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Optimized Pacing of Continuous Reheating Furnaces

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***Abstract*— A pacing algorithm for optimizing process times in continuous reheating furnaces of a rolling mill is presented. The problem is solved by an iterative algorithm that uses quadratic programming in each iteration loop. The method takes into account relevant time constraints and the variable local reheating power based on a mathematical furnace model. An example problem taken from an industrial application demonstrates the feasibility of the proposed method.**

I. Introduction

In the steel industry, hot rolling mills reshape billets, blooms, slabs, or other intermediate formats into final products like sectional steels, heavy plates, and coiled strips. In such plants, fuel-fired continuous furnaces are often used to reheat the material in preparation for the hot rolling process. In the current paper, we deal with the time scheduling problem (also referred to as *pacing*) for parallel reheating furnaces of a heavy-plate rolling mill. Since there is no buffer between the reheating furnaces and the rolling mill, the slabs have to be delivered to the mill on time and with the correct rolling temperature.

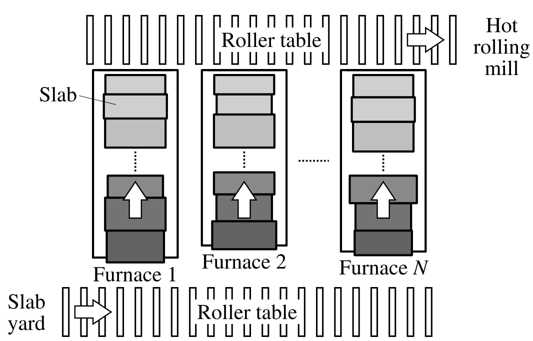

Fig. 1. Top view of slab reheating furnaces.

In the furnaces shown in Fig. 1, the products are slowly conveyed from the entry side (bottom) towards the exit side (top). For instance, the slabs may be pushed through the furnace or they may be conveyed by reciprocating beams. Important parameters of the slab reheating process are the geometries, the steel grades, the charging temperatures, and the desired final temperatures of the slabs. In the considered plant, the sequence of slabs is mainly determined by the order situation and the rolling process. Ideally, the sequence of slabs should ensure that each furnace contains slabs with similar desired final temperatures and similar required reheating times, which simplifies the time scheduling problem. Finding the optimum reheating times, which is the task of a pacing algorithm and the topic of the current paper, is important for minimum energy consumption, maximum productivity, maximum product quality, and hence higher profits.

A review of scheduling methods for integrated steel production is given in [1]. The review is focussed on direct linkages between the continuous caster and the hot rolling mill, where reheating furnaces are bypassed or only used with reduced heating load to save energy. A review focussing more on mathematical methods for production planning and scheduling in integrated steel plants is given in [2].

In [3], the production schedule of a hot strip mill with two identical reheating furnaces is generated by means of the tabu search heuristic based on a price collecting traveling salesperson problem. Each slab is assigned a desired conveying velocity in the reheating section and each furnace is operated at the desired velocity of the slowest slab it contains.

A heuristic scheduling system for a single slab reheating furnace and a rolling mill is presented in [4]. The system uses a modified greedy algorithm with small decision trees and total enumeration. The algorithm is based on an objective function that minimizes empty space in the furnace, the delivery times of slabs, and deviations from ideal slab reheating times, which depend only on the slab thicknesses.

A model-based production scheduling algorithm for a reheating furnace and a hot rolling mill is reported in [5]. A sequencing problem is formulated using a set of scheduling constraints and a cost function that captures time and fuel consumption. The problem is solved by enumeration and a modified branch and pruning procedure. Multiple scheduling constraints ensure that the problem dimension remains in an acceptable range. The heuristic model of the reheating furnace requires that the slabs stay in each furnace zone for a certain minimum time. Only one furnace is considered.

In [6], a model-based scheduling system for a single walking beam reheating furnace is developed. The furnace model computes the inhomogeneity and the mean error of slab temperatures upon discharge. These values are used in an objective function that is minimized by means of a genetic algorithm.

In the early days of furnace temperature control, the optimum furnace temperatures and conveying velocities for a certain product type and discharge temperature were often tabulated for steady-state furnace operation [7], [8]. In [9], a nonlinear PI-type furnace controller is proposed that uses only the slab velocity as a control input. A nonlinear model predictive controller for a small furnace with two zone

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temperatures and the conveying speed as control inputs is presented in [10]. These approaches deal only with a single furnace and do not take into account time constraints of upstream or downstream process steps.

Published solutions for the control of reheating furnaces fall into the two general classes *production scheduling* and *furnace temperature control*. Many production scheduling systems use fixed reheating times for each product, which neglects the non-uniformly distributed reheating power of the furnaces and the impact of furnace temperature control on the effective reheating time of the products. Other systems solve the temperature control problem for a single furnace and assume that the production schedule and especially the process times are predefined. However, we have not found a solution in the literature that suits our problem of pacing for several parallel furnaces considering non-steady-state operation and transient heat transfer into the products. In the current paper, we attempt to fill this gap by a pacing algorithm that uses mathematical furnace models and temperature planning and optimization systems for the furnaces. Our research is therefore right at the interface between scheduling, production optimization, and control.

The paper is organized as follows: The furnace pacing task and criteria for sufficient reheating are described in Section II. In Section III, we formulate the pacing algorithm and discuss its implementation. An example problem in Section IV demonstrates the feasibility of the method and its suitability for real-time operation.

II. Definition of the Task

A. Requirements of a furnace pacing algorithm

A furnace pacing algorithm has to be carried out in regular intervals—at least each time a product is discharged from the furnace. The algorithm requires that the sequence of products is defined in advance. This sequence is usually generated and optimized for the whole plant by means of some superordinate planning entity [11], which we refer to as *plant controller*. It uses simplified, stylized models of each processing step and a set of requirements to be satisfied by the production sequence. The plant controller typically produces, as a byproduct, a rough time schedule. Moreover, it has a load-balancing function wherever the flow of products splits up into several lines, e. g., at parallel slab reheating furnaces (cf. Fig. 1).

The main task of a furnace pacing algorithm is to create and recurrently refine the time schedule. The pacing algorithm may use the rough time schedule of the plant controller as an initial guess. The furnace pacing algorithm should take into account the following *requirements*:

- The temperature trajectories of the products should stay within given bounds. Especially, when the products are discharged from their furnace, their temperature profiles should accurately reach the desired values and they should be homogeneous.
- The throughput rate of processed steel should be maximized, which trivially implies that the conveying velocity should be as high as possible.
- Time constraints of upstream or downstream process steps should be respected.

Especially for the first requirement, an exact model of the reheating power and the temperature-control behavior of the furnaces is indispensable.

B. Formal description of the pacing task

Some important variables used in this paper are summarized in Table I. Given this nomenclature, the task of a furnace pacing algorithm is to define optimum times t_i for the conveying events (times when slabs are moved) so that the above requirements are satisfied.

In the algorithm proposed in this paper, we will iteratively improve the conveying times t_i. Let $\Delta t_{i,min}$ define the minimum required change of the time interval between the conveying events $i-1$ and i, i. e., $\Delta t_i \geq \Delta t_{i,min}$. The reasonable choice $\Delta t_{i,min} > t_{i-1} - t_i$ ensures that the algorithm cannot modify the sequence of conveying events and the sequence of discharging slabs from the furnaces. If $\Delta t_{i,min} = 0$, the algorithm is limited to delaying the reheating process. In this case, unforeseen reductions of the slab reheating times are not permitted, which can simplify the furnace temperature control task.

For a concise notation, we introduce the vectors $\boldsymbol{\Delta H}_{ref} = [\Delta H_{j,ref}]$, $\boldsymbol{t} = [t_i]$, $\boldsymbol{\Delta t} = [\Delta t_i]$, $\boldsymbol{\Delta t}_{min} = [\Delta t_{i,min}]$, the sparse matrix $\boldsymbol{q} = [q_{j,i}]$ with the total heat flux

$$q_{j,i} = \begin{cases} \lim_{\tau \to 0^-} q_j(t_i + \tau) & \text{if } i \in N_j \\ 0 & \text{else} \end{cases} \tag{1}$$

into the slab j at the time t_i (heat flux into bottom and top slab surface), and the set $N_j = \{i \in \mathbb{N} \,|\, t_{j,in} < t_i \leq t_{j,ref}\}$. Here, the index j refers to all slabs reheated during the considered planning period, and the index i refers to all conveying events occurring in this period.

TABLE I

Definition of variables.

Variable	Description	Unit
i	Index of conveying event	-
t_i	Time of conveying event i	s
Δt_i	Change of the time interval between the conveying event $i-1$ and the conveying event i	s
$\Delta t_{i,min}$	Minimum allowed value for Δt_i	s
j	Index of slab	-
$t_{j,in}$	Charging time of slab j	s
$t_{j,hom}$	Start of soaking period of slab j	s
$t_{j,out}$	Discharging time of slab j	s
$t_{j,res}$	Reheating time of slab j	s
$t_{j,ref}$	Reference time of slab j for determination of enthalpy error	s
$q_j(t)$	Total heat flux into slab j	$\mathrm{W/m^2}$
$\Delta H_{j,ref}$	Enthalpy error of slab j at time $t_{j,ref}$	$\mathrm{J/m^2}$
D_j	Thickness of slab j	m
y	Spatial coordinate in vertical direction	m
$T_j(y,t)$	Temperature profile of slab j	K
$\bar{T}_j(t)$	Mean temperature of slab j	K
$\tilde{T}_{j,out}$	Desired final temperature of slab j (uniform distribution over y)	K
ρ_j	Mass density of slab material	$\mathrm{kg/m^3}$
$c_j(T_j)$	Specific heat capacity (temperature-dependent)	J/kg/K

C. Criterion for sufficient reheating

If a product does not stay sufficiently long in the reheating furnace, the potential negative consequences are as follows:

- The final product temperature may fall below $\tilde{T}_{j,out}$
- Temperature constraints during the soaking phase $[t_{j,hom}, t_{j,out}]$ may be violated.
- The final temperature profile $T_j(y, t_{j,out})$ may be inhomogeneous.

These problems can often be avoided by increasing the effective reheating time $t_{j,res} = t_{j,out} - t_{j,in}$ of a slab j. As the reheating capacity varies with respect to both time and location in the furnaces, it is vital to implement the time delay at the optimum instant.

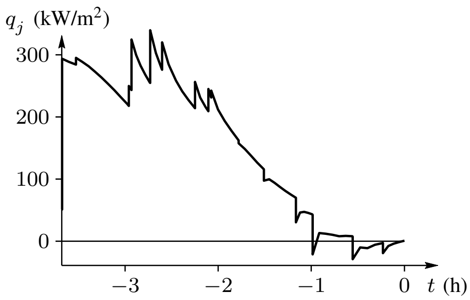

Fig. 2. Total heat flux into a slab.

Fig. 2 shows a typical heat flux curve of a 340 mm thick slab while it is reheated in a pusher-type slab furnace. The shape of the curve depends on the surface properties and temperatures of the slab and its environment. As thermal radiation is the dominant mode of heat transfer, the curve in Fig. 2 has been computed by means of the net-radiation method [12]. The curve is discontinuous at times t_i when the slabs change their position, which is assumed to happen instantaneously. Towards the end of the reheating period, there are downward discontinuities because the hot slab is pushed into colder sections of the furnace, which homogenizes the slab temperature profile. Consequently, the heat flux into the slab may become negative in these furnace sections.

If the slab considered in Fig. 2 were too cold at the exit time $t_{j,out} = 0$ h, increasing the interval between two conveying events by Δt_i is most effective at those times when $q_j(t)$ has a high value. The typical shape shown in Fig. 2 elucidates why the left-hand limit is used in (1).

As an aggregate quantity describing whether a slab j is sufficiently reheated, we consider the enthalpy error

$$\Delta H_{j,ref} = D_j \rho_j \int_{\tilde{T}_{j,out}}^{\mathrm{mean}_y\{T_j(y,t_{j,ref})\}} c_j(T) \mathrm{d}T \qquad (2)$$

at the user-defined reference time $t_{j,ref} \in (t_{j,in}, t_{j,out})$. It defines the difference between the enthalpy per unit slab area expected at the reference time $t_{j,ref}$ and the desired enthalpy when the slab is discharged. Ideally $\Delta H_{j,ref} \geq 0$, so that the remaining time interval $(t_{j,ref}, t_{j,out})$ can be used for homogenizing the temperature profile without further increasing the mean temperature. The value $t_{j,ref} = 1/2(t_{j,hom} + t_{j,out})$ (midpoint of the soaking period) was found to be a good reference time. Other values for $t_{j,ref}$ are also possible. Very small values for $t_{j,ref}$ are however not recommendable as this would induce maximum reheating of the slab early in the furnace, which may impair the efficiency of the furnace and increase the formation of scale at the slab surface. Vice versa, if $t_{j,ref}$ is too close to $t_{j,out}$, the remaining time for homogenizing the slab temperature profile may be insufficient.

The negative enthalpy error $-\Delta H_{j,ref}$ defines precisely the energy that remains to be supplied to the slab until it is discharged at the time $t_{j,out}$. It is a reasonable and empirically corroborated assumption that $\Delta H_{j,ref} = 0$ allows the desired final temperature $\tilde{T}_{j,out}$ to be reached, the temperature constraints to be satisfied, and the final slab temperature $T_j(y, t_{j,out})$ to be sufficiently uniform.

III. Pacing algorithm

The task of a pacing algorithm is to find optimum (minimum) time changes $\boldsymbol{\Delta t}$ such that the constraints

$$\boldsymbol{\Delta H}_{ref} + \boldsymbol{q}\boldsymbol{\Delta t} \geq \boldsymbol{0} \qquad (3a)$$

$$\boldsymbol{\Delta t} \geq \boldsymbol{\Delta t}_{min} \qquad (3b)$$

are satisfied. Here, the inequality sign is to be applied to each component of the respective vectors. Additional time constraints of upstream or downstream process steps can be added on demand.

The rationale of constraint (3a) is to ensure sufficient reheating of the slabs. This would be exactly achieved if the values $q_{j,i}$ remained constant during the extra times Δt_i. In reality, however, $q_j(t)$ is generally getting smaller because the increasing slab surface temperature decreases the heat flux into the slab. Moreover, the extra time Δt_i entails not only a time shift of all signals but a general lowering of $q_j(t)$ after the inserted time slot. These higher-order effects should be captured by a pacing algorithm, which is why the following algorithm works iteratively.

A. Iterative algorithm

We propose the following iterative pacing algorithm:

1) For given conveying times t_i compute for each slab j the heat flux $q_j(t)$ and the enthalpy error $\Delta H_{j,ref}$.
2) Stop if the expected temperature trajectories of the slabs are sufficiently accurate (termination condition).
3) Determine the optimum changes Δt_i for all conveying times t_i so that the throughput and the reheating accuracy of the slabs are high.
4) Use the update formula $t_i \leftarrow t_i + \sum_{k=1}^{i} \Delta t_k$ for the conveying times and start again at step 1.

As will be shown in Section III-C, the optimization problem in step 3 of the above algorithm can be exactly solved. Nevertheless, it is hard to prove optimality of our iterative optimization strategy in the global sense. The considered objectives of the algorithm, i. e., maximum throughput and maximum product quality, may be antagonistic. The operating point with the highest throughput may, for instance, be suboptimal in terms of uniformity of slab temperature profiles. Moreover, high throughput can entail undesirable higher specific energy consumption because also burners with a lower efficiency have to be utilized.

B. Computation of heat flux and enthalpy error

To calculate the heat flux $q_j(t)$ and the enthalpy error $\Delta H_{j,ref}$, we use a mathematical furnace model and a trajectory planning and optimization algorithm presented in [13], [14]. For each slab, the 1-dimensional heat conduction equation with temperature-dependent material parameters is solved by means of the Galerkin weighted residual method with polynomial trial functions up to second order. Thermal radiation is the dominant mode of heat exchange inside a reheating furnace. Therefore, the heat flux values at the slab surfaces are computed based on the net radiation method [12]. This determines $q_j(t)$ and the boundary conditions of the heat conduction equations for the slabs. Based on the slab temperature, $\Delta H_{j,ref}$ follows from evaluating (2).

The local furnace temperatures, which are control variables with limited slopes, are chosen by a trajectory planning and optimization algorithm described in [14]. The algorithm uses the mathematical furnace model in a dynamic optimization routine. It minimizes a cost function that takes into account the slab reheating quality, constraints on the slab temperature trajectories, and constraints on the furnace temperatures, which have a direct impact on the energy consumption of the process. The model-based optimization routine has a sufficiently long planning horizon meaning that it can cover the same planning period as the pacing algorithm. The routine attempts to reach the most accurate slab reheating result and reliably indicates if slabs are too cold or too hot as they are discharged from their furnace.

The temperature optimization routine and the pacing algorithm should be simultaneously executed because the maximum reheating power of a furnace is in fact a dynamic quantity, which depends on the properties of the currently charged slabs and on previous furnace temperatures. Hence, there is no one-to-one relation between the desired slab temperature rise $\tilde{T}_{j,out} - T(y, t_{j,in})$ and the required reheating time $t_{j,res}$. This is all the more true if some slabs are hot charged [15], meaning that thermal energy from continuous casting is recovered by directly forwarding hot slabs from the casting machine to the reheating furnaces.

For slabs that are already in a furnace when the pacing algorithm is executed, the temperature optimization routine should get an accurate estimate of the current slab temperatures. Such estimates may be calculated by means of the mathematical furnace model [13] or a state estimator like the Kalman filter presented in [16].

C. Quadratic program

The optimum time shifts Δt_i in step 3 of the above algorithm are determined by the quadratic program

$$\min_{\boldsymbol{\Delta t},\boldsymbol{s}} \Big\{ \boldsymbol{r}^T\boldsymbol{r} + k_1(\boldsymbol{\Delta t})^T\boldsymbol{\Delta t} + k_2(\boldsymbol{\Delta t})^T\boldsymbol{D}^T\boldsymbol{D}\boldsymbol{\Delta t} + k_3(\boldsymbol{\Delta t})^T\boldsymbol{D}^T\boldsymbol{D}(\boldsymbol{\Delta t} + 2\boldsymbol{D}\boldsymbol{t}) + k_4\boldsymbol{s}^T\boldsymbol{s} \Big\} \tag{4a}$$

$$\boldsymbol{\Delta t} \geq \boldsymbol{\Delta t}_{min} \tag{4b}$$

$$\boldsymbol{s} \geq \boldsymbol{0} \tag{4c}$$

with the residuum $\boldsymbol{r} = \boldsymbol{\Delta H}_{ref} + \boldsymbol{q}\boldsymbol{\Delta t} - \boldsymbol{s}$, the slack variable $\boldsymbol{s}$, and the dynamically sized difference operator

$$\boldsymbol{D} = \begin{bmatrix} 1 & -1 & 0 & \cdots & 0 \\ 0 & 1 & -1 & & \vdots \\ \vdots & & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & 1 & -1 \end{bmatrix}.$$

In (4a), the first term $\boldsymbol{r}^T\boldsymbol{r}$ is used as a soft constraint instead of the inequality (3a), which might cause undesirably large time delays Δt_i if it were strictly implemented. The second term with the weighting factor $k_1 \geq 0$ should minimize the absolute time shifts $|\Delta t_i|$. The third term with the weighting factor $k_2 \geq 0$ homogenizes the time shifts Δt_i, i. e., neighboring time shifts Δt_i and Δt_{i+1} should have similar values. The fourth term with the weighting factor $k_3 \geq 0$ homogenizes the periods between conveying events, i. e., it attempts to make the distribution of the times t_i more uniform. The fifth term with the weighting factor $k_4 \geq 0$ prevents an immoderate growth of the slack variable $\boldsymbol{s}$. It therefore causes negative values Δt_i, i. e., a compression of the time schedule, if $\Delta H_{j,ref}$ contains large positive values (slabs are too hot).

The quadratic program (4) is one of the key findings of this work. Based on the abbreviations

$$\boldsymbol{x} = \begin{bmatrix} \boldsymbol{\Delta t} \\ \boldsymbol{s} \end{bmatrix}, \qquad \boldsymbol{x}_{min} = \begin{bmatrix} \boldsymbol{\Delta t}_{min} \\ \boldsymbol{0} \end{bmatrix}$$

$$\boldsymbol{H} = \begin{bmatrix} \boldsymbol{q} & -\boldsymbol{I} \\ \boldsymbol{I} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{I} \end{bmatrix}^T \begin{bmatrix} \boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & k_1\boldsymbol{I} + (k_2 + k_3)\boldsymbol{D}^T\boldsymbol{D} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & k_4\boldsymbol{I} \end{bmatrix} \begin{bmatrix} \boldsymbol{q} & -\boldsymbol{I} \\ \boldsymbol{I} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{I} \end{bmatrix}$$

$$\boldsymbol{f} = (\boldsymbol{\Delta H}_{ref})^T \begin{bmatrix} \boldsymbol{q} & -\boldsymbol{I} \end{bmatrix} + k_3\boldsymbol{t}^T\boldsymbol{D}^T \begin{bmatrix} \boldsymbol{D}^T\boldsymbol{D} & \boldsymbol{0} \end{bmatrix},$$

(4) can be rewritten in the standard form

$$\min_{\boldsymbol{x}} \left\{ \frac{1}{2}\boldsymbol{x}^T\boldsymbol{H}\boldsymbol{x} + \boldsymbol{f}\boldsymbol{x} \right\}$$
$$\boldsymbol{x} \geq \boldsymbol{x}_{min}.$$

This quadratic program is always feasible and has a unique global solution because $\boldsymbol{H}$ is positive definite.

D. Implementation and parameterization

Depending on the chosen units, it may be beneficial for the numeric condition of the quadratic program to normalize the weighting coefficients and optimization variables beforehand. The dimension of the problem grows linearly with the number of slabs, which is generally proportional to the length of the optimization interval. To solve the quadratic program, we use the open source solver `qpOASES`, Version 3.0beta, which uses an online active set method (cf. [17] and `www.qpOASES.org`).

The solution of the quadratic program requires less than 25 % of the overall CPU time. The biggest part of the CPU load is associated with the computation of the heat flux $q_j(t)$ and the enthalpy error $\Delta H_{j,ref}$ (cf. Section III-B). Advantageously, this computation can be parallelized because it is carried out individually for each furnace.

Consider that the pacing algorithm is executed at the time t. Those slabs j for which $t \in [t_{j,ref}, t_{j,out}]$ holds, i. e., slabs that are discharged soon, should not be considered by the pacing algorithm. The rationale of this strategy is that the concerned slabs are already in a section of the furnace where the installed heating power is rather low (soaking section). If these slabs have not yet absorbed sufficient thermal energy ($\Delta H_{j,ref} \ll 0$), they will not become hot enough even if the production process is delayed by large positive values Δt_i.

The coefficients k_1 to k_4 used in the quadratic cost function (4a) have to be tuned by the user. We found that $k_4 \ll 1$ is a reasonable choice to approach optimum reheating times from below. Then the conveying process is more often retarded than accelerated until the algorithm finds a solution where all slabs are accurately reheated. The pacing algorithm should therefore be initialized with reheating times that are too short; it will automatically find the optimum values ensuring the highest possible throughput.

The convergence of the algorithm towards an optimum solution is fast at the beginning but rather slow as the final solution is approached. Therefore, we use the number of iteration loops as a termination criterion and stop the algorithm always after 4 loops. This is a good compromise between accuracy and CPU time.

IV. Example problem

We study an example problem where three pusher-type slab furnaces supply a heavy-plate rolling mill with two four-high reversing stands. The furnaces are operated by Aktiengesellschaft der Dillinger Hüttenwerke and have different designs and slightly different heating capacities. Their lengths are 33 m, 34 m, and 35 m. Their total capacity is 670 t/h and their total power is 390 MW. The rolling mills reduce the thickness of the slabs from up to 450 mm to the final product thickness in the range 6 mm to 400 mm.

We consider a real operating scenario where more than 1000 slabs (39 000 t of steel) have been reheated within 132 h. In our simulation, we apply the pacing algorithm presented in the previous section to analyze whether the furnaces would facilitate a higher throughput. The dimension of the quadratic program equals the number of slabs. As initial guess of the iterative algorithm, we use $t_{j,res} \approx 2.5$ h. For simplicity, we do not consider time constraints attributed to the rolling mill. Such constraints could be easily implemented as linear inequalities in (4).

Fig. 3 shows that the enthalpy errors $\Delta H_{j,ref}$ decrease in the course of the iterations. After iteration 4, the error is small enough to stop the algorithm. In terms of the mean slab temperature, the remaining maximum error (at the reference time $t_{j,ref}$) is smaller than 85 K. The remaining average absolute error is smaller than 10 K. The sensitivity

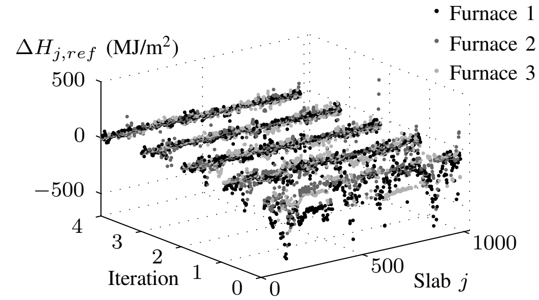

Fig. 3. Enthalpy errors of slabs.

of $\Delta H_{j,ref}$ with respect to $t_{j,res}$ decreases as the optimal solution is approached because the underlying furnace temperature controller aims at maximizing the reheating quality of the slabs, which ideally yields $\Delta H_{j,ref} = 0$.

Fig. 4 shows the total time required to reheat the considered slabs. Instead of 132 h as in the real plant, the slabs can be reheated within just 86 h. This result indicates that the furnaces are underutilized.

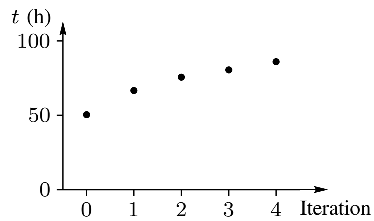

Fig. 4. Total processing time.

Fig. 5 shows the heat flux into a representative slab. The time axis is shifted to make the results of the different iterations comparable. For the same slab, Fig. 6 shows the temperature trajectory (thin lines for minimum and maximum temperatures, thick lines for mean temperatures). This slab can be reheated to its desired final temperature $\tilde{T}_{j,out}$ without violation of any constraints. Most other slabs are reheated with a similarly high accuracy. Fig. 7 corroborates this; it shows for all iterations the final temperatures of some slabs that are consecutively reheated in the same furnace.

The calculation was carried out on a standard PC (2.4 GHz dual core, 2 GB RAM), where it required 3.9 h CPU time. If parallel processing as suggested in Section III-D is used and if the length of the planning horizon is limited to 6 h, which is

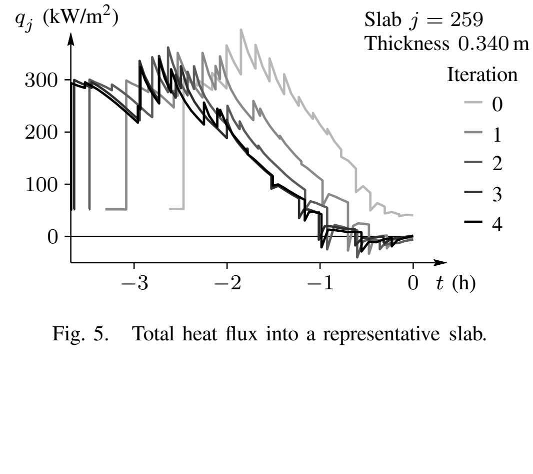

Fig. 5. Total heat flux into a representative slab.

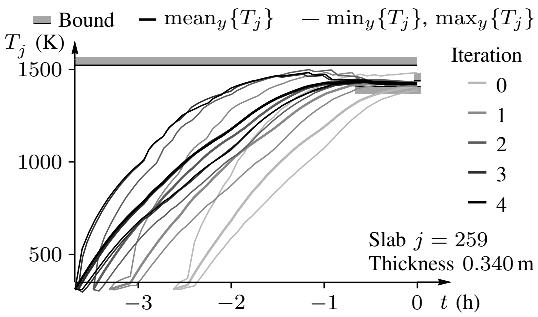

Fig. 6. Temperature trajectory of a representative slab.

practically sufficient, the pacing algorithm is suitable for real-time execution with sampling periods in the range of 8 min. Encouraged by the good simulation results, the proposed pacing system is currently implemented in the real plant.

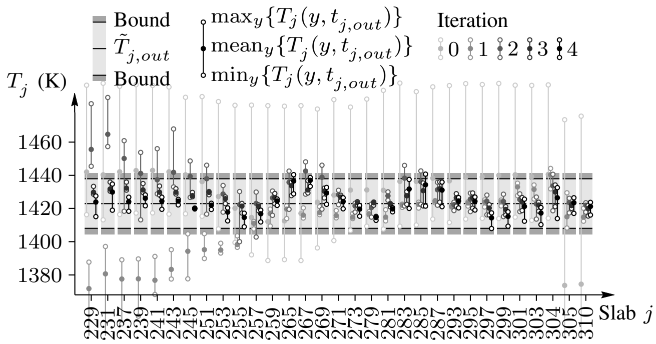

Fig. 7. Final slab temperatures.

V. Conclusions

The main conclusions from this work are as follows:

1) A real-time pacing algorithm that takes into account the heating capacity of the slab furnaces has been developed. The method bridges between specialized furnace temperature control systems and production scheduling systems with standardized slab reheating times. The feasibility of the proposed algorithm has been demonstrated by means of an example scenario.
2) The heating power of a furnace depends on its design, the installed burners, previous furnace temperatures, and the properties of the current stock of slabs, especially their geometry, their charging temperature, and their desired final temperature. Hence, the real heating capacity of a furnace is a dynamic quantity.
3) The heating power is not uniformly distributed over the length of a furnace. It is therefore important to take into consideration the respective position of a slab if the conveying process is retarded or accelerated.
4) In the considered plant, the furnaces are underutilized because the rolling mill itself constitutes a bottleneck. It may thus save energy if certain furnaces are temporarily switched off. The presented methods can be used as analytical tools for deciding about such temporary shut-down strategies.
5) The enthalpy error $\Delta H_{j,ref}$ is a useful indicator for sufficient reheating of the slabs. The sensitivity of $\Delta H_{j,ref}$ with respect to the slab reheating time $t_{j,res}$ decreases as the optimal solution is approached.

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