

# Physics-based Modeling and Identification for HVAC Systems<sup>\*</sup>

Francesco Scotton, Lirong Huang, Seyed A. Ahmadi, Bo Wahlberg

**Abstract**—Heating, ventilation and air conditioning (HVAC) systems are among the largest energy consumers in many buildings. As is known, modeling and identification play important roles in the study of HVAC systems. A good model is very helpful for improving efficiency of the HVAC system. Very recently, a physics-based model of room temperature was proposed. Motivated by this inspiring work, this paper, based on the physical dynamical systems, proposes, identifies and validates three models for CO<sub>2</sub> concentration, temperature and humidity of a test-bed room, respectively. Particularly, our models take into account the effect of occupiers, since the indoor air quality (IAQ) is evidently affected by the number of occupiers. A test-bed has been set up for experiments in a laboratory room on KTH campus. Experimental results verify that our proposed method improves the performance of the physics-based linear parametric models.

## I. INTRODUCTION

Heating, Ventilation and Air Conditioning (HVAC) systems, which consist of components working together to introduce, distribute and condition air in a building, play a major role in the control of Indoor Air Quality (IAQ) and thermal comfort. Poor ventilation and improper temperature or humidity will make a bad indoor environment. When people spend 80-90% of their time indoors (see, e.g., [1] and [2]), they will be less productive and more often get sick in such environment, since poor IAQ can cause irritation of eyes and noses, fatigue, headache and shortness of breath (see [3], [4] and the references therein).

Over the past few years energy saving has become an important topic (see, e.g., [5], [6]). The percentage of buildings contribution to the total energy consumption is between 20% and 40% in developed countries and is rapidly increasing, as the population grows and the demand for building services and comfort levels is rising ([7] and [2]). HVAC systems account for the greatest amount of the energy consumption in a building. Indeed, this has markedly grown over recent years, since comfortability is not considered luxury anymore but as required. Hence, improving efficiency of the HVAC systems will be very helpful for the energy saving. In fact, it has been verified that an intelligent control could reduce HVAC systems energy consumption by 20-30% [8] or even more [9]. This work is to propose and identify physics-based

models for HVAC systems, which can be used in intelligent control algorithms that guarantee human comfort indoors and energy saving at the same time.

As is well known, mathematical modeling of HVAC components plays an important role in control design and fault detection of the HVAC systems. Modeling and identification for HVAC systems have been intensively studied in literature. Generally, previous works can be classified into two groups: black box (no a priori information) and grey box (based on some physical knowledge) approaches. Due to the difficulties arising in modeling of thermodynamics, black box is the most common choice in the references: linear parametric models, such as ARX, ARMAX, BJ and OE, have been implemented to model HVAC systems. Chi-Man Yiu *et al.* [10] dealt with a black box identification for an air conditioning system: a MIMO ARMAX model was estimated, parameters of which were evaluated using the Recursive Extended Least Squares Method (RELS), and compared it with a SISO ARMAX model. Mustafaraj *et al.* [11] tested different temperature and humidity models for an office: BJ, ARX, ARMAX and OE models were employed and identified with the black box technique. Mustafaraj *et al.* further developed their work in [12]: Nonlinear AutoRegressive models with exogenous inputs (NNARX) for temperature and humidity were estimated, and their performances were compared with linear ARX models. They also considered the effect of the carbon dioxide concentration in the models, since there is an obvious correlation between occupancy and CO<sub>2</sub> level, which, as was shown, helps improve performance of the model. Qi and Deng [13] studied a MIMO control strategy for the Air Conditioning system (A/C) to control both indoor air temperature and humidity. The model of the A/C system was derived from energy and mass conservation principles. Maasoumy [14] estimated a temperature model for three rooms of a building and designed an optimal control algorithm for HVAC systems, where the thermal circuit method, analogue of electric circuits, was employed. However, the black box methods (including nonparametric approach) do not consider physical characteristics of the concerned systems, which may not be satisfactory from the viewpoint of physics.

Recently, Wu and Sun [15] proposed a physics-based linear parametric model of room temperature in an office building, where thermodynamics equations were used to determine the structure and the order of a linear regression model and they showed with experimental data that their physics-based ARMAX (pbARMAX) model works better than the black-box ones. This is an inspiring approach since it incorporates some architectural characteristics into the

<sup>\*</sup> This work was partially supported by the Swedish Research Council and the Linnaeus Center ACCESS at KTH. The research leading to these results has received funding from The European Research Council under the European Community's Seventh Framework program (FP7 2007-2013) / ERC Grant Agreement N. 267381.

F. Scotton, L. Huang, S. A. Ahmadi were and B. Wahlberg is with ACCESS Linnaeus Center and Automatic Control Lab, School of Electrical Engineering, KTH Royal Institute of Technology, SE-100 44 Stockholm, Sweden. Email addresses: huangli@control.ee.ethz.ch (L. Huang), bo.wahlberg@ee.kth.se (B. Wahlberg).

models and associates their parameters with some physical processes. Motivated by the work [15], this paper, based on the physical dynamical systems, proposes three MISO ARMAX models for CO<sub>2</sub> level, temperature and humidity of the test-bed room, respectively. Particularly, our models take into account the effect of occupiers, which was considered as disturbance in the pbARMAX model of room temperature presented by [15], since the IAQ is evidently affected by the number of occupiers (see, e.g., [16]). Experimental results of validation verify that our models work very well, even though with a short-term training, and our temperature model outperforms the one in the reference [15].

## II. PHYSICS-BASED MODELING AND IDENTIFICATION

Based on some underlying physical dynamics, three multi-input-single-output (MISO) ARMAX models are proposed for CO<sub>2</sub> concentration, temperature and humidity of the test-bed room in the Q-building on KTH campus, respectively. The general form of MISO ARMAX models is given by (see, e.g., [17])

$$A(q, \theta)y(t) = \sum_{i=1}^{n_i} B_i(q, \theta)u_i(t - n_{ki}) + C(q, \theta)e(t), \quad (1)$$

where  $\theta$  is the vector of the parameters to be estimated and  $n_i$  the number of inputs.

### A. CO<sub>2</sub> Concentration

The CO<sub>2</sub> concentration depends on the number of occupiers and also on the states of the air inlet and outlet in the room. When inlets and outlets are on, air in the room is discharged through the outlets while fresh air with lower CO<sub>2</sub> concentration flows in at the same time. Therefore the dynamics of CO<sub>2</sub> level in the room can be described by

$$\frac{d}{dt}C(t) = kN(t) + \beta_{IN} [I(t) - C(t)] + d(t), \quad (2)$$

where  $C(t)$  is the CO<sub>2</sub> concentration of the room air,  $N(t)$  is the number of occupiers,  $I(t)$  is the CO<sub>2</sub> concentration of the air flowing in from the inlet duct,  $k > 0$  and  $\beta_{IN} > 0$  are the unknown parameters and  $d(t)$  is the disturbance.

The forward difference approximation gives

$$\frac{d}{dt}C(t-1) \approx \frac{C_n - C_{n-1}}{\Delta t}, \quad (3)$$

where  $\Delta t$  is the sampling interval,  $C_n$  and  $C_{n-1}$  are the sampled value  $C(n\Delta t)$  and  $C((n-1)\Delta t)$ . Substituting (3) into (2) we obtain

$$C_n - (1 - \beta_{IN}\Delta t)C_{n-1} = k\Delta tN_{n-1} + \beta_{IN}\Delta tI_{n-1} + \Delta td_{n-1}, \quad (4)$$

which can be rewritten as

$$(1 + a_1q^{-1})C_n = K_N N_{n-1} + K_I I_{n-1} + \Delta td_{n-1}, \quad (5)$$

where  $K_N = k\Delta t$ ,  $K_I = \beta_{IN}\Delta t$  and  $a_1 = K_I - 1$ . Obviously, this can be considered as a difference equation of the ARMAX model (1) and, to identify the parameters, the

numbers of occupiers  $N(t)$  will be regarded input signal and measured as well as the CO<sub>2</sub> concentration of the inflowing air  $I(t)$ .

Then, from (5), it follows

$$N_{n-1} \approx \frac{C_n + a_1 C_{n-1} - K_I I_{n-1}}{K_N}. \quad (6)$$

With this approximation, we can use measurements of CO<sub>2</sub> concentration instead of the number of occupiers in the temperature and humidity models. For these applications, it is not required to obtain very accurate estimate of the number of occupiers. This is an original idea proposed in this paper. Experimental results verify that this improves performance of the pbARMAX models (see Section III-C).

### B. Temperature

Before presenting the temperature model, let us introduce the concepts of thermal resistance, which is an analogue of the electrical resistance (see, e.g., [14] and the references therein). Under the steady-state condition, the heat flow for conduction is given by

$$q_{cond} = \frac{kA}{L}(T_1 - T_2), \quad (7)$$

where  $k \left(\frac{W}{mK}\right)$  is the thermal conductivity (which depends on the wall material),  $A$  is the surface of the wall,  $L$  is the thickness of the wall,  $T_1$  and  $T_2$  are the temperatures of two sides of the same wall. The thermal resistance for conduction is defined by  $R_{cond} \left(\frac{K}{W}\right)$  as  $R_{cond} = \frac{L}{kA}$  and then (7) becomes

$$q_{cond} = \frac{T_1 - T_2}{R_{cond}}. \quad (8)$$

Similarly, regardless of the particular nature of the convection heat transfer process, the appropriate rate equation is of the form [14]

$$q_{conv} = hA(T_s - T_\infty), \quad (9)$$

where  $h \left(\frac{W}{m^2K}\right)$  is the convection heat transfer coefficient,  $A$  the surface of the wall,  $T_s$  is the temperature of the wall surface,  $T_\infty$  is the temperature of the air inside (*INT*) or outside (*EXT*) the room (see Figure 2 in [14]). Denoting  $R_{conv} = \frac{1}{hA} \left(\frac{K}{W}\right)$ , we see an equation similar to (8). Because the conduction and convection resistances are in series and may be summed, as depicted by Figure 2 in [14], the total thermal resistance  $R_{total} \left(\frac{K}{W}\right)$  for heat transfer through a plane wall is given by

$$R_{total} = \frac{1}{h_1A} + \frac{L}{kA} + \frac{1}{h_2A}, \quad (10)$$

where  $T_{INT}$ ,  $T_{EXT}$ ,  $T_{S,1}$ ,  $T_{S,2}$  are the inside, outside, internal surface and external surface temperatures, respectively, and  $q_x$  is the overall heat given by  $q_x = \frac{T_{INT} - T_{EXT}}{R_{total}}$ .

For a room in steady-state condition, the energy balance can be expressed as (see, e.g., [15] and [14])

$$\begin{aligned} \rho V C \frac{d}{dt} T_i &= \dot{m}_{AC} C_{AC} (T_{AC} - T_i) + \dot{m}_{air} C_{air} (T_{air} - T_i) \\ &+ \sum_{j \in N_i} R_{ji}^{-1} (T_j - T_i) \\ &+ \sum_{k \in N_{wall}} h_{wk} S_{wk} (T_{wk} - T_i) \\ &+ d(t, N(t - \Delta t), \phi(t)), \end{aligned} \quad (11)$$

where  $T_i$  is the temperature of the air in the test-bed,  $T_{AC}$  is the temperature of the air of the AC duct,  $T_{air}$  is the temperature of the air inlet duct,  $N$  is the number of occupiers,  $\phi$  represents the effect of solar flux,  $d$  is the overall disturbance,  $V$  is the room volume,  $\rho$  is the room air density,  $C \left( \frac{J}{kg \cdot K} \right)$  is the specific heat capacity of the air in the room,  $\dot{m}_{AC} \left( \frac{kg}{s} \right)$  is the mass flow rate of the AC duct,  $\dot{m}_{air} \left( \frac{kg}{s} \right)$  is the mass flow rate of the air inlet duct,  $R_{ji} \left( \frac{K}{W} \right)$  the thermal resistance of the wall that divides two adjacent rooms,  $N_{wall}$  is the set of the walls,  $h_{wk} \left( \frac{W}{m^2 K} \right)$  is the convection heat transfer coefficient of the wall  $k$ ,  $S_{wk}$  is the surface of the wall  $k$ . Taking  $d$  as a linear function with respect to  $N$  and substituting (6) into (11) yield

$$\begin{aligned} \rho V C \frac{d}{dt} T_i &= \dot{m}_{AC} C_{AC} (T_{AC} - T_i) + \dot{m}_{air} C_{air} (T_{air} - T_i) \\ &+ \sum_{j \in N_i} R_{ji}^{-1} (T_j - T_i) \\ &+ \sum_{k \in N_{wall}} h_{wk} S_{wk} (T_{wk} - T_i) \\ &+ \left[ c_0 C(t) + c_1 C(t - \Delta t) - f_1 I(t - \Delta t) \right] \\ &+ v(t), \end{aligned} \quad (12)$$

where  $v(t)$  is the disturbance. The weighted version of the central difference approximation is

$$\frac{d}{dt} T_i \approx k_1 \frac{T_{i,n+1} - T_{i,n}}{\Delta t} + k_2 \frac{T_{i,n} - T_{i,n-1}}{\Delta t} \quad (13)$$

where  $k_1 > 0$ ,  $k_2 > 0$  and  $k_1 + k_2 = 1$ . Applying the approximation (13) to (12) we obtain

$$\begin{aligned} (1 + \beta_{i1} q^{-1} + \beta_{i2} q^{-2}) T_n &= \alpha_{AC} T_{AC,n-1} + \alpha_{air} T_{air,n-1} + \sum_{j \in N_i} \alpha_{ji} T_{j,n-1} \\ &+ \sum_{k \in N_{wall}} \alpha_{wk} T_{wk,n-1} + (\alpha_0 + \alpha_1 q^{-1}) C_{n-1} \\ &+ \alpha_I I_{n-2} + \alpha v_{n-1}, \end{aligned} \quad (14)$$

where  $\alpha_{AC} = \alpha \dot{m}_{AC} C_{AC}$ ,  $\alpha_{air} = \alpha \dot{m}_{air} C_{air}$ ,  $\alpha_{ji} = \frac{\alpha}{R_{ji}}$ ,  $\alpha_{wk} = \alpha h_{wk} S_{wk}$ ,  $\alpha_0 = \alpha c_0$ ,  $\alpha_1 = -\alpha c_1$ ,  $\alpha_I = -\alpha f_1$ ,  $\alpha = \frac{\Delta t}{\rho V C k_1}$ ,  $\beta_{i1} = \frac{(1-2k_1)}{k_1} + \alpha_{AC} + \alpha_{air} + \sum_{j \in N_i} \alpha_{ji} + \sum_{k \in N_{wall}} \alpha_{wk}$ ,  $\beta_{i2} = \frac{k_1-1}{k_1}$ .

### C. Humidity

The model of humidity is obtained in a similar way as that for the temperature. The humidity in the room depends on that of the air coming out from the inlet and the AC ducts, the room temperature, the number of occupiers and some others. This is described as

$$\begin{aligned} \frac{d}{dt} H &= \lambda_{AC} (H_{AC} - H) \\ &+ \lambda_{air} (H_{air} - H) + h(t, T_i, N), \end{aligned} \quad (15)$$

where  $H_{AC}$  and  $H_{air}$  are the humidities of the air coming from the AC and the inlet, respectively, and  $h(t, T_i, N)$  is a function of time, room temperature and number of occupiers.

Approximating  $h(t, T_i, N)$  with a function that is linear in  $T_i$  and  $N$  and using (6) for estimate of  $N$  give

$$\begin{aligned} \frac{d}{dt} H(t) &\approx \lambda_{AC} (H_{AC} - H) + \lambda_{air} (H_{air} - H) \\ &+ \left[ b_0 C(t) + b_1 C(t - \Delta t) - g_1 I(t - \Delta t) \right] \\ &+ \lambda_T T_i + z(t), \end{aligned} \quad (16)$$

where  $z(t)$  is the disturbance,  $\lambda_{AC}$  and  $\lambda_{air} \left( \frac{1}{s} \right)$  are unknown gains,  $\lambda_T \left( \frac{\%RH}{sK} \right)$  is the unknown gain for temperature.

Applying the weighted central difference approximation as (13) in the temperature model, we obtain

$$\begin{aligned} (1 + \delta_{i1} q^{-1} + \delta_{i2} q^{-2}) H_n &= \gamma_{AC} H_{AC,n-1} + \gamma_{air} H_{air,n-1} + \gamma_T T_{i,n-1} + \gamma_I I_{n-2} \\ &+ \gamma z_{n-1} + (\gamma_0 + \gamma_1 q^{-1}) C_{n-1}, \end{aligned} \quad (17)$$

where  $\gamma_{AC} = \lambda_{AC} \gamma$ ,  $\gamma_{air} = \lambda_{air} \gamma$ ,  $\gamma_0 = b_0 \gamma$ ,  $\gamma_1 = -b_1 \gamma$ ,  $\gamma_I = -g_1 \gamma$ ,  $\gamma = \frac{\Delta t}{k_{H1}}$ ,  $\delta_{i1} = \frac{1-2k_{H1}}{k_{H1}} + \gamma_{AC} + \gamma_{air}$ ,  $\delta_{i2} = \frac{k_{H1}-1}{k_{H1}}$ ,  $k_{H1}$  is the weighted constant of the difference approximation.

### D. Test-bed

Experiments were done in the test-bed set up in the room A 225 (LAB3) on the 2nd floor of the Q-building on KTH campus. This room has an area of about 80 m<sup>2</sup>, a volume of about 270 m<sup>3</sup>, four windows of 0.64 × 4 m<sup>2</sup>

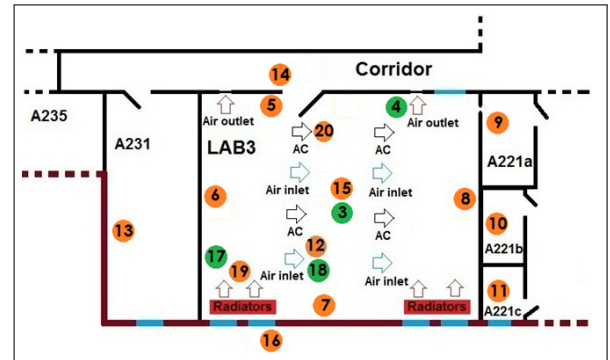


Fig. 1. Test-bed LAB3: temperature and humidity sensors (orange circle), CO2 level sensors (green circle), air outlet ducts (red arrow), fresh air inlet ducts (blue arrow), Air Conditioning ducts (black arrow), Radiators (red arrow), External wall (Brown line), windows (blue line).

on the external wall and one window on one internal wall of 2.5 m<sup>2</sup>. It is equipped with an HVAC system and a Wireless Sensor Network (WSN). Fig. 1 illustrates the spots where sensors are located inside and outside the test-bed room: orange circles stand for temperature and humidity sensors and green one for CO<sub>2</sub> level. Numbers in the circles are the identifiers of the sensors. Locations of the ducts of air inlet, air outlet and AC, the radiator and the windows are also indicated. Moreover, different colours are used to distinguish the “external” walls (brown) from the “internal” ones (black). All the measurements used in this paper (for identification and validation) were taken with sampling interval  $\Delta t = 3$  minutes in this test-bed during May and June, 2012.

### E. Identification of Models

The models were identified with a set of data measured over 45 hours in May 2012. Prediction Error Method was employed to identify the unknown parameters using the MATLAB® System Identification toolbox [18]. The identified parameters of the models (5), (14) and (17) are listed in Tables I, II and III, respectively. Since, in our test-bed, the effects from the neighbors are negligible, there are no parameters  $\alpha_{ij}$  listed in Table II.

Table I: Parameters of CO<sub>2</sub> model (5)

$a_1$	$K_N$	$K_I$
-0.8964	2.108	0.0579

Table III: Parameters of humidity model (17)

$\delta_{i1}$	-1.6696
$\delta_{i2}$	0.6844
$\gamma_{AC}$	$-2.83 \times 10^{-4}$
$\gamma_{air}$	0.0147
$\gamma_T$	$2.16 \times 10^{-4}$
$\gamma_0$	0.0016
$\gamma_1$	-0.0018
$\gamma_I$	$4.98 \times 10^{-5}$

Table II: Parameters of temperature model (14)

$\beta_{i1}$	-1.037
$\beta_{i2}$	0.1036
$\alpha_{AC}$	0.0086
$\alpha_{air}$	0.1331
$\alpha_0$	0.0022
$\alpha_1$	-0.0018
$\alpha_I$	$-4.66 \times 10^{-4}$
$\alpha_{w1}$	-0.0502
$\alpha_{w2}$	0.03
$\alpha_{w3}$	-0.0451
$\alpha_{w4}$	-0.004

## III. VALIDATION OF MODELS

### A. Validation with measurements in May

The validation metric employed in this paper is *fit* presented in [18], which is defined as

$$fit := \left( 1 - \frac{\|\hat{\mathbf{y}} - \mathbf{y}\|}{\|\mathbf{y} - \frac{1}{N} \sum_{i=1}^N y(i)\|} \right) \times 100, \quad (18)$$

where  $N$  is the number of components of the validation dataset,  $\mathbf{y}$  is the  $N \times 1$  vector of the output measurements and  $\hat{\mathbf{y}}$  is the  $N \times 1$  vector of the simulated model output. More specifically, each component of  $\hat{\mathbf{y}}$  is the output of the model calculated with initial conditions and current and past values of input measurements.

Figures 2, 3 and 4 show the performance of the identified models for CO<sub>2</sub> concentration, temperature and humidity, respectively. The models for temperature and humidity work very well. As for the CO<sub>2</sub> model, we had difficulties in obtaining accurate measurements from some sensors when we switched off the air inlet: those sensors are subject to

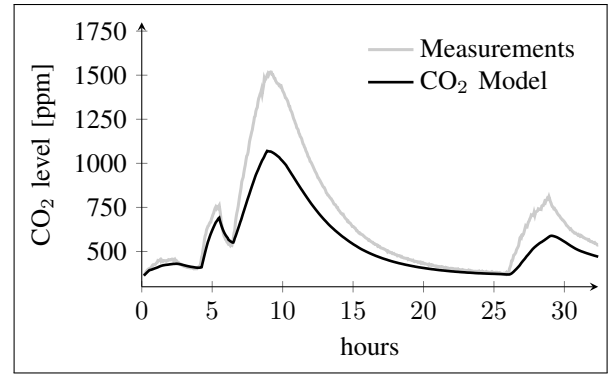


Fig. 2. Validation of CO<sub>2</sub> model (May, 2012): real measurements and outputs of CO<sub>2</sub> model, **fit 46.5%**.

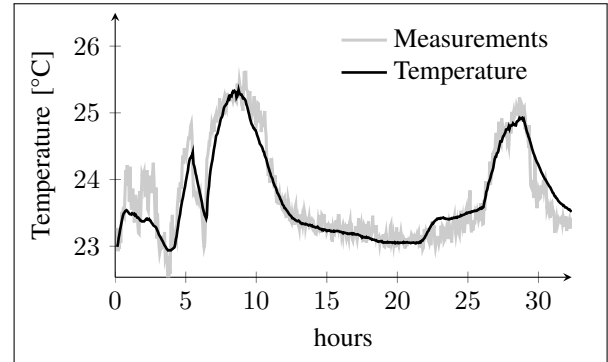


Fig. 3. Validation of the temperature model (May, 2012): real measurements and outputs of temperature model, **fit 64.5%**.

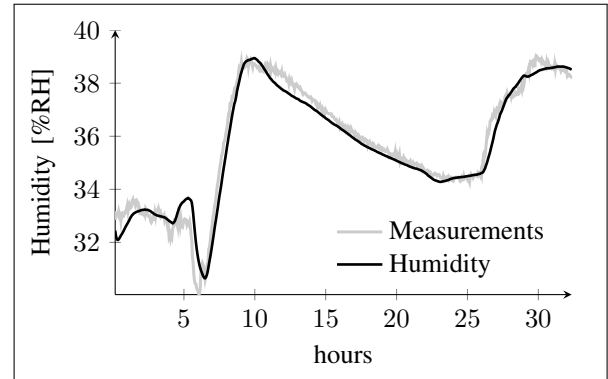


Fig. 4. Validation of the humidity model (May, 2012): real measurements and outputs of humidity model, **fit 80.5%**.

big disturbance due to the small airflow of fresh air from the central ventilation system of KTH campus. This is also one of our motivations to propose hybrid models for future work. However, taken into account the short-term training and the technical difficulties, the performance of the CO<sub>2</sub> model is acceptable.

### B. Validation with measurements in June

The models have been identified and validated with the data measured in May, 2012. It could be interesting to see how the models identified with data measured in May work in a different month, June, when the experiment environments, e.g., the weather conditions, have changed. Figures

5, 6 and 7 show their performance in June. As is seen, their performances in June are not as good as those in May (see Figs. 2-4). This is reasonable since the weather conditions were different. It is also noticed that the humidity model identified in May did not work well in June, which suggests that, under different weather conditions, some parameters of the physics-based models may be different, e.g., they could be very different between the models identified in summer and winter, respectively.

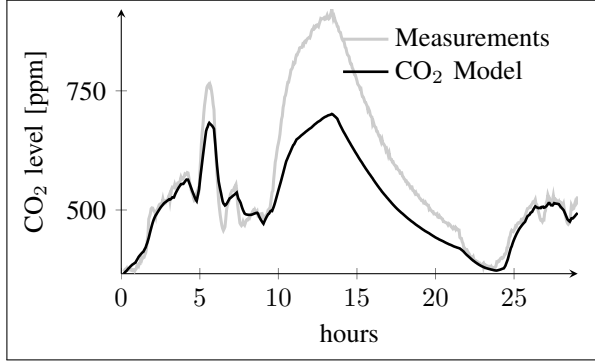


Fig. 5. Validation of CO<sub>2</sub> model (June, 2012): real measurements and outputs of CO<sub>2</sub> model, **fit 40.7%**, using validation data of a different month.

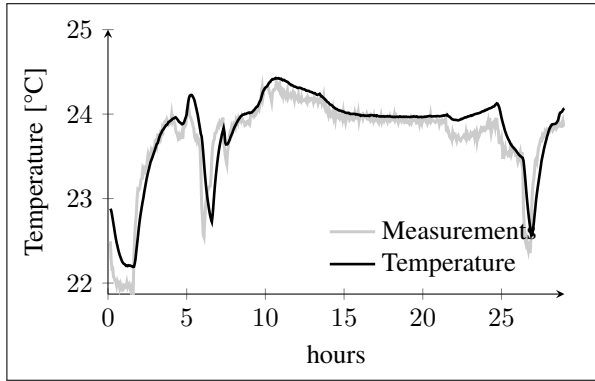


Fig. 6. Validation of temperature model (June, 2012): real measurements and outputs of temperature model, **fit 57.1%**, using validation data of a different month.

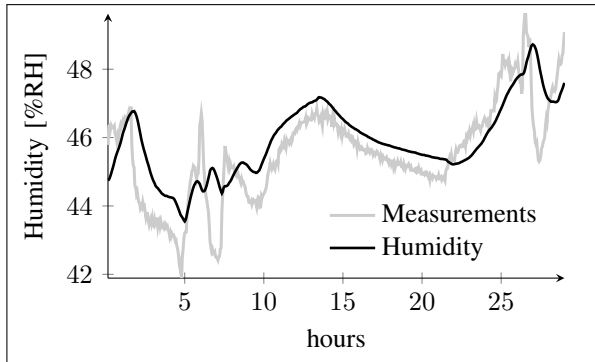


Fig. 7. Validation of the humidity model (June, 2012): real measurements and outputs of humidity model, **fit 34.3%**, using validation data of a different month.

Table IV: Parameters of temperature model (20)

$P_{rm1}$	$P_{rm2}$	$P_{AC}$	$P_{inl}$	$P_{out}$
-0.7556	-0.1406	0.0046	0.0389	0.0665

### C. Comparison with the temperature model in [15]

Our work is motivated by [15]. However, our models consider the effects from the occupiers, which was treated as disturbance in the pbARMAX model of temperature presented by [15], since the IAQ is evidently affected by the number of occupiers in the room (see, e.g., [16]). It is natural that we compare our temperature model with the one in the reference [15] to verify the effectiveness of our proposed models. It is noticed that the pbARMAX model (7) in [15] is obtained by discretizing (6) in [15], which is simplified version of (1) in [15]. Since, in our test-bed, the effects from the neighbors can be neglected, we should apply central difference scheme to (1) instead of (5) in [15] and consider the obtained model

$$T_{rm}(n+1) = T_{rm}(n-1) + 2\Delta t [P_{rm}T_{rm}(n) + P_{disch}T_{disch}(n) + P_{wai}T_{wai}(n) + P_{wdi}T_{wdi}(n) + \Delta E(n)], \quad (19)$$

where  $T_{rm}$  is the temperature of the room,  $T_{disch}$  is the discharge air temperature,  $T_{wai}$  is the temperature of the internal surface of the external wall,  $T_{wdi}$  is the temperature of the internal surface of the window on the external wall,  $\Delta t$  is the sampling interval,  $\Delta E$  is the disturbance and  $P_{rm}$ ,  $P_{disch}$ ,  $P_{wai}$  and  $P_{wdi}$  are parameters to be identified. The reason for only temperatures of the external wall and its windows considered is that, in average, the temperature difference between the room and the outside air is 2200% greater than the difference between the room and its neighbor rooms, and the temperatures of external wall and its windows could be more involved while those of the internal walls can be neglected. Moreover, we may consider applying the weighted central difference approximation instead of the normal one used above. Then, when model (19) is further customized for our test-bed, we have the following

$$(1 + P_{rm1}q^{-1} + P_{rm2}q^{-2})T_{rm}(n) = P_{AC}T_{AC}(n-1) + P_{inl}T_{inl}(n-1) + P_{out}T_{out}(n-1) + \Delta \tilde{E}(n-1), \quad (20)$$

where  $P_{rm1}$ ,  $P_{rm2}$ ,  $P_{AC}$ ,  $P_{inl}$  and  $P_{out}$  are the parameters to be identified. The unknown parameters  $P_{rm1}$ ,  $P_{rm2}$ ,  $P_{AC}$ ,  $P_{inl}$  and  $P_{out}$  were identified by the Least Squares method with the same set of measurements (taken in May, 2012) as used in Section II-E, which are listed in Table IV.

In Figure 8, the performance of our temperature model (3) is compared with that of model (20) using the same set of data (measured in May 2012) as used in Section III-A for validation, which verifies that the result of our proposed method is an improvement.

## IV. CONCLUSIONS

This paper has proposed, identified and validated three physics-based models for CO<sub>2</sub> level, temperature and humid-

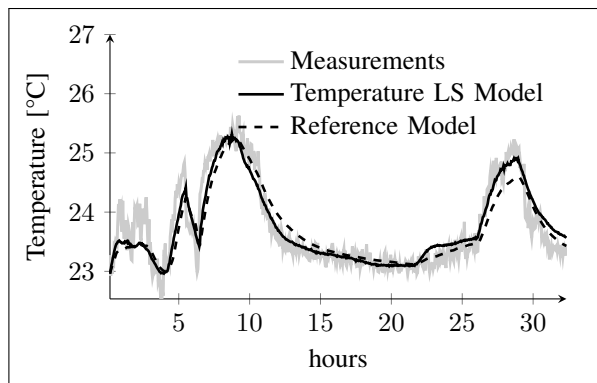


Fig. 8. Comparison with [15] using measurements taken in May, 2012: real measurements, outputs of temperature model (3) (with parameters identified by LS method) (fit 64.4%) and outputs of model (20) customized from [15] (fit 56%)

ity of a room in the Q-building on KTH campus, respectively. Motivated by a recent work [15], however, we consider the effect of occupiers in our models, which was treated as disturbance in the pbARMAX model of room temperature in [15], since the IAQ is evidently affected by the number of occupiers. Particularly, we have presented the formula (6) to estimate number of occupiers and applied it to establish models for temperature and humidity. With this formula, one does not need to count the number of occupiers but use measurements of CO<sub>2</sub> concentration when considering the effect of occupiers in the temperature and humidity models. Experimental results have verified that our models work very well even with a short-term training and our temperature model works better than the one in the reference [15], which indicates that our proposed method has improved the performance of the pbARMAX models.

Undoubtedly, the models should be trained with more data, which may improve their performance. It is also found that some parameters of the physics-based models could be very different if they are identified under different weather conditions (see Section III-B). Moreover, as in the reference [15], the order of the ARMAX model, e.g., the order of  $C(q, \theta)$ , is not justified in this paper. One of the future works is to justify the order of the model with some standard model selection criteria when more data are available. Last but not least, to cope with the technical difficulties (see Section III-A), we suggest the hybrid approach for modeling in future, e.g., to establish two models according to the two states (on and off) of the air inlet, respectively. In fact, though it is more challenging to identify the switched systems, good hybrid models are very useful and desired in our practical situation.

## REFERENCES

- [1] J. Burnett, "City buildings—eco-labels and shades of green!" *Journal of Landscape and Urban Planning*, vol. 83, pp. 29–38, 2007.
- [2] M.-C. P. Georgiadou and T. Hacking, "Futured-proofed design for sustainable urban settlements: Integrating futures thinking into the energy performance of housing developments," in *Proc. World Renewable Energy Congress (Sustainable Cities and Regions)*, 2011.
- [3] E. Bas, *Indoor Air Quality A Guide for Facility Managers*, 2nd ed. Lilburn and Ga and New York, 2004.
- [4] C. Teodosiu, R. Hohota, G. Rusaou, and M. Woloszyn, "Numerical prediction of indoor air humidity and its effect on indoor environment," *Building and Environment*, vol. 38, pp. 655–664, 2003.
- [5] D. Gyalistras, M. Gwerder, F. Oldewurtel, C. Jones, M. Morari, B. Lehmann, K. Wirth, and V. Stauch, "Analysis of energy savings potentials for integrated room automation," in *Proc. Clima - RHEVA World Congress*, 2010.
- [6] S. A. Ahmadi, I. Shames, F. Scotton, L. Huang, H. Sandberg, K. Johansson, and B. Wahlberg, "Towards more efficient building energy management systems," in *Proc. Knowledge, Information and Creativity Support Systems (KICSS'12)*, 2012.
- [7] L. Perez-Lombard, J. Ortiz, and C. Pout, "A review on buildings energy consumption information," *Energy and Buildings*, vol. 40, pp. 394–398, 2008.
- [8] H. Yoshida and S. Kumar, "Rarx algorithm based model development and application to real-time data for on-line fault detection," in *Proc. IBPSA - Building Simulation*, 1999.
- [9] R. Z. Freire, G. H. Oliverira, and N. Mendes, "Predictive controllers for thermal comfort optimization and energy savings," *Energy and Buildings*, vol. 40, pp. 1353–1365, 2008.
- [10] J. C.-M. Yiu and S. Wang, "Multiple ARMAX modeling scheme for forecasting air conditioning system performance," *Energy Conversion and Management*, vol. 48, pp. 2276–2285, 2007.
- [11] G. Mustafaraj, J. Chen, and G. Lowry, "Development of room temperature and relative humidity linear parametric models for an open office using BMS data," *Energy and Buildings*, vol. 42, pp. 348–356, 2010.
- [12] —, "Prediction of room temperature and relative humidity by autoregressive linear and nonlinear neural network models for an open office," *Energy and Buildings*, vol. 43, pp. 1452–1460, 2011.
- [13] Q. Qi and S. Deng, "Multivariable control of indoor air temperature and humidity in a direct expansion (DX) air conditioning (A/C) system," *Building and Environment*, vol. 44, pp. 1659–1667, 2009.
- [14] M. Maasoumy, "Modeling and optimal control algorithm design for HVAC systems in energy efficient buildings," Master's thesis, Berkeley University, California, 2011.
- [15] S. Wu and J.-Q. Sun, "A physics-based linear parametric model of room temperature in office buildings," *Building and Environment*, vol. 50, pp. 1–9, 2012.
- [16] T. E. Toolbox. Hvac systems—calculating indoor temperature and humidity loads. [Online]. Available: <http://www.engineeringtoolbox.com>
- [17] L. Ljung, *System Identification: Theory for the user*, 2nd ed. Prentice Hall, 1999.
- [18] —. System identification toolbox. [Online]. Available: <http://www.mathworks.it/products/sysid/>