2013 European Control Conference (ECC)
July 17-19, 2013, Zürich, Switzerland.

Adaptive Mobile Robots Formation Control using Neural Networks*

Cesáreo Raimúndez[1] and Enrique Paz[2]

***Abstract*— In this paper we present the tracking problem of controlling a particular formation among mobile robots, using feedback linearization techniques. Reference tracking will be made using look ahead control. Look ahead control will be obtained by feedback linearization. To cancel the modeling errors or/and external perturbations, the closed loop will incorporate an adaptive element performed by a one neural network. The adaptive controller, implemented through a hidden layer feed-forward neural network, has its weights realtime updated to cope with external perturbations as well as modeling errors. The control procedures required for tracking control, are inspired in the Lyapunov stability theory.**

I. INTRODUCTION

In this paper we present the tracking problem of controlling a particular formation among mobile robots, using feedback linearization techniques. Reference tracking will be made using look ahead control. Look ahead control will be obtained by linearization feedback. To cancel the modeling errors or external perturbations, the closed loop will incorporate an adaptive element performed by a neural network. To move in formation is to follow a leader, keeping a relative positioning geometry between individuals. The required information is local to each follower and can be obtained through rangefinder sensors. The platoon configuration can be changed with information adquired by the leader. It is assumed that there is a single lead robot for the whole platoon. The leader path is established externally. The platoon depends cascade-wise on the leader so that the references to follow are passed from individual to following individual. The cart dynamics as a whole will be considered instead of the simple kinematic model as in other works [3], [4]. The adaptive controller, implemented through a hidden layer feed-forward neural network, has its weights realtime updated to cope with external perturbations as well as modeling errors. The control procedures required for tracking control, are inspired in the Lyapunov stability theory. In Section II we develop the robot model, in Section III is formulated the formation problem, in Section IV we present the main ideas about input-output linearization, in Section V we show how to handle tracking, in Section VI we present a simulation concerning a case study and finally with Section VII we conclude this paper.

*This work was supported by the Spanish Ministry of Science and Education, grant DPI-2007-66455-C02-02.

[1]C. Raimúndez is with Escola de Enxeñeria Industrial, Depto. Enx. Sistemas e Automática, University of Vigo, R / Maxwell,9 - 36310, Spain cesareo@uvigo.es

[2] E. Paz is with Escola de Enxeñeria Industrial, Depto. Enx. Sistemas e Automática, University of Vigo, R / Maxwell,9 - 36310, Spain epaz@uvigo.es

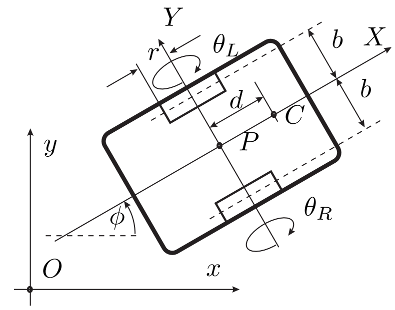


Fig. 1. Two wheeled mobile robot

II. NONHOLONOMIC ROBOT MODELING

Consider a two wheeled robot moving in the plane as shown in figure 2. The robot is characterized by its pose (x, y, ϕ) which is measured according to a local referential frame with origin at the moving basis geometric center P.

The wheel rotation angles are measured regarding a normal to the movement plane, having values (θ_L, θ_R) respectively. Each wheel is independently actuated and spins without slipping.

Let m be the total robot mass, J its moment of total inertia about a vertical axis through P and J_w the inertia moment associated to each wheel regarding its rotation axis. Then $J = J_c + m_c d^2 + 2m_w b^2$ where J_c is the inertial moment without the wheels, regarding the mass center C. The cart mass without wheels is m_c and m_w is the wheel mass. Also $m = m_c + 2m_w$. The C point location and its velocity are obtained doing.

$$P_c = P + d(\cos\phi, \sin\phi),\ V_c = V + \dot{\phi} d(-\sin\phi, \cos\phi) \quad (1)$$

Setting to facilitate the notation:

$$m = m_c + 2m_w, \quad J = J_c + m_c d^2 + 2m_w b^2 \quad (2)$$

the Lagrangian is:

$$L = \frac{1}{2} m \langle V_c, V_c \rangle + \frac{1}{2} J \dot{\phi}^2 + \frac{1}{2} J_w (\dot{\theta}_L^2 + \dot{\theta}_R^2) \quad (3)$$

where $< \cdot, \cdot >$ indicates dot product. There are three constraints. The first one is that the mobile robot can not move in lateral direction, i.e.,

$$\dot{y}\cos\phi - \dot{x}\sin\phi = 0 \quad (4)$$

The other two constraints are that the two driving wheels roll and do not slip:

$$\begin{aligned} \dot{x}\cos\phi + \dot{y}\sin\phi + b\dot{\phi} &= r\dot{\theta}_L \\ \dot{x}\cos\phi + \dot{y}\sin\phi - b\dot{\phi} &= r\dot{\theta}_R \end{aligned} \quad (5)$$

Considering the constraints (4) and (5) we can characterize two constraints as nonholonomic and one as holonomic. To obtain the holonomic constraint, we subtract the first equation of (5) from the last of (5) obtaining $\dot{\phi} = r/(2b)(\dot{\theta}_L - \dot{\theta}_R)$. Integrating the above equation and properly choosing the initial condition of ϕ, θ_L, θ_R we have $\phi = c(\theta_L - \theta_R)$ with $c = r/(2b)$ which is clearly a holonomic constraint equation. The two nonholonomic constraints are

$$\begin{array}{rcl} \dot{y}\cos\phi - \dot{x}\sin\phi & = & 0 \\ \dot{x}\cos\phi + \dot{y}\sin\phi & = & cb(\dot{\theta}_L + \dot{\theta}_R) \end{array} \quad (6)$$

Considering the configuration variables $q = (x, y, \theta_L, \theta_R)^\top$, the restrictions (6) can be presented as:

$$A(q)\dot{q} = \begin{pmatrix} -\sin\phi & \cos\phi & 0 & 0 \\ \cos\phi & \sin\phi & -cb & -cb \end{pmatrix} \dot{q} = 0 \quad (7)$$

The movement equations can be easily obtained, solving the Lagrange equations of motion for the nonholonomic mobile robot, with the aid of a symbolic processor [7], [8].

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} + A(q)^\top \lambda = E(q)\tau \quad (8)$$

here $\lambda = (\lambda_1, \lambda_2)^\top$ are the so called Lagrange multipliers regarding the kinematic restriction described in (7), $\tau = (\tau_R, \tau_L)^\top$ are the drive torques, and $E(q) = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}^\top$. So the movement equations are:

$$M(q)\ddot{q} + V(q, \dot{q}) + A(q)^\top \lambda = E(q)\tau \quad (9)$$

with

$$M(q) = \begin{pmatrix} m & 0 & -mcd\sin\phi & mcd\sin\phi \\ 0 & m & mcd\cos\phi & -mcd\cos\phi \\ -mcd\sin\phi & mcd\cos\phi & J_1 & J_2 \\ mcd\sin\phi & -mcd\cos\phi & J_2 & J_1 \end{pmatrix} \quad (10)$$

with $J_1 = J_w + c^2(J + md^2)$ and $J_2 = -c^2(J + md^2)$. Also with

$$V(q, \dot{q}) = \begin{pmatrix} -c^2 dm\dot{\phi}^2 \cos\phi \\ -c^2 dm\dot{\phi}^2 \sin\phi \\ 0 \\ 0 \end{pmatrix} \quad (11)$$

Solving equation (9) regarding $\ddot{q}$ we obtain the explicit movement equations. Defining now

$$N_A(q) = \begin{pmatrix} cb\cos\phi & cb\cos\phi \\ cb\sin\phi & cb\sin\phi \\ 1 & 0 \\ 0 & 1 \end{pmatrix} \quad (12)$$

such that $A(q)N_A(q) = 0$ and pre-multiplying equation 9 by $N_A(q)^\top$ we obtain.

$$N_A(q)^\top M(q)\ddot{q} + N_A(q)^\top V(q, \dot{q}) = N_A(q)^\top E(q)\tau \quad (13)$$

Now [1] because $A(q)N_A(q) = 0$, exists $\eta = (\eta_1, \eta_2)^\top$ such that $\dot{q} = N_A(q)\eta$. By the nature of $N_A(q)$ can easily be verified that $\eta = (\dot{\theta}_L, \dot{\theta}_R)^\top$. Then calculating the derivative of $\dot{q} = N_A(q)\eta$ regarding time we obtain

$$\ddot{q} = \dot{N}_A(q)\eta + N_A(q)\dot{\eta} \quad (14)$$

Considering now an *ad hoc* partition such that $q = (P, \Theta)$ with $P = (x, y)$ and $\Theta = (\theta_L, \theta_R)$ we can write (14) as

$$\ddot{q} = \dot{N}_A(q)\dot{\Theta} + N_A(q)\ddot{\Theta} \quad (15)$$

After substituting (15) in (13) we get

$$\ddot{\Theta} = f_\theta(q, \dot{q}) + g_\theta(q)\tau \quad (16)$$

where

$$\begin{array}{rcl} f_\theta(q, \dot{q}) & = & -\left(N_A^\top M N_A\right)^{-1}\left(N_A^\top M \dot{N}_A \dot{\Theta} + N_A^\top V\right) \\ g_\theta(q) & = & \left(N_A^\top M N_A\right)^{-1} N_A(q)^\top E(q) \end{array} \quad (17)$$

reminding that $q = (P, \Theta)$ we obtain also

$$\ddot{P} = f_p(q, \dot{q}) + g_p(q)\tau \quad (18)$$

The set of equations (16) and (18) represent the so called explicit dynamic movement equations. Defining now $Y = P_c = P + d(\cos\phi, \sin\phi) = h(q)$ as the output function, the system equations are:

$$\begin{array}{rcl} \ddot{P} & = & f_p(q, \dot{q}) + g_p(q)\tau \\ \ddot{\Theta} & = & f_\theta(q, \dot{q}) + g_\theta(q)\tau \\ Y & = & h(q) \end{array} \quad (19)$$

To verify if the system (19) is input-output linearizable with Y as output equation, we compute the derivatives of Y until the appearance of the inputs τ. Adopting the derivation representation using the Lie formalism [2] we obtain

$$\ddot{Y} = F(q, \dot{q}) + G(q)\tau \quad (20)$$

with

$$\begin{array}{rcl} F(q, \dot{q}) & = & (L_{f_p}^2 + L_{f_\theta}^2)h(q) \\ G(q) & = & (L_{g_p}L_{f_p} + L_{g_\theta}L_{f_\theta})h(q) \end{array} \quad (21)$$

The input-output linearization process, involves details that must be considered with some care. Reference [1] covers satisfactorily the required explanations regarding coordinate transformations and zero-dynamics stability.

III. Formation

Platoon formations in this paper, follow the specifications given in [3], more precisely, with focus in the $l - \psi$ subordinate link. Defining a subordinate $l - \psi$ link in which $\{l_{12}, \psi_{12}\}$ is a pair (distance, relative atitude) to be maintained and according to figure (2)

$$\begin{array}{rcl} P_{c_1} & = & P_1 + (\cos\phi_1, \sin\phi_1)^\top d \\ P_{c_2} & = & P_2 + (\cos\phi_2, \sin\phi_2)^\top d \end{array} \quad (22)$$

The link configuration dependence is easily obtained:

$$\begin{array}{rcl} l_{12} & = & |P_{c_1} - P_{c_2}| \\ \psi_{12} & = & \arctan(\Delta x, \Delta y) - \phi_2 \\ \Delta x & = & x_1 - x_2 + (\cos\phi_1 - \cos\phi_2)d \\ \Delta y & = & y_1 - y_2 + (\sin\phi_1 - \sin\phi_2)d \end{array} \quad (23)$$

Let l^r, ψ^r be a reference pair: the tracking error regarding platoon formation can be defined as $e_l = l^r - l$ and $e_\psi =$

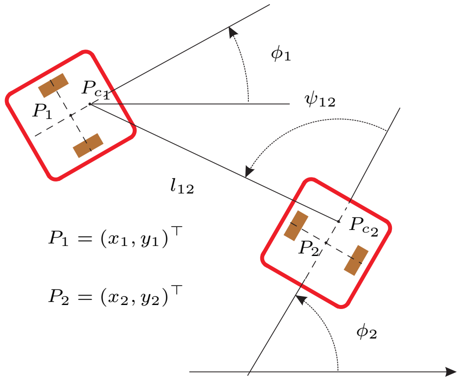


Fig. 2. Typical $l - \psi$ link with subordinate cart

$\psi^r - \psi$. With the purpose of ensuring stability for the error dynamics, we will choose

$$\begin{aligned} \ddot{e}_l + k_d^l \dot{e}_l + k_p^l e_l &= 0 \\ \ddot{e}_\psi + k_d^\psi \dot{e}_\psi + k_p^\psi e_\psi &= 0 \end{aligned} \tag{24}$$

Given $\ddot{q} = f(q, \dot{q}) + g(q)\tau$ the cart movement equations and $\ddot{e}_l = \ddot{l}^r - \ddot{l}$ after some straightforward calculations we get

$$\begin{aligned} \ddot{e}_l &= \ddot{l}^r - \dot{q}^\top \frac{\partial^2 l}{\partial q^2} \dot{q} - \frac{\partial l}{\partial q} f(q, \dot{q}) - \frac{\partial l}{\partial q} g(q) \tau_2 \\ &= \ddot{l}^r - f_l(q, \dot{q}) - g_l(q) \tau_2 \end{aligned} \tag{25}$$

in the same way

$$\begin{aligned} \ddot{e}_\psi &= \ddot{\psi}^r - \dot{q}^\top \frac{\partial^2 \psi}{\partial q^2} \dot{q} - \frac{\partial \psi}{\partial q} f(q, \dot{q}) - \frac{\partial \psi}{\partial q} g(q) \tau_2 \\ &= \ddot{\psi}^r - f_\psi(q, \dot{q}) - g_\psi(q) \tau_2 \end{aligned} \tag{26}$$

analogously can be defined $e_{12} = (e_{l_{12}}, e_{\psi_{12}})$ and $e_{13} = (e_{l_{13}}, e_{\psi_{13}})$

IV. Input-Output Linearization

One common method for controlling nonlinear dynamical systems is based on approximate feedback linearization [2], which depends on the relative degree of each controlled variable. For newtonian systems like the dynamical equations (20), the regulated variables of interest here represented as the vector η, have relative degree two. The control variables are represented by vector u.

$$\ddot{\eta} = f(\eta, \dot{\eta}, u) \tag{27}$$

A pseudo control ν is defined such that the dynamic relation between it and the system state is linear.

$$\ddot{\eta} = \nu \tag{28}$$

where

$$\nu = f(\eta, \dot{\eta}, u) \tag{29}$$

Since the function $f(\eta, \dot{\eta}, u)$ is not known exactly, an approximation is used which is invertible regarding u

$$\nu = \hat{f}(\eta, \dot{\eta}, u) \tag{30}$$

resulting in

$$\ddot{\eta} = \nu + \Delta(\eta, \dot{\eta}, u) \tag{31}$$

where the modeling error is represented by

$$\Delta(\eta, \dot{\eta}, u) = f(\eta, \dot{\eta}, u) - \hat{f}(\eta, \dot{\eta}, u) \tag{32}$$

So the effective actuator displacement can be calculated as

$$\hat{u} = \hat{f}^{-1}(\eta, \dot{\eta}, \nu) \tag{33}$$

Supposing in (31) that $\Delta(\eta, \dot{\eta}, u) = 0$ we can proceed in the stabilization problem, choosing a linear controller, a PD for instance, that locally will solve the regulation problem. A single hidden layer neural network with weights conveniently adapted, will be responsible for the modeling error cancelation. Let $\eta = Y = P + d(\cos\phi, \sin\phi)^\top, \ u = \tau$. The input-output linearization will be done in the mobile robot dynamic equations as follows.

$$\begin{aligned} \ddot{Y} &= F(q, \dot{q}) + G(q)\tau \\ &= f(\eta, \dot{\eta}, u) \end{aligned} \tag{34}$$

V. Tracking

Consider the path following and tracking problem. Let the path reference be Y_r. Calling $e = Y_r - Y$ after

$$\ddot{Y} = f(\eta, \dot{\eta}, u) = \nu \tag{35}$$

follows:

$$\ddot{e} = \ddot{Y}_r - \ddot{Y} = \ddot{Y}_r - \nu \tag{36}$$

Let $\nu = \ddot{Y}_r + \nu_{PD} = \ddot{Y}_r - (k_p e + k_d \dot{e})$ then follows

$$\ddot{e} + k_p e + k_d \dot{e} = 0 \tag{37}$$

which is, in the absence of modeling errors, the desired dynamic error behavior during the tracking process. Those conclusions assume null dynamic inversion error $\Delta = 0$. In practice this error can be substantial so we provide a cancelation therm ν_{ad} to cope with this problem. In case of imperfect cancelation as in

$$\ddot{Y} = \nu + \Delta \tag{38}$$

the pseudo control will include

$$\nu = \ddot{Y}_r + \nu_{PD} + \nu_{ad} \tag{39}$$

where $\nu_{PD} = -(k_p e + k_d \dot{e})$ and ν_{ad} is the adaptive component responsible for the cancelation of Δ. Figure 3 shows the controller architecture. Here $\mathcal{C}$ represents the path commander, $\mathcal{F}$ represents a suitable filter to obtain derivatives, with the structure

$$\ddot{Y}_r = \mathcal{F}(Y_r, \dot{Y}_r, Y_c) \tag{40}$$

where Y_c are defined by the path commander. PD is the proportional plus derivative controller responsible for ν_{PD} and $\mathcal{NN}$ represents the adaptive element responsible for ν_{ad}. Velocities are observed according to [6].

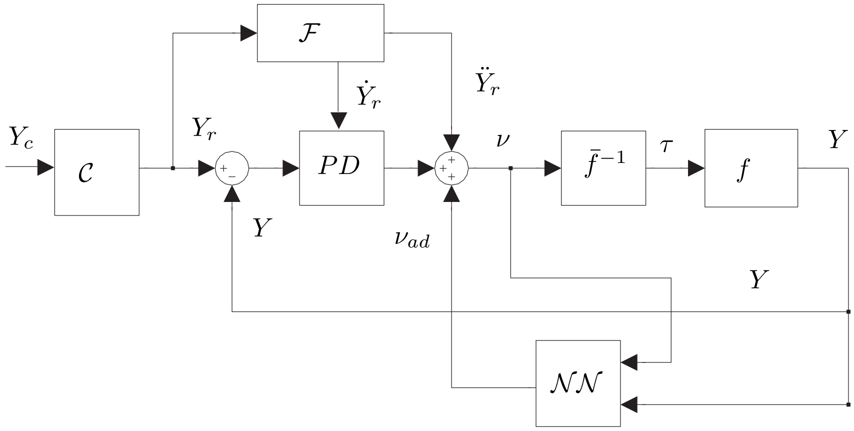


Fig. 3. Controller diagram

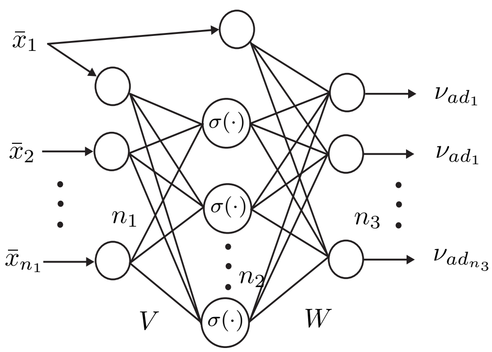


Fig. 4. $\mathcal{NN}$-Adaptive element structure

A. Adaptive element

The adaptive element is implemented by a feed-forward neural network with conveniently tuned weights V, W with structure as depicted in figure 4 and such that

$$\nu_{ad} = W^\top \bar{\sigma}(V^\top \bar{x}) \tag{41}$$

where $\bar{x} = (1, \nu, \eta)$ is the input to the neural network, comprehending the constant 1 the state η and the actuation ν. W, V are the network weights with the structure.

$$\begin{aligned} V &= \begin{pmatrix} v_{0,1} & v_{0,2} & \cdots & v_{0,n_2} \\ v_{1,1} & v_{1,2} & \cdots & v_{1,n_2} \\ \vdots & \vdots & \ddots & \vdots \\ v_{n_1,1} & v_{n_1,2} & \cdots & v_{n_1,n_2} \end{pmatrix} \\ W &= \begin{pmatrix} w_{0,1} & w_{0,2} & \cdots & w_{0,n_3} \\ w_{1,1} & w_{1,2} & \cdots & w_{1,n_3} \\ \vdots & \vdots & \ddots & \vdots \\ w_{n_2,1} & w_{n_2,2} & \cdots & w_{n_2,n_3} \end{pmatrix} \end{aligned} \tag{42}$$

Here n_1, n_2, n_3 are number of inputs, inner layer neurons and number of outputs respectively. Also $\bar{\sigma}(\xi) = (1, \sigma(\xi_1), \cdots, \sigma(\xi_{n_1}))^\top$. The scalar function σ is the sigmoidal activating function .

$$\sigma(\xi) = \frac{1}{1 + e^{-\alpha \xi}} \tag{43}$$

Here e is the basis of natural logarithms. The transformation in (41) must be contractive regarding ν_{ad}. As we know Δ depends on ν_{ad} through ν, and ν_{ad} is produced to cancel Δ. So it will be assured the existence of a fixed point solution for $\nu_{ad} = \Delta(q, \dot{q}, \nu_{ad})$. A sufficient condition to assure that the mapping $\nu_{ad} \mapsto \Delta(q, \dot{q}, \nu_{ad})$ is a contraction is the verification of $\|\partial \Delta / \partial \nu_{ad}\| < 1$. This condition is equivalent to [5].

$$0 < \frac{1}{2} \left| \frac{\partial f}{\partial u} \right| < \left| \frac{\partial \hat{f}}{\partial u} \right| < \infty \tag{44}$$

B. Tracking Error Boundedness

The tracking error dynamics is given by

$$\dot{e} = Ae + B(\nu_{ad} - \Delta) \tag{45}$$

where $e = \eta_r - \eta$ with

$$A = \begin{bmatrix} O & I \\ -k_p & -k_d \end{bmatrix}, \quad B = \begin{bmatrix} O \\ I \end{bmatrix} \tag{46}$$

where I and O are a suitable identity and null matrices respectively. Consider the system (27), the inverse law (33) and the contractibility property, as well as the adaptation laws

$$\begin{aligned} \dot{W} &= -\left((\bar{\sigma} - \bar{\sigma}' V^\top \bar{\eta}) r^\top + \kappa \|e\| W\right) \Gamma_W \\ \dot{V} &= -\Gamma_V \left(\bar{\eta}(r^\top W^\top \bar{\sigma}') + \kappa \|e\| V\right) \end{aligned} \tag{47}$$

where

$$\bar{\sigma}'(\hat{z}) = \frac{\partial \bar{\sigma}(z)}{\partial z}\Big|_{z = \hat{z}} \tag{48}$$

is the Jacobian matrix of $\bar{\sigma}(z)$ and $r = e^\top PB$. Also $P \succ 0$ is the unique positive definite solution for the Lyapunov equation

$$A^\top P + PA + Q = 0 \tag{49}$$

for any convenient $Q \succ 0$. A and B are defined in (46). Given (47) with $\Gamma_W \succ 0$, $\Gamma_V \succ 0$ and $\kappa > 0$, according to [5] the tracking error e uniform boundedness is assured.

C. Obtaining the Adaptation Laws

Let us consider the Lyapunov function

$$\begin{aligned} \mathcal{V}(e, \tilde{V}, \tilde{W}) &= \frac{1}{2} \left(e^\top P e\right. \\ &+ \left. \text{tr}\left(\tilde{W}^\top \Gamma_W^{-1} \tilde{W}\right) + \text{tr}\left(\tilde{V}^\top \Gamma_V^{-1} \tilde{V}\right)\right) \end{aligned} \tag{50}$$

with $\tilde{W} = W - W^*$, $\tilde{V} = V - V^*$ and $\text{tr}(\cdot)$ is the trace operation. Here W^*, V^* are the optimum values that best approximate Δ. Also P solves the equation

$$A^\top P + PA + Q = 0, \quad A = \begin{pmatrix} O & I \\ -k_p & -k_d \end{pmatrix} \tag{51}$$

with $-Q$ and P definite positive. In order to obtain the adaptation equations (47) we must follow the steps required to proof that, on the error orbits, the following condition is satisfied:

$$\dot{\mathcal{V}} \leq 0 \tag{52}$$

as explained in [5]. The following steps are given in order to show the parameters regarding an adequate tuning of the controller. The details of the proof of convergence follow the above mentioned reference. Let us consider

$$\epsilon = \nu_{ad}^* - \Delta = W^{*\top} \bar{\sigma}(V^{*\top} \bar{q}) - \Delta \tag{53}$$

The error dynamics is

$$\dot{e} = Ae + B\left(W^{*\top}\bar{\sigma}(V^\top \bar{q}) - W^\top \bar{\sigma}(V^{*\top}\bar{q}) + \epsilon\right) \tag{54}$$

Using the Taylor series expansion of σ with respect to V in the neighborhood of V^*, which is the optimum value, we obtain

$$\dot{e} = Ae + B\left(\tilde{W}^\top(\sigma - \sigma' V^\top \bar{q}) + W^\top \sigma' \tilde{V}^\top \bar{q} + w\right) \tag{55}$$

with

$$w = \epsilon - W^{*\top}\left(\sigma^* - \sigma + \sigma' \tilde{V}^\top \bar{q}\right) + \tilde{W}^\top \sigma' V^{*\top} \bar{q} \tag{56}$$

Substituting now (47) and (55) in the expression of $\dot{\mathcal{V}}$ we have

$$\dot{\mathcal{V}} = -\frac{1}{2} e^\top Q e + e^\top P B w - \kappa \|e\| \, \text{tr}\left(\tilde{Z}^\top Z\right) \tag{57}$$

where

$$Z = \begin{pmatrix} V & 0 \\ 0 & W \end{pmatrix}, \quad \tilde{Z} = Z - Z^* \tag{58}$$

Using $\text{tr}(\tilde{Z}^\top Z) \leq \left\|\tilde{Z}\right\| \|Z^*\| - \left\|\tilde{Z}\right\|^2$ and following [5] there exist $a_0, a_1, c_3, \kappa > \|PB\| \, c_3$ such that

$$\begin{aligned} \dot{\mathcal{V}} &= -\frac{1}{2}\lambda_{min}(Q)\|e\|^2 - (\kappa - \|PB\| \, c_3)\|e\| \left\|\tilde{Z}\right\|^2 + \\ &+ a_0 \|e\| + a_1 \|e\| \left\|\tilde{Z}\right\| \end{aligned} \tag{59}$$

and, with $Z_m = \frac{a_1 + \sqrt{a_1^2 + 4a_0(\kappa - \|PB\| c_3)}}{\kappa - \|PB\| c_3}$,

$$\|e\| \geq \frac{a_0 + a_1 Z_m}{\frac{1}{2}\lambda_{min}(Q)} \Rightarrow \dot{\mathcal{V}} \leq 0 \tag{60}$$

Thus for convenient initial conditions, the tracking error e is ultimately uniformly bounded.

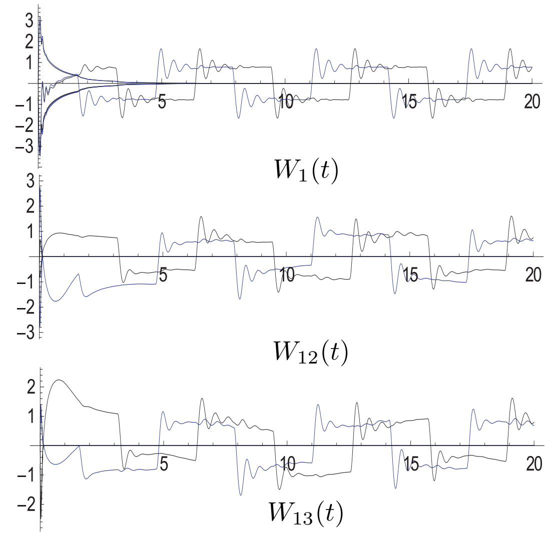


Fig. 6. W weights evolution

VI. Case Study

Simulations were run with a platoon triangle-shaped with a leader and two followers. In figure (5) one can observe typical a maneuver. Dashed lines indicate the ideal path to follow. The red line, is the leader path: at both sides are the followers. In figure (6) we can see the output layer weights evolution concerning the neural network. The linking attitudes are respectively,

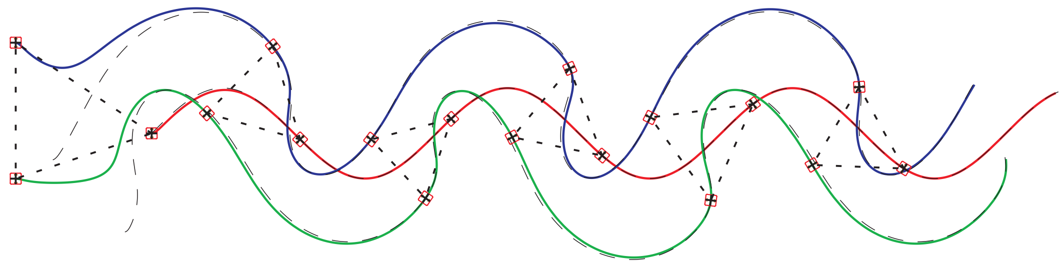
$$\begin{aligned} \{x_1^r, \, y_1^r\} &= \{t, \, 8 + \sin(t)\} \\ \{l_{12}^r, \, \psi_{12}^r\} &= \{2 + \cos(t/2)/4, \, 5/6 \cdot \pi\} \\ \{l_{13}^r, \, \psi_{13}^r\} &= \{2 + \cos(t/2)/4, \, -5/6 \cdot \pi\} \end{aligned} \tag{61}$$

The considered perturbations are of additive type with the structure $\delta^i = (\delta_x^i, \, \delta_y^i, \, \delta_L^i, \, \delta_R^i)^\top$, with $\delta_x^i = 0.8\text{sign}(\sin t)$, $\delta_y^i = 0.8\text{sign}(\cos t)$, $\delta_L^i = \delta_R^i = 0$ and entering the equations of motion as follows

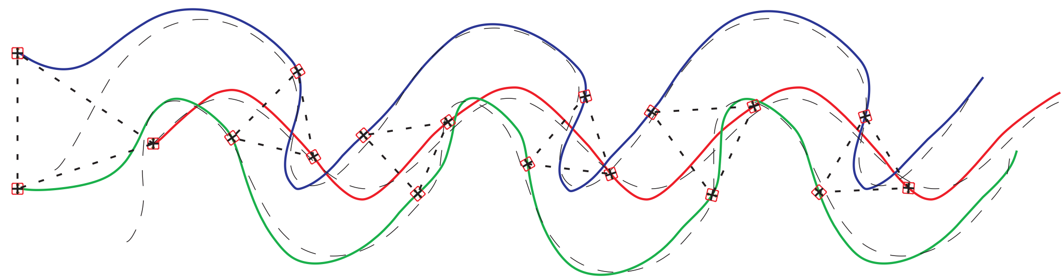
$$\ddot{q}^i = f^i + g^i \tau_i + \delta^i, \;\; i = 1, 2, 3. \tag{62}$$

VII. Conclusions

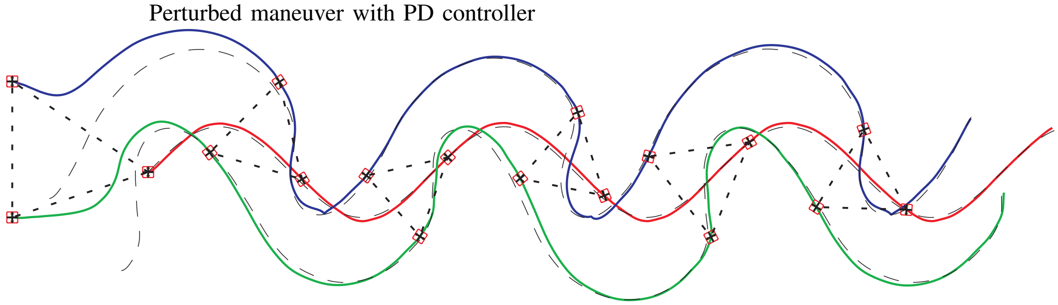
With the consideration of the dynamic model of mobile robots it is possible to obtain better results in tracking problems. With the introduction of dynamics in the platoon problem, we were able to incorporate an adaptive component to correct unmodeled disturbances, gaining robustness during tracking.



Unperturbed maneuver with PD controller



Perturbed maneuver with PD controller



Perturbed maneuver with PD + Adaptive controller

Fig. 5. Performance comparison

REFERENCES

[1] Xiaoping Yun and Yoshio Yamamoto (1993) Internal Dynamics of a Wheeled Mobile Robot Proceedings of the 1993 IEEE/RSJ International Conference on Intelligent Robots and Systems Yokohama, pp. 1288–1294.

[2] Isidori, A. (1995). Nonlinear Control Systems. Springer-Verlag ed.. London.

[3] Jaydev P. Desai and Jim Ostrowski and Vijay Kumar, Controlling Formations of Multiple Mobile Robots, in Proceedings of the IEEE International Conference on Robotics and Automation, pp (2864–2869) 1998.

[4] Jaydev P. Desai, James P. Ostrowski, and Vijay Kumar (2001), Modeling and Control of Formations of Nonholonomic Mobile Robots, IEEE Trans. on Robotics and Automation, vol. 17, no. 6, dec. 2001

[5] Kim, Nakwan (2003). Improved Methods in Neural Network-Based Adaptive Output Feedback Control, with Applications to Flight Control. Georgia Institute of Technology.

[6] Elżbieta Jarzębowska (2011). A velocity observer design for tracking task-based motions of unicycle type mobile robots. Commun Nonlinear Sci Numer Simulat 16 (2011) 23012307. www.elsevier.com/locate/cnsns.

[7] Stephen Wolfram (2003), The Mathematica Book, 5th ed., Wolfram Media, 2003.

[8] Maple User Manual (2005), Maplesoft, Waterloo Maple Inc.