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An Active Vibration Control System as a Benchmark on Adaptive Regulation

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***Abstract*— The adaptive regulation is an important issue with a lot of potential for applications in active suspension, active vibration control, disc drives control and active noise control. One of the basic problems from the "control system" point of view is the rejection of multiple unknown and time varying narrow band disturbances without using an additional transducer for getting information upon the disturbances. An adaptive feedback approach has to be considered for this problem. Industry needs a *state of the art* in the field based on a solid experimental verification. The paper presents a benchmark problem for suppression of multiple unknown and/or time-varying vibrations and an associated active vibration control system using an inertial actuator on which the experimental verifications have been done.**
The benchmark has three levels of difficulty and the associated control performance specifications are presented[1]. An extensive comparison of the results obtained by various approaches will be presented[2].

I. Introduction

One of the basic problems in control is the attenuation (rejection) of unknown disturbances without measuring them. The common framework is the assumption that the disturbance is the result of a white noise or a Dirac impulse passed through the *model of the disturbance*. The knowledge of this model allows to design an appropriate controller. When considering the model of a disturbance, one has to address two issues: 1) its structure (complexity, order of the parametric model) and 2) the values of the parameters of the model. In general, one can assess from data the structure for such *model of disturbance* (using spectral analysis or order estimation techniques) and assume that the structure does not change. However the parameters of the model are unknown and may be time varying. This will require to use an adaptive feedback approach.

The classical adaptive control paradigm deals essentially with the construction of a control law when the parameters of the plant dynamic model are unknown and time varying ([14]). However, in the present context, the plant dynamic model is almost invariant and it can be identified and the objective is the rejection of disturbances characterized by unknown and time varying disturbance models. It seems reasonable to call this paradigm *adaptive regulation*. In *adaptive regulation* the objective is to asymptotically suppress (attenuate) the effect of unknown and time-varying disturbances. Therefore adaptive regulation focuses on adaptation of the controller parameters with respect to variations in the disturbance model parameters. The plant model is assumed to be known. It is also assumed that the possible small variations or uncertainties of the plant model can be handled by a robust control design. The problem of adaptive regulation as defined above has been previously addressed in a number of papers ([5], [2], [17], [16], [13], [10], [4], [6], [11]) among others.
The objective of the proposed benchmark is to evaluate on an experimental basis the available techniques for adaptive regulation in the presence of unknown/time varying multiple narrow band disturbances. Active vibration control constitutes an excellent example of a field where this situation occurs. Solutions for this problem in active vibration control can be extrapolated to the control of disc drives and active noise control (see [11]). The benchmark specifically will focus in testing: 1) performances, 2) robustness and 3) complexity.
The test bed is an active vibration control system using an inertial actuator located at GIPSA-Lab, Grenoble (France).

II. An active vibration control system using an inertial actuator

A. System structure

The structure of the system used for the benchmark is presented in figure 1. A general view of the whole system including the testing equipment is shown figure 2. It consists of a passive damper, an inertial actuator, a load, a transducer for the residual force, a controller, a power amplifier and a shaker. The inertial actuator will create vibrational forces which can counteract the effect of vibrational disturbances. The equivalent control scheme is shown in figure 3. The system input, $u(t)$ is the position of the mobile part (magnet) of the inertial actuator (see figures 1 and 3), the output $y(t)$ is the residual force measured by a force sensor. The transfer function ($q^{-d_1}\frac{C}{D}$), between the disturbance force, $u_p(t)$, and the residual force $y(t)$ is called *primary path*. In our case (for testing purposes), the primary force is generated by a shaker driven by a signal delivered by the computer. The plant transfer function ($q^{-d}\frac{B}{A}$) between the input of the inertial actuator, $u(t)$, and the residual force is called *secondary path*. The control objective is to reject the effect of unknown narrow band disturbances on the output of the system (residual force), i.e. to attenuate the vibrations transmitted from the machine to the chassis. The physical parameters of the system are not available. The system has to be considered as a *black box* and the corresponding models for

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[1]The GIPSA-LAB team has done the experiments for all the contributors.
[2]Results for some of the approaches are included in the proceedings of ECC 13[3], [8], [9], [7], [1], [18].

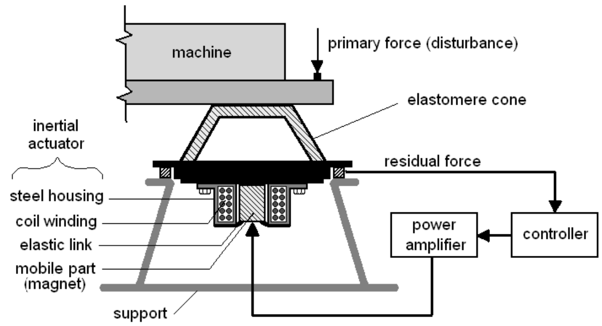

Fig. 1. Active vibration control using an inertial actuator (scheme).

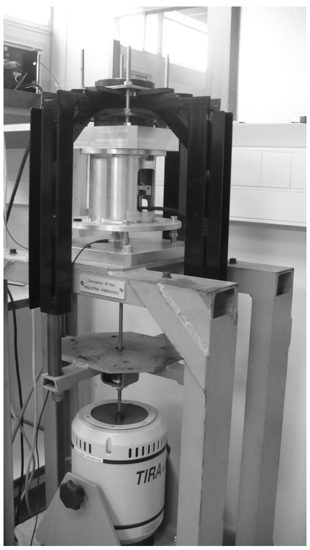

Fig. 2. Active vibration control system (photo).

control design should be identified. The sampling frequency is $F_s = 800$ Hz. Data used for system identification are available on the benchmark website (`http://www.gipsa-lab.grenoble-inp.fr/~ioandore.landau/benchmark_adaptive_regulation/index.html`).

B. Simulator

A ***black box*** discrete time simulator of the active suspension built on MATLAB©Simulink (2007 version) has been provided (can be downloaded from the benchmark website). The simulator has been used by the participants to the benchmark to set the appropriate control scheme and test the performance.

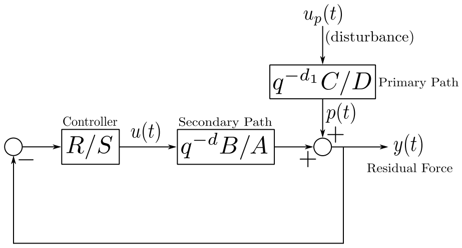

Fig. 3. Block diagram of active vibration control systems.

C. Real time implementation

The real time implementation uses the MATLAB xPC Target environment (2007). The procedure compiles the algorithms directly from the Simulink scheme provided by the participants.

III. PLANT/DISTURBANCE REPRESENTATION AND CONTROLLER STRUCTURE

The structure of the linear time invariant discrete time model of the plant - the secondary path - used for controller design is:

$$G(z^{-1}) = \frac{z^{-d}B(z^{-1})}{A(z^{-1})} = \frac{z^{-d-1}B^*(z^{-1})}{A(z^{-1})} \ , \qquad (1)$$

with: d = time delay in number of sampling periods

$$A = 1 + a_1 z^{-1} + \cdots + a_{n_A} z^{-n_A} \ ;$$
$$B = b_1 z^{-1} + \cdots + b_{n_B} z^{-n_B} = z^{-1} B^* \ ;$$
$$B^* = b_1 + \cdots + b_{n_B} z^{-n_B+1} \ ,$$

where $A(z^{-1})$, $B(z^{-1})$, $B^*(z^{-1})$ are polynomials in the complex variable z^{-1} and n_A, n_B and $n_B - 1$ represent their orders[3]. The model of the plant may be obtained by system identification ([15], [12]).

Since the benchmark is focused on regulation, the controller to be designed is a *RS*-type polynomial controller (or an equivalently state space controller + observer) ([14], [15]) - see also figure 3). The output of the plant $y(t)$ and the input $u(t)$ may be written as:

$$y(t) = \frac{q^{-d}B(q^{-1})}{A(q^{-1})} \cdot u(t) + p(t) \ ; \qquad (2)$$

$$S(q^{-1}) \cdot u(t) = -R(q^{-1}) \cdot y(t) \ , \qquad (3)$$

where q^{-1} is the delay (shift) operator and $p(t)$ is the resulting additive disturbance on the output of the system. $R(z^{-1})$ and $S(z^{-1})$ are polynomials in z^{-1} having the orders n_R and n_S, respectively, with the following expressions:

$$R(z^{-1}) = r_0 + r_1 z^{-1} + \ldots + r_{n_R} z^{-n_R} = R'(z^{-1}) \cdot H_R(z^{-1}) \ ; \qquad (4)$$

$$S(z^{-1}) = 1 + s_1 z^{-1} + \ldots + s_{n_S} z^{-n_S} = S'(z^{-1}) \cdot H_S(z^{-1}) \ , \qquad (5)$$

where H_R and H_S are pre-specified parts of the controller.

Suppose that $p(t)$ is a deterministic disturbance, so it can be written as

$$p(t) = \frac{N_p(q^{-1})}{D_p(q^{-1})} \cdot \delta(t) \ , \qquad (6)$$

where $\delta(t)$ is a Dirac impulse and $N_p(z^{-1})$, $D_p(z^{-1})$ are coprime polynomials in z^{-1}, of degrees n_{N_p} and n_{D_p}, respectively. In the case of stationary disturbances the roots of $D_p(z^{-1})$ are on the unit circle (which will be the case for the disturbances considered in the benchmark). The energy of the disturbance is essentially represented by D_p.

Figure 4 gives the frequency characteristics of the identified parametric models for the primary and secondary path (the excitation signal was a PRBS)[4]. The system itself in the absence of the disturbances will feature a number of low damped vibration modes as well as low damped complex

[3]The complex variable z^{-1} will be used for characterizing the system's behavior in the frequency domain and the delay operator q^{-1} will be used for describing the system's behavior in the time domain.

[4]These models have been used in the simulator

zeros (anti-resonance). This will make the design of the controller difficult for rejecting disturbances close to the location of low damped complex zeros. The parametric models of both the secondary and primary path are of significant high order ($n_A = 23, n_B = 26$ and $n_C = 17, n_D = 16$ respectively).

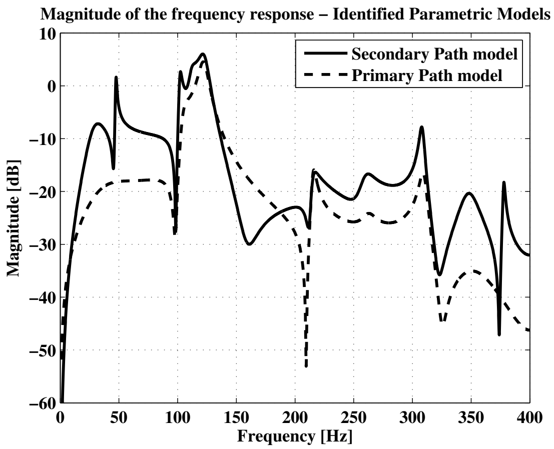

Fig. 4. Frequency response (magnitude) for the primary and the secondary paths models.

IV. Control specifications

The narrow band disturbances are located in the range 50 to 95 Hz. There are three levels of difficulty corresponding to one, two or three unknown time varying narrow band disturbances. In order to test the required performances, 3 protocols have been defined:

Protocol 1. Tuning capabilities: Evaluation in steady state operation after application of the disturbance once the adaptation settles. *This is the most important aspect of the benchmark.*

Protocol 2. Transient performance in the presence of step application of the disturbance and step changes in the frequency of the disturbances.

Protocol 3. Chirp changes in frequency.

The loop is closed before the disturbances are applied for all the above tests.

The complexity of the procedures proposed have been evaluated by measuring the average *Task Execution Time* on the real-time system.

V. Structure of the controllers

All the controllers configurations except Karimi *et al.* use different types of Youla - Kucera parametrization. Callafon *et al* and Wu *et al* provided a single controller configuration valid for all the three levels. The other contributors provided a specific controller configuration for each level.

VI. Measurements for Performance Analysis

A. Measurements for Simple Step test

The benchmark protocol for the *Simple Step* test defines the time period for the disturbance application. The disturbance is applied at $t = 15$ seconds, while the entire experiment duration is 30 seconds. In this context, the *transient* behavior will be considered in the first 3 seconds after the disturbance is applied. For measuring the *steady state* behavior the last 3 seconds of the test (before the disturbance is removed), will be used since is expected that the algorithm has converged at this time. The measurements considered in time domain are:

- The *square of the truncated two norm* of the residual force defined by
$$N^2T = \sum_{i=1}^{m} y(i)^2,$$
where $y(i)$ is a sample of the discrete-time signal to evaluate.
- The *maximum value* measured in millivolts and defined by
$$MV = \max_n |y(i)|.$$

The measurements in frequency domain (steady state behaviour) are:

- *Global Attenuation* (*GA*) measured in dB and defined by
$$GA = 20\log_{10}\frac{N^2Y_{ol}}{N^2Y_{cl}},$$
where Y_{ol} and Y_{cl} correspond to the last 3 seconds of the measured output in open and closed loop, respectively.
- *Disturbance Attenuation* (*DA*) measured in dB and defined as follows:
$$DA = \min\left(\mathrm{PSD}_{cl} - \mathrm{PSD}_{ol}\right).$$
where *PSD* stands for the Power Spectral Density of the residual force in open loop (*ol*) and closed loop (*cl*)
- *Maximum Amplification* (*MA*) measured in dB, is defined as
$$MA = \max\left(\mathrm{PSD}_{cl} - \mathrm{PSD}_{ol}\right).$$

For all the frequency domain measurements, only the last 3 seconds of the test are considered.

TABLE I
Control specifications

Control specifications	Level 1	Level 2	Level 3
Transient duration	$\leq$ 2 sec	$\leq$ 2 sec	$\leq$ 2 sec
Global attenuation	$\geq$ 30 dB*	$\geq$ 30 dB	$\geq$ 30 dB
Minimum disturbance attenuation	$\geq$ 40 dB	$\geq$ 40 dB	$\geq$ 40 dB
Maximum amplification	$\leq$ 6 dB	$\leq$ 7 dB	$\leq$ 9 dB
Chirp speed	10 Hz/sec	6.25 Hz/sec	3 Hz/sec
Maximum value during chirp	$\leq$ 0.1 V	$\leq$ 0.1 V	$\leq$ 0.1 V

* For this level, the specification of 30 dB is for the range between 50 and 85 Hz, for 90 Hz is 28 dB and for 95 Hz is 24 dB.

B. Measurements for Step Frequency Changes

For the *Step Frequencies Changes* only time domain measurements were considered. Based on the protocol for this test, a frequency step change occurs every 3 seconds. During this time period the following measurements are considered

- Square of the truncated two norm of the transient N^2T.
- Maximum value of the transient MV.

C. Chirp Frequency Change

For the *Chirp* Test only time domain measurements were considered. The measurements are:

- Mean Square of the residual force defined as
$$MSE = \frac{1}{m}\sum_{i=1}^{m} y(i)^2 = \frac{1}{m}N^2T,$$
where m correspond to the number of output samples evaluated.
- Maximum value MV measured in millivolts.

VII. Evaluation Criteria

The results of each group will be evaluated with respect to the benchmark specifications. The simulation results will give us information upon the potential of the design methods under the assumption: *design model = true plant model*. The real-time results will tell us in addition what is the robustness of the design with respect to plant model uncertainties and real noise.

A. Steady State Performance (Tuning capabilities)

For the steady state performance, which is evaluated only in the *simple step test*, the variable k, with $k = 1,\ldots,3$, will indicate the *level* of the benchmark. In several criteria a mean of certain variables will be considered. The number of measurements, M, is used to compute the mean. This number depend upon the level of the benchmark as follows:

$$M = 10, \text{ if } k = 1 \ ; M = 6, \text{ if } k = 2 \ ; M = 4, \text{ if } k = 3.$$

The performances can be evaluated with respect to the benchmark specifications. The benchmark specifications will be in the form: XXB, where XX will denote the evaluated variable and B will indicate the benchmark specification. ΔXX will represent the error with respect to the benchmark specification.

1) Global Attenuation - GA: The benchmark specification corresponds to $GAB_k = 30$ dB, for all the levels and frequencies, except for 90 Hz and 95 Hz at $k = 1$, for which GAB_1 is 28 dB and 24 dB respectively. One defines:

$$\Delta GA_i = GAB_k - GA_i \quad \text{if} \quad GA_i < GAB_k$$
$$\Delta GA_i = 0 \quad \text{if} \quad GA_i \geq GAB_k$$

with $i = 1,\ldots,M$.
Global Attenuation Criterion

$$J_{\Delta GA_k} = \frac{1}{M}\sum_{j=1}^{M}\Delta GA_i \qquad (7)$$

2) Disturbance Attenuation - DA: The benchmark specification corresponds to $DAB = 40$ dB, for all the levels and frequencies. One defines:

$$\Delta DA_{ij} = DAB - DA_{ij} \quad \text{if} \quad DA_{ij} < DAB$$
$$\Delta DA_{ij} = 0 \quad \text{if} \quad DA_{ij} \geq DAB$$

with $i = 1,\ldots,M$ and $j = 1,\ldots,j_{max}$, where $j_{max} = k$.
Disturbance Attenuation Criterion

$$J_{\Delta DA_k} = \frac{1}{M}\sum_{i=1}^{M}\sum_{j=1}^{j_{max}}\Delta DA_{ij} \qquad (8)$$

3) Maximum Amplification - MA: The benchmark specifications depend on the level, and are defined as

$$MAB_k = 6, \text{ if } k = 1 \ ; MAB_k = 7, \text{ if } k = 2 \ ; MAB_k = 9, \text{ if } k = 3.$$

One defines:

$$\Delta MA_i = MA_i - MAB_k, \quad \text{if} \quad MA_i > MAB_k$$
$$\Delta MA_i = 0, \quad \text{if} \quad MA_i \leq MAB_k$$

with $i = 1,\ldots,M$.
Maximum Amplification Criterion

$$J_{\Delta MA_k} = \frac{1}{M}\sum_{i=1}^{M}\Delta MA_i \qquad (9)$$

4) ***Global criterion of steady state performance for one level****:*

$$J_{SS_k} = \frac{1}{3}[J_{\Delta GA_k} + J_{\Delta DA_k} + J_{\Delta MA_k}] \qquad (10)$$

5) ***Benchmark Satisfaction Index for Steady State Performance****:* The *Benchmark Satisfaction Index* is a performance index computed from the *average* criteria $J_{\Delta GA_k}$, $J_{\Delta DA_k}$ and $J_{\Delta MA_k}$. The *Benchmark Satisfaction Index* is 100%, if these quantities are "0" (full satisfaction of the benchmark specifications) and it is 0% if the corresponding quantities are half of the specifications for GA, and DA or twice the specifications for MA. The corresponding reference quantities are summarized below:

$$\Delta GA_{index} = 15; \ \Delta DA_{index} = 20; \Delta MA_{index} = 6, \text{ if } k = 1;$$
$$\Delta MA_{index} = 7, \text{ if } k = 2 \ ; \Delta MA_{index} = 9, \text{ if } k = 3.$$

The computation formulas are

$$GA_{index,k} = \left(\frac{\Delta GA_{index} - J_{\Delta GA_k}}{\Delta GA_{index}}\right)100\%$$
$$DA_{index,k} = \left(\frac{\Delta DA_{index} - J_{\Delta DA_k}}{\Delta DA_{index}}\right)100\%$$
$$MA_{index,k} = \left(\frac{\Delta MA_{index,k} - J_{\Delta MA_k}}{\Delta MA_{index,k}}\right)100\%.$$

Then the *Benchmark Satisfaction Index* (BSI), is defined as

$$BSI_k = \frac{GA_{index,k} + DA_{index,k} + MA_{index,k}}{3} \qquad (11)$$

The results for J_{SS_k} and BSI_k obtained both in simulation and real-time for each participant and all the levels are summarized in Table II, and represented graphically in figure 5 (for BSI_k).

TABLE II
BENCHMARK SATISFACTION INDEX FOR ALL THE PARTICIPANTS

Participant	LEVEL 1				LEVEL 2				LEVEL 3			
	Simulation		Real Time		Simulation		Real Time		Simulation		Real Time	
	J_{SS_1}	BSI₁	J_{SS_1}	BSI₁	J_{SS_2}	BSI₂	J_{SS_2}	BSI₂	J_{SS_3}	BSI₃	J_{SS_3}	BSI₃
Aranovskiy *et. al.*	0.87	86.94%	1.20	80.22%	1.77	76.33%	2.04	73.58%	0.84	90.65%	1.41	84.89%
Callafon *et. al.*	2.12	89.21%	6.74	49.37%	5.02	72.89%	11.01	29.08%	17.14	51.74%	31.47	8.40%
Karimi *et. al.*	1.33	91.92%	2.17	72.89%	3.42	76.13%	7.43	44.33%	-	-	-	-
Wu *et. al.*	0.11	98.31%	1.31	**83.83%**	0.13	98.48%	1.35	84.69%	0.18	98.01%	1.34	91.00%
Xu *et. al.*	**0.00**	**100.00%**	**1.00**	**86.63%**	**0.00**	**100.00%**	1.37	**86.65%**	**0.04**	**99.78%**	1.45	92.52%
Airimitoaie *et. al.*	0.08	98.69%	1.23	81.11%	0.11	98.38%	**0.94**	**88.51%**	0.11	99.44%	1.58	90.64%
Castellanos *et. al.*	0.50	93.30%	1.35	80.87%	0.29	97.29%	1.20	**89.56%**	0.17	99.13%	**0.43**	**97.56%**

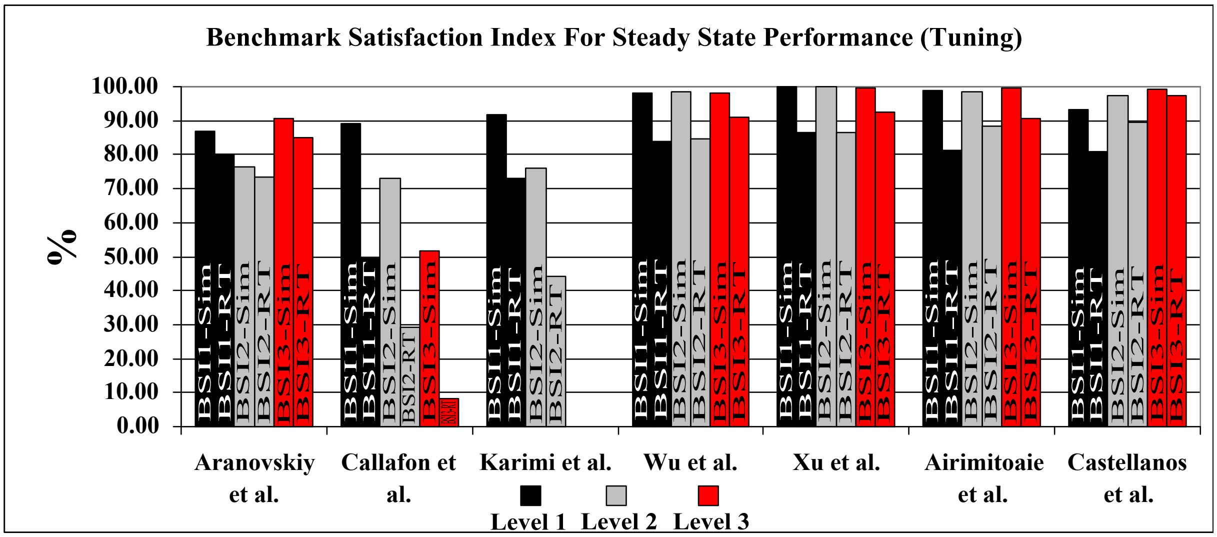

Fig. 5. Benchmark Satisfaction Index for Steady State Performance (BSI_{ss}) for all levels and all participants, both in simulation and real-time.

B. Simulation Results

Consider the *simulation results* in terms of the BSI. Clearly the *benchmark specifications* are achievable since Xu *et. al.* have achieved 100% for Level 1 and 2 and for Level 3 Xu et al. and Airimitoaie *et. al.* have achieved respectively 99.78% and 99.44%.

C. Real Time Results

The results which are provided for the BSI in Table II have to be considered with an associate uncertainty of about $+/-$ 4% (the physical system is not a "deterministic system"). The consequence is that we can not classify results within this uncertainty range. From Table II it result that for Level 1 the best results have been obtained by Xu *et. al.* and Wu *et al.*. For Level 2 the best results have been obtained by Castellanos *et al.*, Airimitoaie et al. and Xu *et al.*. For Level 3 the best results have been obtained by Castellanos *et al.*[5].

D. Transient Performance

The basic specification for transient performance is the requirement that the transient duration when a disturbance is applied, be smaller than 2 sec. Details of the measurement procedure can be found on the website. Similar to the steady state performance a BSI index for transient duration has been established (a transient duration of 4 sec corresponds to 0%). Table III gives the results obtained for the various approaches. Most of the approaches have met the specifications or are very close.

The transient performances have been further investigated in order to compare the various approaches. Simple step test, step changes in frequencies and chirp tests have been considered. A compounded index J_{TRAV_k} which integrate all these measurement has been defined for each level (details can be found on the website). Table IV gives the values of J_{TRAV_k} for all levels and participants, both in simulation and real-time. For this criterion lower values means a better transient behaviour.

[5]All these mentioned results differ by less than 4% with respect to the highest value obtained

VIII. EVALUATION OF THE COMPLEXITY

For complexity evaluation, the measure of the *average Task Execution Time* (TET) in the xPC Target environment will be used ($ATET$) . It is however interesting to asses the $ATET$ specifically associated to the controller by subtracting from the measured $ATET$ in closed loop operation, the average TET in open loop operation (this quantity is called ΔTET). An average value of ΔTET for each level have been defined by considering the various types of tests (simple step, step frequency changes, chirp). Table V summarizes the results obtained by each participant for all the levels. All the

TABLE III

BENCHMARK SATISFACTION INDEX FOR TRANSIENT PERFORMANCE(FOR SIMPLE STEP TEST)

Participant \ Index	BSI_{Trans_1}		BSI_{Trans_2}		BSI_{Trans_3}	
	Simulation	Real Time	simulation	Real Time	Simulation	Real Time
Aranovsikiy *et al.*	100%	100%	100%	100%	100%	100%
Callafon *et al.*	100%	100%	100%	100%	100%	81.48%
Karimi *et al.*	100%	85.74%	100%	91.79%	-	-
Wu *et al.*	100%	99.86%	94.85%	100%	100%	92.40%
Xu *et al.*	100%	100%	100%	100%	100%	100%
Airimitoaie *et al.*	100%	99.17%	83.33%	100%	100%	100%
Castellanos *et al.*	100%	96.45%	100%	95.74%	100%	100%

TABLE IV

AVERAGE GLOBAL CRITERION FOR TRANSIENT PERFORMANCE FOR ALL THE PARTICIPANTS

Participant	**JTRAV1**		**JTRAV2**		**JTRAV3**	
	Simulation	**Real Time**	**Simulation**	**Real Time**	**Simulation**	**Real Time**
Aranovskiy *et al.*	0.76	0.89	0.57	0.72	0.51	0.61
Callafon *et al.*	0.44	0.54	**0.26**	**0.40**	**0.22**	0.52
Karimi *et al.*	**0.35**	**0.40**	0.34	0.49	-	-
Wu *et al.*	0.50	0.56	0.36	0.46	0.34	**0.37**
Xu *et al.*	0.39	0.55	0.76	0.81	0.63	0.74
Airimitoaie *et al.*	0.93	0.85	0.60	0.71	0.42	0.49
Castellanos *et al.*	0.55	0.61	0.48	0.60	0.90	0.98

values are in microseconds. Higher values indicate higher complexity. One can conclude that the lowest complexity structures for Level 1 are provided by Karimi *et al.*, Xu *et al.*, Castellanos *et al.* and Aranovskiy *et al.*, for Level 2 by Karimi *et al.*,Castellanos *et al.* and Aranovskiy *et al.* and for Level 3 by Aranovskiy *et. al* and Castellanos *et al.*.

TABLE V

TASK EXECUTION TIME FOR ALL LEVELS AND PARTICIPANTS

Participant	**ΔTET**		
	L1	**L2**	**L3**
Aranovskiy *et al.*	3.71	4.18	**4.92**
Callafon *et al.*	210.68	209.90	212.62
Karimi *et al.*	**2.37**	4.08	-
Wu *et al.*	14.73	14.65	14.74
Xu *et al.*	2.96	9.11	14.27
Airimitoaie *et al.*	254.24	203.83	241.22
Castellanos *et al.*	3.26	**3.90**	5.60

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