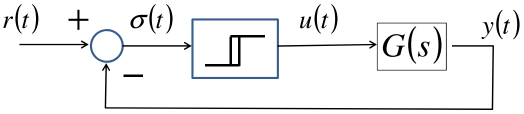

Fig. 1. Block scheme of the considered perturbed SISO system

If the linear plant $G(s)$ is stable and non-integrating, so that the value of the amplitude of the self-excited oscillations in the system is limited, then for every possible value of ω one can always find sufficiently high amplitude a_r of the external signal (1) which will result in the suppression (*quenching*) of the self excited oscillations and locking of the oscillation in the loop to the frequency of the external excitation. In other words, there exists some minimum or critical amplitude a_r^{cr} of $r(t)$ above which the self-excited oscillations are quenched and locked to the frequency of the external signal. This critical amplitude can be found for every particular system and every frequency of the external excitation as suggested in [19], [21], and [10]. The diagram showing the dependence of the critical amplitude of the external harmonic signal on the frequency of this signal for a certain system proposed by Pospelov [19] is provided in Figure 2. The point Ω_0 in the abscissa axis of Figure 2 corresponds to the frequency of the self-excited oscillations in the unforced system.

The diagram in Figure 2, which also lists certain different modes of operation depending on the frequency and amplitude of the external input, can be interpreted as follows. For each value of ω there exist a "critical amplitude" $a_r^{cr} = a_r^{cr}(\omega)$ (the dashed line in Figure 1) such that if $a_r \geq a_r^{cr}(\omega)$ then an oscillation of single frequency ω is observed in the system output y. In other words, the self excited oscillation can always be suppressed by a sufficiently large (in magnitude) exogenous harmonic input. Corresponding mode of operation is referred to as the "periodic mode".

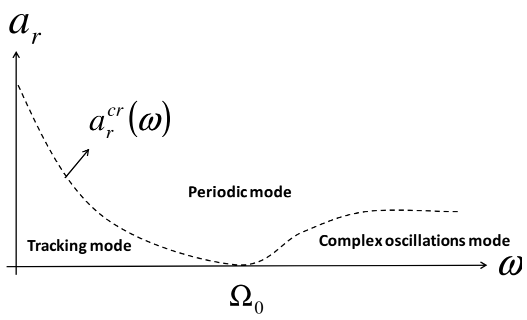

Fig. 2. Exemplifying diagram of the perturbed SISO system behaviour

In the alternative situation when $a_y < a_r^{cr}(\omega)$, two different modes of operation can take place, the "tracking mode", attained when $\omega < \Omega_0$, and the "complex oscillations" mode, happening when $\omega > \Omega_0$. It is interesting to note that the required critical amplitude at the frequency Ω_0 of the self-excited oscillations in the unforced system is equal to zero: $a_r^{cr}(\Omega_0) = 0$, which is consistent with the fact that an Ω_0-periodic mode is established if the external forcing signal has the frequency Ω_0 and arbitrary, possibly zero, magnitude. This fairly obvious observation leads to the conclusion that the closer the frequencies of the self-excited oscillations in the fully independent loops to each other the more likely the system happens to be in the single-frequency mode, as only a very low interaction between the loops is required to bring them to this mode, as follows from Fig. 2.

In the "Tracking mode", the output is characterized by **two distinct frequencies**, one of which is ω and the second one is close to Ω_0. The output can be analyzed as a sum of two periodic components. A detailed and accurate analysis of the tracking mode was made in [5] by means of the LPRS and the equivalent gain concepts. An accurate method for evaluating the magnitudes of the slow (ω-periodic) and fast (Ω_0-periodic) output components was suggested. This mode of operation is widely used in relay servomechanisms designed to track external harmonic signals [6].

In the "Complex oscillations" mode, because the frequency of the external excitation is higher than Ω_0, that usually implies higher degree of filtering-out of the forced component, and with sufficiently small amplitudes a_r the motion in the loop may be "almost periodic" of frequency Ω_0. When the amplitude a_r is high the system may reveal virtually irregular oscillations in which both frequencies: Ω_0 and of the external signal can be identified. The output still retains some form of periodicity but at the relay it has multiple switches within one period of the unforced self-excited oscillation.

III. MULTIVARIABLE RELAY SYSTEMS

We consider the two-input-two-output (TITO) relay control system described by the input output closed loop model

$$\begin{aligned} y_1 &= G_{11}(s)u_1 + G_{12}(s)u_2, \\ y_2 &= G_{21}(s)u_1 + G_{22}(s)u_2, \\ u_1 &= -c_1 R(y_1; b_1), \\ u_2 &= -c_2 R(y_2; b_2), \end{aligned} \tag{2}$$

where $R(\cdot)$ represents the conventional hysteretic relay with unit magnitude and hysteresis value b_i $(i = 1, 2)$, $G_{ij}(s)$ are transfer functions with relative degree higher than one for all $i, j = 1, 2$, and notation $y = G(s)u$ stands for representing that $y = y(t)$ is the output of the SISO process having transfer function $G(s)$ supplied by the input $u = u(t)$.

By relying on simple block manipulations, a block diagram of the closed loop system can be drawn as that in the Figure 3.

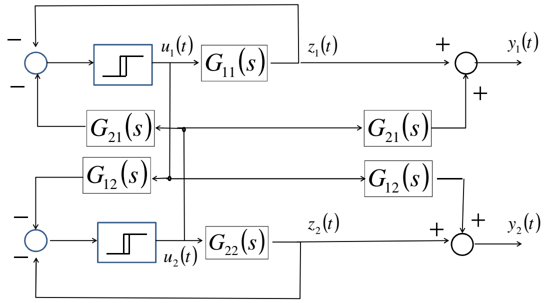

Fig. 3. Block scheme representation of the TITO system (2)

The representation of Figure 3 is very useful since it brings each feedback loop of the MIMO system in the form of a perturbed SISO loop formally analogous to the scheme of Figure 1. From this viewpoint, both the tracking mode and the complex oscillations mode can therefore take place in the loops of a MIMO relay systems with low or moderate degree of interaction, It is also well understood why the Mode 1 is likely to occur in presence of strong interaction between the loops causing the quenching of the self excited oscillation modes.

Without loss of generality, we can therefore investigate the oscillatory behaviour of the "artificial output variables" $z_1(t)$ and $z_2(t)$ by noticing that the corresponding characteristics of the original output variables can be trivially retained by exploiting the simple relations

$$\begin{aligned} y_1(t) &= z_1(t) + G_{21}(s)u_2(t), \\ y_2(t) &= z_2(t) + G_{12}(s)u_1(t), \end{aligned} \tag{3}$$

which yield, after simple manipulations

$$\begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} = \begin{bmatrix} 1 & G_{21}(s)G_{22}^{-1}(s) \\ G_{12}(s)G_{11}^{-1}(s) & 1 \end{bmatrix} \begin{bmatrix} z_1(t) \\ z_2(t) \end{bmatrix} \tag{4}$$

A. Main result

The task is to analyze the operation of a weakly-coupled TITO system in Modes 2 and 3 - as presented by Figure 3. We shall quantitatively define a *weakly coupled TITO system* as the one in which the signals that come to the input of a loop from the opposite loop (Fig. 3) do not result in a change of the switching pattern of the relay, i.e. the relay still has two switches on the period of the self-excited oscillation. The signals that come from the other loop change only the timing of the switch and through this mechanism they act as some kind of modulating signals. Under this assumption, the loop having the lower frequency of the self-excited oscillations has only the low-frequency component (and its harmonics) in the relay output signal, which results in the absence of the frequency Ω_2 in $z_1(t)$. The second loop will be obviously operating in the Tracking mode, also with a low level of interaction, and its output will contain both the frequency of the oscillation of the first loop the frequency of the oscillation of the second loop. Therefore, the mathematical model for the scenario under investigation is

$$\begin{aligned} z_1(t) &= z_{11}(t), \\ z_2(t) &= z_{21}(t) + z_{22}(t), \end{aligned} \tag{5}$$

where

$$z_{11}(t) = \sum_{k=1,3,5,\ldots}^{\infty} \{a_{11}^{(k)} \cos(k\Omega_1^* t) + b_{11}^{(k)} \sin(k\Omega_1^* t)\} \tag{6}$$

is a periodic output signal of the first loop of frequency Ω_1^*.

$$z_{21}(t) = \sum_{k=1,3,5,\ldots}^{\infty} \{a_{21}^{(k)} \cos(k\Omega_1^* t) + b_{21}^{(k)} \sin(k\Omega_1^* t)\} \tag{7}$$

is a periodic component of frequency Ω_1^* in the output of the second loop, and

$$z_{22}(t) = \sum_{k=1,3,5,\ldots}^{\infty} \{a_{22}^{(k)} \cos(k\Omega_2^* t) + b_{22}^{(k)} \sin(k\Omega_2^* t)\} \tag{8}$$

is a periodic component of frequency Ω_2^* in the output of the second loop.

We aim to find an estimation of the amplitude and frequency values of the oscillation model (5)-(8). The proposed procedure is outlined stepwise as follows:

Step 1. Compute the fundamental frequencies Ω_1 and Ω_2 of the self-excited oscillations of the uncoupled loops.

The "Locus of a Perturbed Relay System" (in short LPRS [5]) approach offers a simple way to compute the exact frequency value of the self excited oscillation (the same computed by means of the Tsypkin method) while giving additional informations that shall be discussed later on.

The LPRS associated to the plant transfer functions $G_{11}(s)$ and $G_{22}(s)$, respectively, take the form

$$\begin{aligned} J_{ii}(\omega) &= \sum_{k=1}^{\infty} (-1)^{k+1} \mathtt{Re}\, G_{ii}(k\omega) \\ &+ j \sum_{k=1}^{\infty} \frac{\mathtt{Im}\, G_{ii}((2k-1)\omega)}{2k-1}, \quad i = 1, 2. \end{aligned} \tag{9}$$

The frequency values Ω_1 and Ω_2 are evaluated in accordance with the relations

$$\mathrm{Im} J_{11}(\Omega_1) = -\frac{\pi b_1}{4 c_1}, \quad \mathrm{Im} J_{22}(\Omega_2) = -\frac{\pi b_2}{4 c_2}. \tag{10}$$

Let, without loss of generality, $\Omega_1 < \Omega_2$.

Step 2. Compute the equivalent gain k_{n2} associated to the relay of the second loop

This computation is also made via the LPRS approach, by exploiting the previously mentioned "additional informations" which are stored in the real part of the LPRS.

The equivalent gain of the relay is computed according to the following relation (see [6], [5]))

$$k_{n2} = \frac{1}{-2\, \mathtt{Re}\ [J_{22}(\Omega_2)]}. \tag{11}$$

Earlier formulations of the same concept in the setting of DF based analysis led to a different construction of the equivalent gain using the plant harmonic response rather than the associated LPRS, along with an expression analogous to (11). It is worth to stress that the formulation (11) is more accurate than the corresponding DF-based one(s). The meaning and role of the equivalent gain will be discussed in the next step of analysis.

Step 3. Estimation of frequency Ω_1^*.

This step is carried out by exploiting the useful properties of the equivalent gain and particularly the sensible approximation of substituting the relay block of the z_2 loop with the corresponding equivalent gain k_{n2}, yielding the system outlined in the Figure 4.

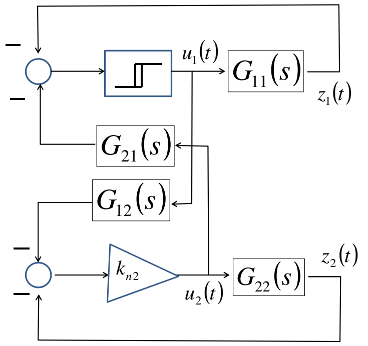

Fig. 4. Equivalent system of the considered 2 × 2 MIMO dynamics for the propagation of the slow component in the z_2 loop

Once replaced the relay of the z_2 loop with the associated gain, the obtained linearized system (due to the known effect of "chatter smoothing" [11]) well describes the propagation of exogenous components slow in comparison with the frequency of the self excited oscillation. This is exactly the case of the z_2 loop, which has the self excited oscillations of frequency Ω_2 and receives from the z_1 loop a harmonic input of lower frequency.

After standard block algebra, the system in Figure 4, can be represented according to a standard unforced single-loop relay control scheme with the linear process transfer function

$$G^*_{11}(s) = G_{11}(s) + G_{21}(s)G_{12}(s)\frac{k_{n2}}{1 + k_{n2}G_{22}(s)}. \quad (12)$$

With reference to this partially linearized "equivalent" relay SISO system one can compute the associated frequency Ω^*_1 of the self exited oscillations by means of LPRS method, in the same way as it was done in the Step 1, leading to the relation

$$ImJ^*_{11}(\Omega^*_1) = -\frac{\pi b_1}{4c_1}, \quad (13)$$

where $J^*_{11}(\omega)$ is the LPRS

$$\begin{aligned} J^*_{11}(\omega) &= \sum_{k=1}^{\infty}(-1)^{k+1}\mathtt{Re}\ G^*_{11}(k\omega) \\ &+ j\sum_{k=1}^{\infty}\frac{\mathtt{Im}\ G^*_{11}((2k-1)\omega)}{2k-1}, \quad i = 1, 2. \quad (14) \end{aligned}$$

The frequency value Ω^*_2 is well estimated by the value Ω_2 computed in the step 1, i.e.

$$\Omega^*_2 = \Omega_2 \quad (15)$$

Step 4. Estimation of oscillation amplitude parameters

We refer to the next, more convenient, representation of the mathematical model of oscillation (5)-(8), with the z_{ij} functions in (6)-(8) expressed as

$$z_{ij}(t) = \sum_{k=1,3,5,\ldots}^{\infty} \gamma^{(k)}_{ij}\cos(k\Omega^*_j t + \phi^{(k)}_{ij}). \quad (16)$$

The relation between the parameters of the two representations is

$$a^{(k)}_{ij} = \gamma^{(k)}_{ij}\cos(\phi^{(k)}_{ij}), \quad b^{(k)}_{ij} = -\gamma^{(k)}_{ij}\sin(\phi^{(k)}_{ij}). \quad (17)$$

The amplitude parameters are computed according to the formulae

$$\gamma^{(k)}_{11} = \frac{4c_1}{k\pi}\left|G_{11}(jk\Omega^*_1)\right|, \quad (18)$$

$$\gamma^{(k)}_{21} = \frac{4c_1}{k\pi}\left|G_{12}(jk\Omega^*_1)\right|\frac{k_{n2}\left|G_{22}(jk\Omega^*_1)\right|}{1 + k_{n2}\left|G_{22}(jk\Omega^*_1)\right|}, \quad (19)$$

$$\gamma^{(k)}_{22} = \frac{4c_2}{k\pi}\left|G_{22}(jk\Omega^*_2)\right|. \quad (20)$$

All the above expressions (18)-(20) are readily justified by referring to the scheme of Figure 4, as they simply come from the relevant harmonics propagation analysis of the square wave inputs u_1 and u_2 through the loops.

IV. ANALYSIS AND SIMULATION EXAMPLE

We consider the multivariable relay system as represented in Figure 3 with the transfer functions

$$G_{11}(s) = \frac{1}{(s+1)(s+2)(s+3)}, \quad (21)$$

$$G_{22}(s) = \frac{1}{(s+10)(s+10)(s+10)}, \quad (22)$$

$$G_{12}(s) = G_{21}(s) = \frac{0.001}{(s+2)(s+2)}, \quad (23)$$

with ideal relays without hysteresis and having the same unit magnitude ($c_1 = c_2 = 1$). The time evolutions of the output variables $z_1(t)$ and $z_2(t)$ are shown in Figure 5. Corresponding spectral contents are shown in Figure 6, which clearly reveal that $z_1(t)$ features a single-frequency mode of oscillation while $z_2(t)$ contains two distinct frequencies.

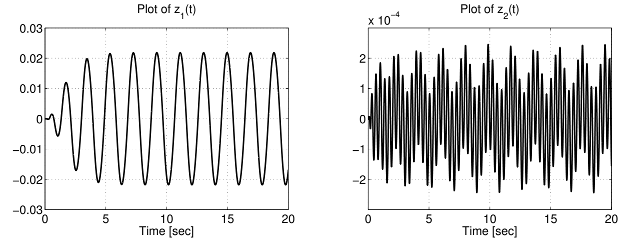

Fig. 5. Time histories of the output variables

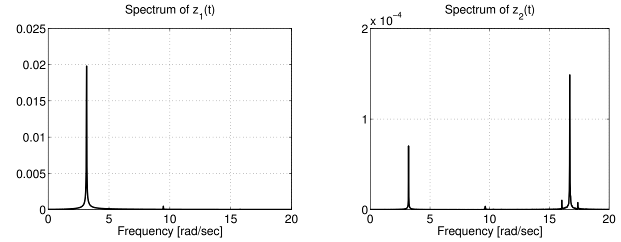

Fig. 6. Frequency spectra of the output variables

The previously outlined procedure is now applied to the system under investigation to compute the frequency and magnitude parameters of the observed periodic motions.

The fundamental frequencies Ω_1 and Ω_2 of the self-excited oscillations that would appear in the decoupled z_1 and z_2 loops are computed (Step 1) according to the relations

$$ImJ_{11}(\Omega_1) = 0, \quad ImJ_{22}(\Omega_2) = 0, \quad (24)$$

that are relations (10) for the case of zero value of the hysteresis parameters b_1 and b_2. For practical computation purposes, only the first 30 harmonics are taken into account in the LPRS computation, thereby leading to a truncated approximate formulation of (9).

Corresponding values of Ω_1 and Ω_2 are derived by considering the imaginary parts of the LPRS functions versus frequency as

$$\Omega_1 = 3.2626\,\mathrm{rad/sec}, \quad \Omega_2 = 17.0707\mathrm{rad/sec}. \quad (25)$$

Condition $\Omega_1 < \Omega_2$ actually holds. The equivalent gain associated to the relay of the second loop (Step 2) is

$$k_{n2} = \frac{1}{-2\ \mathtt{Re}\ [J_{22}(17.07)]} = 433.7. \qquad (26)$$

The LPRS analysis of the transfer function $G_{11}^{*}(s)$ given by (12) (Step 3) is made to compute the frequency Ω_1^{*} satisfying the relation $Im J_{11}^{*}(\Omega_1^{*}) = 0$. The LPRS $J_{11}^{*}(\omega)$ is computed using the first 30 harmonics, as before. By inspecting the imaginary part of $J_{11}^{*}(\omega)$ solution is then obtained in correspondence to the value $\Omega_1^{*} = 3.2685$ rad/sec. Parameter Ω_2^{*} is taken as $\Omega_2^{*} = \Omega_2 = 17.07$ rad/sec according to (15). The values of the oscillation frequency appear in a good agreement with the computed spectra in Fig. 6. Additionally, figure 7 depicts zoomed plots of the output and control signals, showing that in both loops the control signals feature exactly two switches during the period of their self-excited oscillations. This confirms that the system behaves according to the previously defined cross-influencing mode of oscillation, thereby validating the present analysis.

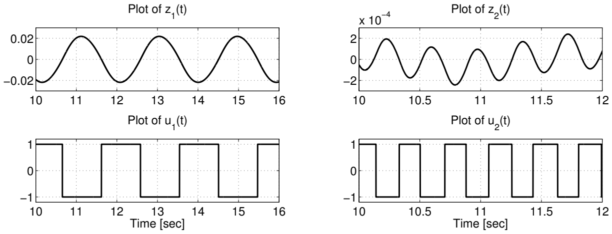

Fig. 7. Switching condition verification

The amplitudes of the first harmonic components are computed according to formulas (18)-(20), for $c_1 = c_2 = 1$ and $k = 1$, as

$$\gamma_{11}^{(1)} = \frac{4}{\pi}\left|G_{11}(j\Omega_1^{*})\right| = 0.0219, \qquad (27)$$

$$\gamma_{21}^{(1)} = \frac{4}{\pi}\left|G_{12}(j\Omega_1^{*})\right| \frac{k_{n2}\left|G_{22}(j\Omega_1^{*})\right|}{1 + k_{n2}\left|G_{22}(j\Omega_1^{*})\right|} = 6.83 \cdot 10^{-5}, \qquad (28)$$

$$\gamma_{22}^{(1)} = \frac{4}{\pi}\left|G_{22}(j\Omega_2^{*})\right| = 1.64 \cdot 10^{-4}. \qquad (29)$$

It ia apparent that the computed amplitude values are also in very good agreement with the computed spectra shown in Figure 6, thereby confirming the accuracy of the proposed analysis and computational procedure.

V. CONCLUSIONS

A suitable model of the oscillatory process in a weakly coupled two-input-two-output (TITO) relay system has been proposed and studied by a frequency-domain approach, and a simple and accurate procedure for the computation of the oscillation parameters in each loop is outlined. The presented analysis and simulation example has led to satisfactorily accurate results in good agreement with the observed trajectories of the numerically simulated plant. Analysis of more general conditions such as the strongly coupled TITO system and the MIMO systems of dimension higher than two remain among other problems to be tackled in the future within the present framework. The application of the present results to identification or controller tuning for linear TITO processes lies among other interesting topics to be addressed in next research.

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