2013 European Control Conference (ECC)
July 17-19, 2013, Zürich, Switzerland.

Optimal $\mathcal{H}_2$ Mode-Independent Filter for Generalized Bernoulli Jump Systems*

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***Abstract*— In this paper we work with the filtering problem for a special class of discrete-time Markov Jump Linear Systems (MJLS) whose transition probability matrix has identical rows. For that class of systems we design, with the help of new necessary and sufficient LMI conditions, $\mathcal{H}_2$ optimal mode-independent filters with the same order of the plant. For cluster availability of the mode, we also show it is possible to design optimal filters. We complete the results with a numerical example.**

Keywords: Discrete-time systems; Markov jump linear systems; Optimal filtering; Linear Matrix Inequalities;

I. Introduction

Dynamic systems that present sudden changes on their structures or parameters have been the subject of several studies in the last decades. Among the several ways to model such a dynamic system, one of increasing interest is the Markovian jump linear system (MJLS). There is a large amount of theory in the literature that extend the usual concepts of stability, observability, controllability, $\mathcal{H}_2$ and $\mathcal{H}_\infty$ norms to this special class, see [5] and references therein. An important assumption to consider for MJLS design problems is if the Markov chain state, often called mode, is available or not to the controller or filter at every instant of time $k \in \mathbb{N}$. Based on that information the design is said to be either mode-dependent or mode-independent.

The problem of determining a strictly proper optimal $\mathcal{H}_2$ mode-dependent filter was solved in [4] with the use of Coupled Algebraic Riccati Equations (CARE) and in [6] with the use of LMIs. For the mode-independent problem, an optimal $\mathcal{H}_2$ filter using augmented matrices based on the Kronecker product was proposed in [3]. The filters obtained using this method are of order Nn, where N is the number of Markov chain modes and n is the plant order. Only strictly proper filters have been considered and the output to be estimated must be independent of the system mode and input noise.

In this paper we work with a particular case of a Markov chain given by a transition probability matrix with identical rows. For $N = 2$, this is equivalent to considering the random variable to be governed by a Bernoulli process. Under that assumption, the probability that the system mode assumes a particular value does not depend on the current one. Such a modeling has been used to describe packet dropouts in several works in the literature of Networked Control Systems [8]. It is a very simple way to model such phenomena, yet there are remarkable properties that can be explored under that assumption. Our main result is that, given those assumptions over the transition probabilities, and considering cluster availability of the mode, it is possible to obtain an optimal $\mathcal{H}_2$ proper filter. That includes both the mode-dependent and mode-independent filters as particular cases. For the mode-independent case, the obtained filter has the same order of the plant, as opposed to the extended order filter in [3]. Through a numerical example, we illustrate our results and compare them with the alternatives in the literature.

The notation used throughout is standard. Capital letters denote matrices and small letters denote vectors. For scalars, small Greek letters are used. For real matrices or vectors ($'$) indicates transpose. For

*This research was supported by grants from “Conselho Nacional de Desenvolvimento Científico e Tecnológico - CNPq” and “Fundação de Amparo à Pesquisa do Estado de São Paulo - FAPESP”, Brazil.

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square matrices $\mathrm{Tr}(X)$ denotes the trace function of X being equal to the sum of its eigenvalues and, for the sake of easing the notation of partitioned symmetric matrices, the symbol $(\bullet)$ denotes generically each of its symmetric blocks. The set of natural numbers is denoted by $\mathbb{N}$ while $\mathbb{K} = \{1, \cdots, N\}$. The symbol $\mathcal{E}\{\cdot\}$ denotes mathematical expectation of $\{\cdot\}$. For any stochastic signal $\xi(k)$, defined in the discrete-time domain $k \in \mathbb{N}$, the quantity $\|\xi\|_2^2 := \sum_{k=0}^{\infty} \mathcal{E}\{\xi(k)'\xi(k)\}$ is its squared norm. The set of signals $\xi(k) \in \mathbb{R}^p$, $k \in \mathbb{N}$ such that $\|\xi\|_2^2 < \infty$ is denoted $\mathcal{L}_2^p$. For $p_1, \cdots, p_N$ being probabilities which must satisfy $p_i \geq 0$ for all $i \in \mathbb{K}$ and $\sum_{i\in\mathbb{K}} p_i = 1$ we denote $p = [p_1 \cdots p_N]' \in \mathbb{R}^N$ the probability vector. Given $P_1, \cdots P_N$ symmetric and positive definite matrices, the mean value or convex combination is denoted as $P_p = \sum_{i\in\mathbb{K}} p_i P_i$.

II. Preliminaries

Consider the following discrete-time system

$$\mathcal{G} : \begin{cases} x(k+1) = A(\theta_k)x(k) + J(\theta_k)w(k) \\ z(k) = C_z(\theta_k)x(k) + E_z(\theta_k)w(k) \\ y(k) = C_y(\theta_k)x(k) + E_y(\theta_k)w(k) \end{cases} \tag{1}$$

where $x(k) \in \mathbb{R}^n$ is the state, $w(k) \in \mathbb{R}^m$ is the external perturbation, $z(k) \in \mathbb{R}^r$ is the output to be estimated and $y(k) \in \mathbb{R}^q$ is the measured output. In the general MJLS framework, we consider that the random variable θ_k takes values in $\mathbb{K}$ according to a Markov chain with transition probability matrix given by $\mathbb{P} = [p_{ij}] \in \mathbb{R}^{N\times N}$ where $p_{ij} = \mathrm{Prob}(\theta_{k+1} = j \mid \theta_k = i)$, which satisfies the normalized constraints $p_{ij} \geq 0$ and $\sum_{j=1}^{N} p_{ij} = 1$ for each $i \in \mathbb{K}$. To ease the presentation, the following notations $A(\theta_k) := A_i$, $J(\theta_k) := J_i$, $C_z(\theta_k) := C_{zi}$, $E_z(\theta_k) := E_{zi}$, $C_y(\theta_k) = C_{yi}$ and $E_y(\theta_k) = E_{yi}$ whenever $\theta_k = i \in \mathbb{K}$ are used. We will also consider that matrices C_{zi} and E_{zi} are orthogonal, that is $C_{zi}'E_{zi} = 0$ for all $i \in \mathbb{K}$. Throughout this paper, an important assumption about the transition probabilities will be taken:

$$p_{ij} = p_j, \ \forall i, j \in \mathbb{K} \times \mathbb{K} \tag{2}$$

This corresponds to a transition probability matrix $\mathbb{P} \in \mathbb{R}^{N\times N}$ with identical rows. For $N = 2$, this is equivalent to defining the random variable $\theta_k \in \mathbb{K}$ with Bernoulli distribution. For $N > 2$ we say it has a generalized Bernoulli distribution. One important characteristic of such distribution is that the mode $\theta_k \in \mathbb{K}$ at every instant of time does not depend on any previous value.

There are several equivalent forms to define stability of the system (1) with $w(k) = 0$ and arbitrary initial condition, like mean-square stability, stochastic or exponential mean-square stability. It has been shown in [9] that those definitions are equivalent for a MJLS, being referred to as second-moment stability, or simply stability. In particular, it is well known that system (1) with $w(k) = 0$ and generalized Bernoulli distribution (2) is stable if and only if there exist symmetric matrices $P_i > 0$ for $i \in \mathbb{K}$ satisfying

$$A_i'P_pA_i - P_i < 0, \ i \in \mathbb{K} \tag{3}$$

which is a set of N LMIs with N matrix variables. Its solution can be found with no difficulty mainly if the number of modes of the generalized Bernoulli chain is small. This stability condition can be reduced, with no loss of generality (that is, keeping its necessary and sufficient nature), to only one LMI expressed through only one matrix variable. The next definition is the generalization of the $\mathcal{H}_2$-norm from discrete-time deterministic systems to the stochastic jump case under consideration.

Definition 1: The $\mathcal{H}_2$-norm of a stable system $\mathcal{G}$ is defined by

$$\|\mathcal{G}\|_2^2 := \sum_{i=1}^{N}\sum_{s=1}^{m} \mu_i \|z^{s,i}\|_2^2 \tag{4}$$

where $\mu_i = \mathrm{Prob}(\theta_0 = i \in \mathbb{K})$ and $z^{s,i}$ represents the output $z(0), z(1), \cdots$ obtained from the input $w(k) = e_s\delta(k)$, where $e_s \in \mathbb{R}^m$ is the s-th column of the $m \times m$ identity matrix, $\delta(k)$ is the discrete impulse function, $x(0) = 0$ and $\theta_0 = i \in \mathbb{K}$.

The calculation of the $\mathcal{H}_2$-norm of a stable system $\mathcal{G}$ can be done by solving a convex programming problem expressed through LMIs. Indeed, it is well known [2] that

$$\|\mathcal{G}\|_2^2 = \inf_{P_i>0} \sum_{i\in\mathbb{K}} \mu_i \mathrm{Tr}\left(J_i'P_pJ_i + E_{zi}'E_{zi}\right) \tag{5}$$

subject to

$$A_i' P_p A_i + C_{zi}' C_{zi} - P_i < 0, \; i \in \mathbb{K} \tag{6}$$

The values to be adopted by the parameters $\mu_1 \cdots \mu_N$ can recast some important practical situations. If $\theta_0 = i \in \mathbb{K}$ is known then $\mu_i = 1$ and $\mu_j = 0$ for all $j \neq i \in \mathbb{K}$ must be chosen. Another possibility is to set $\mu = \pi^* = p$ which represents the stationary distribution.

III. Analysis of Generalized Bernoulli Jump Systems

An alternative LMI condition to calculate the $\mathcal{H}_2$ norm, under assumption (2), is shown next.

Theorem 1: The $\mathcal{H}_2$-norm of system $\mathcal{G}$ defined in (4), under assumption (2) for the transition probabilities, is given by

$$\|\mathcal{G}\|_2^2 = \inf_{Q>0} \sum_{i \in \mathbb{K}} \mu_i \mathrm{Tr}\left(J_i' Q J_i + E_{zi}' E_{zi} \right) \tag{7}$$

subject to

$$\sum_{j \in \mathbb{K}} p_j (A_j' Q A_j + C_{zj}' C_{zj}) - Q < 0 \tag{8}$$

Proof: We start the proof by assuming that inequality (6) is valid for some symmetric matrices $P_i > 0, i \in \mathbb{K}$. Multiplying (6) by $p_i \geq 0$, summing up for $i \in \mathbb{K}$ and with $Q = \sum_{j \in \mathbb{K}} p_j P_j = P_p > 0$, the constraint (8) is also satisfied. Conversely, for $Q > 0$ satisfying (8), define $P_i = A_i' Q A_i + C_{zi}' C_{zi} + \epsilon I > 0$ for all $i \in \mathbb{K}$ and $\epsilon > 0$. Hence,

$$\begin{aligned} P_p - Q &= \sum_{j \in \mathbb{K}} p_j (A_j' Q A_j + C_{zj}' C_{zj}) - Q + \epsilon I \\ &< 0 \end{aligned} \tag{9}$$

for $\epsilon > 0$ small enough. Therefore we can write

$$\begin{aligned} A_i' P_p A_i + C_{zi}' C_{zi} - P_i &\leq A_i' Q A_i + C_{zi}' C_{zi} - P_i \\ &\leq -\epsilon I \\ &< 0 \end{aligned} \tag{10}$$

and the proof is concluded. ■

This theorem shows that for generalized Bernoulli jump systems the number of LMIs and matrix variables can be reduced to only one. Contrarily, this result implies that the $\mathcal{H}_2$ norm can be calculated more efficiently as far as the numerical point of view is concerned. The fact that only one matrix variable is needed will have important consequences to be discussed next.

Inequality (8) can be rewritten in a form more adapted to synthesis by adding supplementary slack variables $R_i, i \in \mathbb{K}$, such that

$$R_p - Q < 0 \tag{11}$$

where they have to satisfy

$$\begin{bmatrix} R_i & \bullet & \bullet \\ Q A_i & Q & \bullet \\ C_{zi} & 0 & I \end{bmatrix} > 0, \; i \in \mathbb{K} \tag{12}$$

Hence, simple algebraic manipulations show that the $\mathcal{H}_2$ norm is

$$\|\mathcal{G}\|_2^2 = \inf \sum_{i \in \mathbb{K}} \mu_i \mathrm{Tr}(W_i) \tag{13}$$

subject to (11), (12) and

$$\begin{bmatrix} W_i & \bullet & \bullet \\ Q J_i & Q & \bullet \\ E_{zi} & 0 & I \end{bmatrix} > 0, \; i \in \mathbb{K} \tag{14}$$

In the LMIs (11), (12) and (14), the system matrices are only involved in products with $Q > 0$, that is independent of the Markov mode.

IV. Optimal Cluster Filtering

The assumption of the Markov mode $\theta_k \in \mathbb{K}$ availability may not be practical. In some applications, it may be more adequate to consider the cluster availability. Consider the set $\mathbb{L} = \{1, 2, \cdots, N_c\}$ with $N_c \leq N$ and define the set of Markov chain states $\mathbb{K}$ as the union of N_c disjoint sets, or clusters, that is, $\mathbb{K} \equiv \cup_{\ell \in \mathbb{L}} \mathbb{U}_\ell$ such that $\mathbb{U}_i \cap \mathbb{U}_j = \emptyset, \forall i \neq j \in \mathbb{L}$. Associated to (1) consider the full order linear filter with cluster observation

$$\mathcal{F} : \begin{cases} x_f(k+1) = A_{f\ell} x_f(k) + B_{f\ell} y(k) \\ \quad z_f(k) = C_{f\ell} x_f(k) + D_{f\ell} y(k) \end{cases} \tag{15}$$

where $x_f(k) \in \mathbb{R}^n$, $x_f(0) = 0$ whenever $\theta_k \in \mathbb{U}_\ell$. This implies that the N modes are split into N_c clusters and we assume it is possible to measure to which cluster $\mathbb{U}_\ell$ a mode i belongs, even if the mode i itself is unknown. The mode-dependent

($N_c = N$) and mode-independent ($N_c = 1$) filter design problems are special cases of this definition. The matrices $A_{f\ell}$, $B_{f\ell}$, $C_{f\ell}$ and $D_{f\ell}$ for all $\ell \in \mathbb{L}$ are of compatible dimensions and the goal is to determine them in such a way that some norm of the estimation error is minimized. The estimation error satisfies

$$\mathcal{G}_f : \begin{cases} \tilde{x}(k+1) = \tilde{A}(\theta_k, \ell)\tilde{x}(k) + \tilde{J}(\theta_k, \ell)w(k) \\ \quad e(k) = \tilde{C}(\theta_k, \ell)\tilde{x}(k) + \tilde{E}(\theta_k, \ell)w(k) \end{cases} \tag{16}$$

where the indicated matrices are given by

$$\tilde{A}_{i\ell} := \begin{bmatrix} A_i & 0 \\ B_{f\ell}C_{yi} & A_{f\ell} \end{bmatrix}, \; \tilde{J}_{i\ell} := \begin{bmatrix} J_i \\ B_{f\ell}E_{yi} \end{bmatrix} \tag{17}$$

and

$$\begin{aligned} \tilde{C}_{i\ell} &:= \begin{bmatrix} C_{zi} - D_{f\ell}C_{yi} & -C_{f\ell} \end{bmatrix}, \\ \tilde{E}_{i\ell} &:= E_{zi} - D_{f\ell}E_{yi} \end{aligned} \tag{18}$$

where $\theta_k = i \in \mathbb{U}_\ell \subset \mathbb{K}$ and the state is given by $\tilde{x}(k) = [x(k)' \;\; x_f(k)']'$. Hence, the problem to be solved is written in the form

$$\min_{A_{f\ell}, B_{f\ell}, C_{f\ell}, D_{f\ell}} \|\mathcal{G}_f\|_2^2 \tag{19}$$

Problem (19) is non-convex, but it is possible to state it in terms of LMIs. Actually, in [7] sufficient LMI conditions were demonstrated for the design of a cluster filter in the from (15) for a MJLS with arbitrary transition probabilities. The next theorem is the main contribution of this paper.

Theorem 2: There exists a filter of the form (15) such that $\|\mathcal{G}_f\|_2^2 < \gamma$, under assumption (2) for the transition probabilities, if and only if there exist symmetric matrices W_i,S_i,H_i,Z, X, and matrices G_i, M_ℓ, L_ℓ, F_ℓ, K_ℓ of compatible dimensions satisfying $\sum_{i\in\mathbb{K}} \mu_i \mathrm{Tr}(W_i) < \gamma$ and the LMIs

$$\begin{bmatrix} W_i & \bullet & \bullet & \bullet \\ ZJ_i & Z & \bullet & \bullet \\ XJ_i + F_\ell E_{yi} & Z & X & \bullet \\ E_{zi} - K_\ell E_{yi} & 0 & 0 & I \end{bmatrix} > 0 \tag{20}$$

$$\begin{bmatrix} S_i & \bullet & \bullet & \bullet & \bullet \\ G_i & H_i & \bullet & \bullet & \bullet \\ ZA_i & ZA_i & Z & \bullet & \bullet \\ \Xi_{i\ell} & XA_i + F_\ell C_{yi} & Z & X & \bullet \\ \Gamma_{i\ell} & C_{zi} - K_\ell C_{yi} & 0 & 0 & I \end{bmatrix} > 0 \tag{21}$$

$$\begin{bmatrix} S_p & \bullet \\ G_p & H_p \end{bmatrix} - \begin{bmatrix} Z & \bullet \\ Z & X \end{bmatrix} < 0 \tag{22}$$

where $\Xi_{i\ell} = XA_i + F_\ell C_{yi} + M_\ell$ and $\Gamma_{i\ell} = C_{zi} - K_\ell C_{yi} + L_\ell$, for all $i \in \mathbb{U}_\ell \subset \mathbb{K}$ and $\ell \in \mathbb{L}$. In the affirmative case, an adequate filter is given by

$$\begin{aligned} A_{f\ell} &= (Z - X)^{-1}M_\ell, \; C_{f\ell} = -L_\ell, \\ B_{f\ell} &= (Z - X)^{-1}F_\ell, \; D_{f\ell} = K_\ell \end{aligned} \tag{23}$$

Proof: For the necessity, we consider that the LMIs (11), (12) and (14) are valid for the estimation error $\mathcal{G}_f$ with state-space matrices (17) and (18). We also consider the following partitions where all blocks are $n \times n$ dimensional

$$Q = \begin{bmatrix} X & \bullet \\ U & \hat{X} \end{bmatrix}, \; Q^{-1} = \begin{bmatrix} Y & \bullet \\ V & \hat{Y} \end{bmatrix}, \; T = \begin{bmatrix} I & I \\ VY^{-1} & 0 \end{bmatrix} \tag{24}$$

If we multiply (14) to the left by $\mathrm{diag}\{I, T', I\}$ and to the right by its transpose and adopt the following changes of variables

$$Z = Y^{-1} \tag{25}$$

$$F_\ell = U'B_{f\ell}, \; \ell \in \mathbb{L} \tag{26}$$

$$K_\ell = D_{f\ell}, \quad \ell \in \mathbb{L} \tag{27}$$

we get (20). Multiplying (12) to the left by $\mathrm{diag}\{T', T', I\}$ and to right by its transpose, adopting the same changes of variables used so far and

$$T'R_iT = \begin{bmatrix} S_i & \bullet \\ G_i & H_i \end{bmatrix}, \; i \in \mathbb{K} \tag{28}$$

$$L_\ell = -C_{f\ell}VZ, \quad \ell \in \mathbb{L} \tag{29}$$

$$M_\ell = U'A_{f\ell}VZ, \quad \ell \in \mathbb{L} \tag{30}$$

we get (21). Finally, if we multiply (11) to the left by T' and to the right by T we get (22). For the sufficiency, we consider (20)-(22) are valid. That implies $X > Z > 0$ and therefore $U = Z - X$ is nonsingular. From this, we can calculate $B_{f\ell} = (Z - X)^{-1}F_\ell$ and $D_{f\ell} = K_\ell$. On the other hand, to satisfy (24) this choice imposes $V = Z^{-1}$ and

$$Q = \begin{bmatrix} X & Z - X \\ Z - X & X - Z \end{bmatrix} > 0, \; T = \begin{bmatrix} I & I \\ I & 0 \end{bmatrix} \tag{31}$$

putting in evidence that (20) can be rewritten as

$$\begin{bmatrix} W_i & \bullet & \bullet \\ T'Q\tilde{J}_{i\ell} & T'QT & \bullet \\ \tilde{E}_{i\ell} & 0 & I \end{bmatrix} > 0, \; i \in \mathbb{K} \tag{32}$$

which, multiplied to the right by $\mathrm{diag}\{I, T^{-1}, I\}$ and to the left by its transpose imply (14). Since the particular choice of $U = Z - X$ implies $V = Z^{-1}$, setting $A_{f\ell} = (Z-X)^{-1}M_\ell$ and $C_{f\ell} = -L_\ell$ inequality (21) can be rewritten as

$$\begin{bmatrix} T'R_iT & \bullet & \bullet \\ T'Q\tilde{A}_{i\ell}T & T'QT & \bullet \\ \tilde{C}_{i\ell}T & 0 & I \end{bmatrix} > 0, \ i \in \mathbb{K} \qquad (33)$$

which, multiplied to the right by $\mathrm{diag}\{T^{-1}, T^{-1}, I\}$ and to left by its transpose imply (12). Finally, it is immediate that multiplying (22) to the right by T^{-1} and to the left by its transpose we get (11) and the claim follows. ■

It is important to emphasize the fact that the LMI conditions from Theorem 2 are necessary and sufficient for the design of cluster $\mathcal{H}_2$ filters. This is different from what was accomplished in [7], where the conditions were only sufficient. The difference between the MJLS that were addressed in [7] and this paper is that, in the first case, arbitrary probability transitions were considered. Here, only generalized Bernoulli transition probabilities respecting (2) are considered. It is also important to make clear that this result is only possible due to the fact that the matrix variable $Q > 0$ does not depend on the index $i \in \mathbb{K}$ which allows us to use a constant matrix $T \in \mathbb{R}^{n\times n}$ to linearize the involved conditions. In this sense, the result of Theorem 2 is entirely based on Theorem 1 which follows from the particular structure of the transition probability matrix of the generalized Bernoulli process.

A. Optimal mode-independent filters

The cluster modeling proposed in this section allows the treatment of the mode-dependent filter design, if $N_c = N$. The mode-independent design can also be seen as a particular case, in which all modes belong to a single cluster, or $N_c = 1$. If no information from the mode is available, filter (15) becomes linear and time-invariant (LTI). The $\mathcal{H}_2$ optimal mode-independent filters can be calculated with the LMI conditions from Theorem 2 and

$$M_\ell = M, \ F_\ell = F, \ L_\ell = L, \ K_\ell = K \qquad (34)$$

In [3], the design of the optimal $\mathcal{H}_2$ mode-independent filter was proposed, for MJLS with arbitrary transition probabilities. Since the LMI conditions from Theorem 2 are necessary and sufficient, it is possible to compare that mode-independent filter design with [3], for the particular case where assumption (2) holds. There are some differences between the two approaches. The filter from [3] is strictly proper with order Nn, where n is the order of (1) and N is the number of Markov modes. Filter (15) is proper with order n. In [3], system (1) is assumed to be proper and $C_{yi} = C_y$ for all $i \in \mathbb{K}$, differently from what is assumed here. Finally, [3] considered the initial distribution was stationary, while in Theorem 2 it is possible to consider any initial probability distribution.

By applying the conditions of Theorem 2, it is possible to obtain a proper filter with the same order of the plant (1), under the assumption that the mode $\theta_k \in \mathbb{K}$ follows a generalized Bernoulli process. To the best of our knowledge, this is the first time an optimal $\mathcal{H}_2$ mode-independent filter other than the one presented in [3] is proposed.

Suboptimal mode-independent were proposed by [6] for $\mathcal{H}_2$ norm. It mentions the fact that, under (2), mode-independent filters could be obtained. The novelty in this paper is that we now show LMI conditions that are not only sufficient but also necessary when the jumps follow a generalized Bernoulli distribution. Therefore, the obtained filter has the best possible $\mathcal{H}_2$ performance among all mode-independent filters.

B. Illustrative Example

Our example has been adapted from [6] in order to respect the hypothesis of [3]. It is based on the closed-loop economic system derived from the Samuelson's multiplier accelerator model [1]. Three operation modes of an economic system are considered and the transition probability matrix is

$$\mathbb{P} = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}' \begin{bmatrix} 0.45 & 0.21 & 0.34 \end{bmatrix}. \qquad (35)$$

Finally, the parameters of each of these modes of operation are given in Table I and $E_{yi} = [0 \ 1]$, $C_{zi} = [1 \ 0]$ and $E_{zi} = [0 \ 0]$ for $i \in \{1, 2, 3\}$.

We considered the case where the initial distribution is equivalent to the stationary one. For

i	A_i	J_i	C_{yi}
1	$\begin{bmatrix} 0 & 1 \\ -0.0158 & 0.9652 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0.5 & 1.0 \end{bmatrix}$
2	$\begin{bmatrix} 0 & 1 \\ 0.0597 & 0.8064 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0.5 & 0.5 \end{bmatrix}$
3	$\begin{bmatrix} 0 & 1 \\ 0.0056 & 0.9051 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 1.0 & 0.5 \end{bmatrix}$

TABLE I

DATA FOR EXAMPLE 1

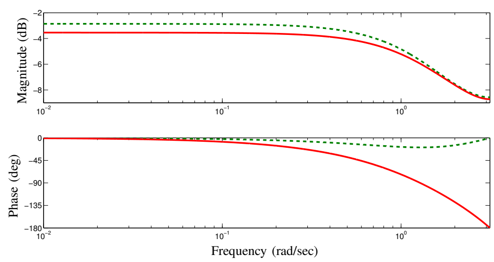

Fig. 1. Bode diagram for filters designed in Example 1

the mode-independent filtering problem, both the 6th order filter from [3] and the 2nd order strictly proper one designed with the results from Theorem 2 imposing $K = 0$ present $\|\mathcal{G}_f\|_2 = 1.1985$. Applying Theorem 2 without the constraint $K = 0$ provides $\|\mathcal{G}_f\|_2 = 0.6827$, a reduction of more than 40% compared to the strictly proper one.

In Figure 1 we show the bode diagram for the three calculated filters. In dashed line, the proper filter designed with Theorem 2. In solid line, with both results coinciding, the strictly proper filter proposed in this paper as well as the one proposed by [3]. It is important to stress that the 6th order filter designed by [3] has 4 zeros and 4 poles close to the origin, that, when canceled, is reduced to the same 2nd order strictly proper filter proposed here.

V. CONCLUSION

This paper is entirely devoted to the solution of the optimal $\mathcal{H}_2$ filter design problem under cluster assumption and generalized Bernoulli jumps. Two important particular cases are the mode dependent and mode independent filters, and optimality is kept for both. The design conditions are expressed in terms of LMIs that follow from new results for $\mathcal{H}_2$ norm calculation of stable Bernoulli jump systems, which present only one convex constraint and matrix variable. The result is compared with the optimal mode-independent filter proposed by [3] and we show that under our assumption for the transition probabilities and their assumptions under the system matrices, we obtain a full order optimal filter instead of an extended order one, both achieving the same performance. Finally, a numerical example illustrates the applicability and the features of the proposed filter design.

Similarly to what has been done to the $\mathcal{H}_2$ norm, the $\mathcal{H}_\infty$ optimal filter can also be designed in the same framework, as stated in the journal version of the present paper.

REFERENCES

[1] W. P. Blair and D. D. Sworder. Feedback control of a class of linear discrete systems with jump parameters and quadratic cost criteria. *International Journal of Control*, 21:833–841, 1975.

[2] O. L. V. Costa, J. B. R. do Val, and J. C. Geromel. A convex programming approach to H_2 control of discrete-time markovian linear systems. *International Journal of Control*, 66:557–579, 1997.

[3] O. L. V. Costa and S. Guerra. Robust linear filtering for discrete-time hybrid Markov linear systems. *International Journal of Control*, 75:712–727, 2002.

[4] O. L. V. Costa and E. F. Tuesta. H_2-Control and the Separation Principle for Discrete-Time Markovian Jump Linear Systems. *Mathematics of Control, Signals and Systems*, 16:320–350, 2004.

[5] O.L.V. Costa, M.D. Fragoso, and R.P. Marques. *Discrete-Time Markov Jump Linear Systems*. Probability and Its Applications. Springer-Verlag, 2005.

[6] A. R. Fioravanti, A. P. C. Gonçalves, and J. C. Geromel. H_2 Filtering of Discrete-time Markov Jump Linear Systems Through Linear Matrix Inequalities. *International Journal of Control*, 81:1221–1231, 2008.

[7] A. P. C. Gonçalves, A. R. Fioravanti, and J. C. Geromel. Markov jump linear systems and filtering through network transmitted measurements. *Signal Processing*, 90:2842–2850, 2010.

[8] J. Hespanha, P. Naghshtabrizi, and Y. Xu. A survey of recent results in networked control systems. *Proceedings of the IEEE*, 95:138–162, 2007.

[9] Y. Ji, H. J. Chizeck, X. Feng, and K. A. Loparo. Stability and control of discrete-time jump linear systems. *Control Theory and Advanced Technology*, 7:247–270, 1991.