

An Interpolation-based Robust MPC Algorithm Using Polyhedral Invariant sets

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Abstract— This article presents an interpolation-based robust MPC algorithm for uncertain polytopic discrete-time systems using polyhedral invariant sets. Two nested polyhedral invariant sets are constructed off-line by solving robust constrained model predictive control optimization problems. The first one is a large set constructed to cover all of the desired operating spaces. The second one is a small target set constructed to drive the terminal state into. The real-time control law is calculated by linear interpolation between the two state feedback gains corresponding to these nested pre-computed polyhedral invariant sets. At each sampling instant, only a computationally low-demanding optimization problem is needed to be solved on-line. The controller design is illustrated with an example. The proposed algorithm can achieve good control performance while on-line computation is still tractable.

I. INTRODUCTION

Robust model predictive control (MPC) has been developed as a strategy to deal with model uncertainty. The synthesis approaches for on-line robust MPC have been studied by many researchers. Robust constrained MPC using linear matrix inequalities (LMI) was proposed by [1]. The goal is to calculate the state feedback gain that robustly stabilizes the closed-loop system. The state feedback gain is derived by minimizing the worst-case performance cost. At each sampling instant, an invariant ellipsoid containing the currently measured state is constructed to guarantee robust stability. Since the entire optimization problem is solved on-line, the algorithm requires high on-line computational time.

Quasi-min-max MPC algorithm was proposed by [2]. The algorithm is seen as an extension of the algorithm proposed by [1] by keeping the first control input as a free decision variable in order to reduce the conservativeness. However, the algorithm still requires high on-line computational time because the entire optimization problem is solved on-line. Moreover, the algorithm is still based on an ellipsoidal approximation of the true polyhedral invariant set. Robust MPC derived by using parameter-dependent Lyapunov function was developed by [3]. The algorithm can achieve less conservative result as compared with those derived by using a single Lyapunov function because the degree of freedom in the optimization problem is increased. However, the algorithm is computationally prohibitive in practical situations because the on-line optimization problem contains many decision variables and constraints.

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Since on-line robust MPC requires high computational time, its application is rather restricted to the relatively slow dynamic processes. For this reason, the synthesis approaches for off-line robust MPC have been widely investigated. An off-line formulation of robust MPC using LMI was developed by [4]. A sequence of explicit control laws corresponding to a sequence of ellipsoidal invariant sets is computed off-line. At each sampling instant, the smallest ellipsoidal invariant set containing the currently measured state is determined and the real-time control law is calculated by linear interpolation between the pre-computed control laws. Although the algorithm substantially reduces on-line computational burdens, the conservative result is obtained because an invariant ellipsoid constructed to guarantee robust stability is only an approximation of the true polyhedral invariant set. An ellipsoidal off-line MPC algorithm for linear parameter varying (LPV) systems was developed by [5]. The algorithm can achieve less conservative result as compared with [4] because the scheduling parameter is included in the controller design. However, the algorithm is still based on an ellipsoidal approximation of the true polyhedral invariant set. Moreover, the algorithm can handle only the uncertainty in the state matrix.

An off-line robust MPC algorithm using polyhedral invariant sets was proposed by [6]. The algorithm computes off-line a sequence of state feedback gains corresponding to a sequence of polyhedral invariant sets. At each sampling instant, the smallest polyhedral invariant set containing the currently measured state is determined and the corresponding state feedback gain is implemented to the process. Although the true polyhedral invariant set is used, the conservativeness is still obtained because the control law implemented at each time step is only an approximation of the true optimal control law. Moreover, the input discontinuities caused by the switching between state feedback control laws are occurred. Therefore, the algorithm requires constructing high number of polyhedral invariant sets in order to improve the control performance and to reduce input discontinuities.

An interpolation technique for polyhedral invariant sets was proposed by [7]. The algorithm uses decomposition variables and solves online optimization on performance index with subject to constraint sets. The paper has shown the potential benefits of using interpolation to generate predictive control algorithm and to enlarge the stabilizable region. Unfortunately, the technique proposed is developed for only the system without model uncertainty.

In this work, we propose an interpolation-based robust MPC algorithm for uncertain polytopic discrete-time systems using polyhedral invariant sets. The basic idea of the algorithm is to use interpolation technique on two nested polyhedral invariant sets constructed by the method

suggested in [6]. Two state feedback gains corresponding to two nested polyhedral invariant sets are computed off-line by considering infinite horizon performance cost. The real-time control law is then calculated by linear interpolation between these two control laws.

The paper is organized as follows. In section 2, the problem description is presented. In section 3, the proposed algorithm is presented. In section 4, we present an example to illustrate the implementation of our algorithm. Finally, in section 5, we conclude the paper.

Notation: For a matrix A , A^T denotes its transpose, A^{-1} denotes its inverse. I denotes the identity matrix. For a vector x , $x(k/k)$ denotes the state measured at real time k , $x(k+i/k)$ denotes the state at prediction time $k+i$ predicted at real time k . The symbol $*$ denotes the corresponding transpose of the lower block part of symmetric matrices. The Lyapunov function $V(i,k)$ is defined as $V(i,k) = x(k+i/k)^T P(i,k)x(k+i/k)$ where $\forall k, \forall i \geq 0, P(i,k) > 0$.

II. PROBLEM DESCRIPTION

The model considered here is the following linear time varying (LTV) system with polytopic uncertainty:

$$\begin{aligned} x(k+1) &= A(k)x(k) + B(k)u(k) \\ y(k) &= Cx(k) \end{aligned} \quad (1)$$

where $x(k)$ is the state of the plant, $u(k)$ is the control input and $y(k)$ is the plant output. We assume that

$$[A(k), B(k)] \in \Omega, \Omega = Co\{[A_1, B_1], [A_2, B_2], \dots, [A_L, B_L]\} \quad (2)$$

where Ω is the polytope, Co denotes convex hull, $[A_j, B_j]$ are vertices of the convex hull. Any $[A(k), B(k)]$ within the polytope is a linear combination of the vertices

$$[A(k), B(k)] = \sum_{j=1}^L \lambda_j(k) [A_j, B_j], \sum_{j=1}^L \lambda_j(k) = 1, 0 \leq \lambda_j(k) \leq 1 \quad (3)$$

where $\lambda(k) = [\lambda_1(k), \lambda_2(k), \dots, \lambda_L(k)]$ is the uncertain parameter vector. The aim of this research is to find the control law

$$u(k+i/k) = g(x(k+i/k)) \quad (4)$$

that robustly stabilizes (1) and satisfies both input and output constraints

$$|u_h(k+i/k)| \leq u_{h,\max}, h=1,2,3,\dots,n_u \quad (5)$$

$$|y_r(k+i/k)| \leq y_{r,\max}, r=1,2,3,\dots,n_y \quad (6)$$

III. THE PROPOSED ALGORITHM

In this section, an interpolation-based robust MPC algorithm for uncertain polytopic discrete-time systems using polyhedral invariant sets is presented. Two nested polyhedral invariant sets are pre-computed off-line in order

to reduce the on-line computational burdens. The first one is constructed to cover all of the desired operating space. The second one is the terminal target set constructed to drive the terminal state into. The real-time control law is calculated by linear interpolation between these two state feedback gains corresponding to the polyhedral invariant sets previously generated. The optimization problem solved at each time step is based on optimization of linear performance index and only a computationally low-demanding optimization problem is required to be solved on-line. An approach to construct the polyhedral invariant set proposed by [8] is adopted in this paper.

Algorithm 3.1

Off-line step 1; Choose two states $x_i, i \in \{1,2\}$, located inside feasible region. For each x_i , solve the following problem to obtain the corresponding state feedback gains $K_i = Y_i Q_i^{-1}$. The first chosen state x_1 should be far from the origin because it is used to calculate K_1 that has the largest possible stabilizable region. The second chosen state x_2 should be near the origin because it is used to calculate K_2 that has the smallest possible stabilizable region.

$$\begin{aligned} \min_{\gamma_i, Y_i, Q_i} \gamma_i \\ \text{s.t.} \begin{bmatrix} 1 & * \\ x_i & Q_i \end{bmatrix} \geq 0 \end{aligned} \quad (7)$$

$$\begin{bmatrix} Q_i & * & * & * \\ A_j Q_i + B_j Y_i & Q_i & * & * \\ \Theta^{\frac{1}{2}} Q_i & 0 & \gamma_i I & * \\ R^{\frac{1}{2}} Y_i & 0 & 0 & \gamma_i I \end{bmatrix} \geq 0, \quad (8)$$

$\forall j = 1, 2, \dots, L$

$$\begin{bmatrix} X & * \\ Y_i^T & Q_i \end{bmatrix} \geq 0, \quad (9)$$

$X_{hh} \leq u_{h,\max}^2, h=1,2,\dots,n_u$

Note that the following condition must be satisfied

$$Q_1^{-1} - (A_j + B_j K_2)^T Q_1^{-1} (A_j + B_j K_2) > 0, \forall j = 1, 2, \dots, L. \quad (10)$$

The optimization problem used to derive the state feedback gains in this step is based on the controller proposed in [1]. It is the minimization of upper bound of infinite horizon worst-case performance. However, the output constraints are relaxed in this step in order to enlarge the stabilizable region and to reduce conservativeness. The output constraints and also input constraints are then properly taken into account during polyhedral invariant set construction in off-line-step 2. The condition (10) is used to assure robust stability satisfaction of a convex combination of K_1 and K_2 .

Off-line step 2; Given the state feedback gains $K_i = Y_i Q_i^{-1}, i = 1, 2$ calculated from step 1. For each K_i , the corresponding polyhedral invariant set $S_i = \{x / M_i x \leq d_i\}$ is constructed by following these steps:

2.1) Set $M_i = [C^T, -C^T, K_i^T, -K_i^T]^T$, $d_i = [y_{\max}^T, y_{\min}^T, u_{\max}^T, u_{\min}^T]^T$ and $m = 1$.

2.2) Select row m from (M_i, d_i) and check $\forall j$ whether $M_{i,m}(A_j + B_j K_i)x \leq d_{i,m}$ is redundant with respect to the constraints defined by (M_i, d_i) by solving the following problem:

$$\begin{aligned} & \max_x W_{i,m,j} \\ & \text{s.t. } W_{i,m,j} = M_{i,m}(A_j + B_j K_i)x - d_{i,m} \\ & \quad M_i x \leq d_i \end{aligned} \quad (11)$$

If $W_{i,m,j} > 0$, the constraint $M_{i,m}(A_j + B_j K_i)x \leq d_{i,m}$ is non-redundant with respect to (M_i, d_i) . Then, add non-redundant constraints to (M_i, d_i) by assigning $M_i = [M_i^T, (M_{i,m}(A_j + B_j K_i))^T]^T$ and $d_i = [d_i^T, d_{i,m}^T]^T$.

2.3) Let $m = m + 1$ and return to step 2.2. If m is strictly larger than the number of rows in (M_i, d_i) then terminate.

On-line: At each sampling time, $x(k)$ is measured.

If $x(k) \in S_1$ and $x(k) \notin S_2$, the problem (12) is solved. This problem is based on the interpolation between the two state feedback gains K_1 and K_2 corresponding to two pre-computed polyhedral invariant sets S_1 and S_2 .

$$\min \lambda \quad (12)$$

$$\text{s.t. } M_1(A_j + B_j(\lambda K_1 + (1-\lambda)K_2))x(k) - d_1 \leq 0, j = 1, 2, \dots, L \quad (13)$$

$$|((\lambda K_1 + (1-\lambda)K_2)x(k))_h| \leq u_{h,\max}, h = 1, 2, 3, \dots, n_u \quad (14)$$

$$0 \leq \lambda \leq 1$$

$$\text{Implement } u(k) = (\lambda K_1 + (1-\lambda)K_2)x(k). \quad (15)$$

$$\text{If } x(k) \in S_2, \text{ implement } u(k) = K_2 x(k). \quad (16)$$

The satisfaction of (8) for the state feedback gain $K_i = Y_i Q_i^{-1}, i = \{1, 2\}$ ensures that

$$\begin{aligned} & x(k+i/k)^T \{[A(k+i) + B(k+i)K_i]^T P(i+1, k) \\ & [A(k+i) + B(k+i)K_i] - P(i, k)\} x(k+i/k) < 0 \end{aligned} \quad (17)$$

Thus, $V(i, k) = x(k+i/k)^T P(i, k)x(k+i/k)$ is a strictly decreasing Lyapunov function and the system is robustly stabilized by state feedback gain K_i .

By solving (11) and iteratively adding non-redundant constraints $M_{i,m}(A_j + B_j K_i)x \leq d_{i,m}$ to (M_i, d_i) by assigning $M_i = [M_i^T, (M_{i,m}(A_j + B_j K_i))^T]^T$ and $d_i = [d_i^T, d_{i,m}^T]^T$, we can find the set of initial states $x_i(k)$ defined by $S_i = \{x_i(k) / M_i x_i(k) \leq d_i\}$ such that all future states are guaranteed to stay within this set without input and output constraints violation. Any initial states outside S_i lead to the future states that violate input and output constraints for at least one realization of the uncertainty. Thus, the set S_i is polyhedral invariant set and the corresponding state feedback gain K_i assures robust constraint satisfaction to the closed-loop system.

Since K_1 and K_2 assure robust stability and robust constraint satisfaction to the closed-loop system, and the condition (10) is satisfied, a convex combination of K_1 and K_2 as shown in (15) also assures robust stability satisfaction to the closed-loop system. That is

$$\begin{aligned} & Q_1^{-1} - (A_j + B_j(\lambda K_1 + (1-\lambda)K_2))^T Q_1^{-1} \\ & (A_j + B_j(\lambda K_1 + (1-\lambda)K_2)) > 0, 0 \leq \lambda \leq 1, \forall j = 1, 2, \dots, L. \end{aligned} \quad (18)$$

By using the control law (15), we can guarantee that any initial states inside S_1 lead to the future states inside S_1 and move toward S_2 , which is close to the origin. By minimization of λ in (12), the maximum state feedback gain is obtained. The output and input constraints are satisfied by (13) and (14), respectively. The control law (16) assures robust constraint satisfaction to closed-loop system for all states inside S_2 .

IV. AN EXAMPLE

In this section, we present an example that illustrates the implementation of the proposed robust MPC algorithm. The numerical simulations have been performed in Intel Core i-5 (2.4GHz), 2 GB RAM, using SeDuMi [9] and YALMIP [10] within Matlab R2008a environment. We will consider the application of our approach to the nonlinear two-tank system [11], which is described by the following equation.

$$\rho S_1 \dot{h}_1 = -\rho A_1 \sqrt{2gh_1} + u \quad (19)$$

$$\rho S_2 \dot{h}_2 = \rho A_1 \sqrt{2gh_1} - \rho A_2 \sqrt{2gh_2}$$

Where h_1 is the water level in tank 1, h_2 is the water level in tank 2 and u is the water flow. The operating parameters are shown in table I.

TABLE I.
THE OPERATING PARAMETERS OF NONLINEAR TWO-TANK SYSTEM

Parameters	Value
S_1	2500 cm ²
S_2	1600 cm ²
A_1	9 cm ²
A_2	4 cm ²
g	980 cm/s ²
ρ	0.001 kg/cm ³

Let $\bar{h}_1 = h_1 - h_{1,eq}$, $\bar{h}_2 = h_2 - h_{2,eq}$ and $\bar{u} = u - u_{eq}$ where subscript eq is used to denote the corresponding variable at equilibrium condition, the objective is to regulate \bar{h}_2 to the origin by manipulating \bar{u} . The input and output constraints are given as follows

$$\left| \bar{u} \right| \leq 1.5 \text{ kg/s}, \left| \bar{h}_1 \right| \leq 13 \text{ cm}, \left| \bar{h}_2 \right| \leq 50 \text{ cm} \quad (20)$$

By evaluating the Jacobian matrix of (19) along the vertices of the constraints set (20), we have that all the solutions of (19) are also the solution of the following differential inclusion

$$\begin{bmatrix} \rho S_1 \dot{\bar{h}}_1 \\ \rho S_2 \dot{\bar{h}}_2 \end{bmatrix} \in \left(\sum_{j=1}^4 p_j A_j \right) \begin{bmatrix} \bar{h}_1 \\ \bar{h}_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \bar{u} \quad (21)$$

where $A_j, j=1, \dots, 4$ are given by

$$A_1 = \begin{bmatrix} -\rho A_1 \sqrt{\frac{2g}{h_{1,\min}}} & 0 \\ \rho A_1 \sqrt{\frac{2g}{h_{1,\min}}} & -\rho A_2 \sqrt{\frac{2g}{h_{2,\min}}} \end{bmatrix}$$

$$A_2 = \begin{bmatrix} -\rho A_1 \sqrt{\frac{2g}{h_{1,\max}}} & 0 \\ \rho A_1 \sqrt{\frac{2g}{h_{1,\max}}} & -\rho A_2 \sqrt{\frac{2g}{h_{2,\min}}} \end{bmatrix}$$

$$A_3 = \begin{bmatrix} -\rho A_1 \sqrt{\frac{2g}{h_{1,\min}}} & 0 \\ \rho A_1 \sqrt{\frac{2g}{h_{1,\min}}} & -\rho A_2 \sqrt{\frac{2g}{h_{2,\max}}} \end{bmatrix}$$

$$A_4 = \begin{bmatrix} -\rho A_1 \sqrt{\frac{2g}{h_{1,\max}}} & 0 \\ \rho A_1 \sqrt{\frac{2g}{h_{1,\max}}} & -\rho A_2 \sqrt{\frac{2g}{h_{2,\max}}} \end{bmatrix} \quad (22)$$

and $p_j, j=1, \dots, 4$ are given by

$$p_1 = \frac{(1/\sqrt{h_{1,\max}}) - (1/\sqrt{h_1})}{(1/\sqrt{h_{1,\max}}) - (1/\sqrt{h_{1,\min}})} \cdot \frac{(1/\sqrt{h_{2,\max}}) - (1/\sqrt{h_2})}{(1/\sqrt{h_{2,\max}}) - (1/\sqrt{h_{2,\min}})}$$

$$p_2 = \frac{(1/\sqrt{h_1}) - (1/\sqrt{h_{1,\min}})}{(1/\sqrt{h_{1,\max}}) - (1/\sqrt{h_{1,\min}})} \cdot \frac{(1/\sqrt{h_{2,\max}}) - (1/\sqrt{h_2})}{(1/\sqrt{h_{2,\max}}) - (1/\sqrt{h_{2,\min}})}$$

$$p_3 = \frac{(1/\sqrt{h_{1,\max}}) - (1/\sqrt{h_1})}{(1/\sqrt{h_{1,\max}}) - (1/\sqrt{h_{1,\min}})} \cdot \frac{(1/\sqrt{h_2}) - (1/\sqrt{h_{2,\min}})}{(1/\sqrt{h_{2,\max}}) - (1/\sqrt{h_{2,\min}})}$$

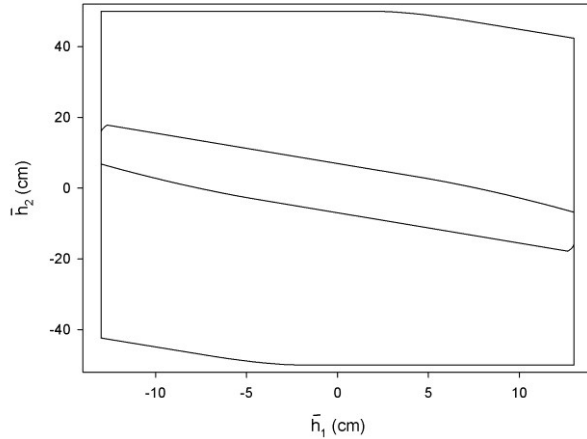
$$p_4 = \frac{(1/\sqrt{h_1}) - (1/\sqrt{h_{1,\min}})}{(1/\sqrt{h_{1,\max}}) - (1/\sqrt{h_{1,\min}})} \cdot \frac{(1/\sqrt{h_2}) - (1/\sqrt{h_{2,\min}})}{(1/\sqrt{h_{2,\max}}) - (1/\sqrt{h_{2,\min}})} \quad (23)$$

The discrete-time model is obtained by discretization of (21) using Euler first-order approximation with a sampling period of 0.1 s and it is omitted here for brevity. The proposed algorithm will be compared with an off-line robust MPC algorithm using polyhedral invariant sets [6]. The

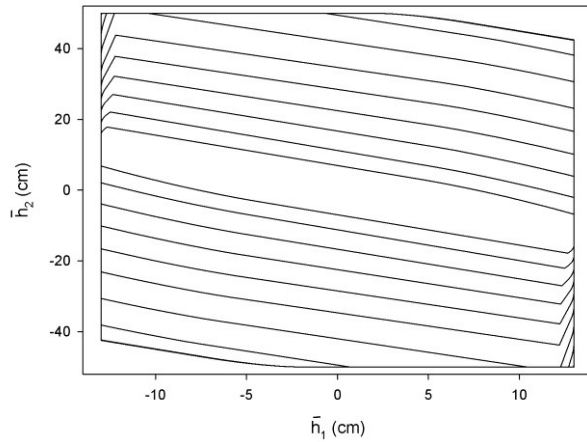
tuning parameters are $\Theta = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$ and $R = 0.01$.

Figure 1 shows the polyhedral invariant sets constructed off-line by the proposed algorithm and an off-line robust MPC algorithm [6]. For an off-line robust MPC algorithm [6], a sequence of ten polyhedral invariant sets is constructed off-line. In comparison, only two polyhedral invariant sets are constructed in the proposed algorithm.

Figure 2(a) shows the regulated output. It is seen that the proposed algorithm can achieve better control performance as compared to an off-line robust MPC algorithm [6] that a sequence of ten polyhedral invariant sets is constructed. This is due to the fact that the proposed algorithm is based on interpolation between the two state feedback gains of the two pre-computed polyhedral invariant sets. In comparison, an off-line robust MPC algorithm [6] has no interpolation between state feedback gains.



(a) Algorithm 3.1



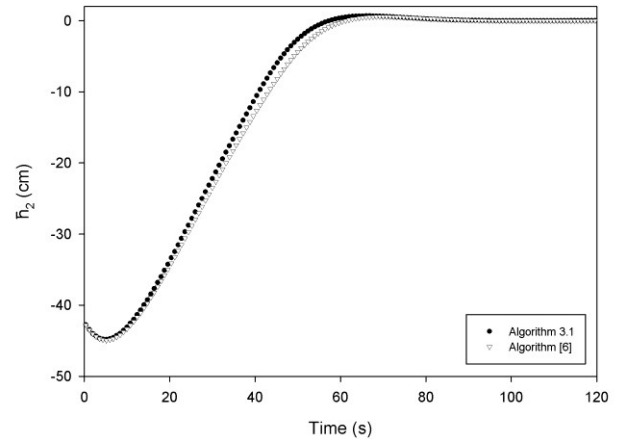
(b) An off-line robust MPC algorithm [6]

Figure 1. (a) The polyhedral invariant sets constructed off-line by algorithm 3.1 and (b) an off-line robust MPC algorithm [6].

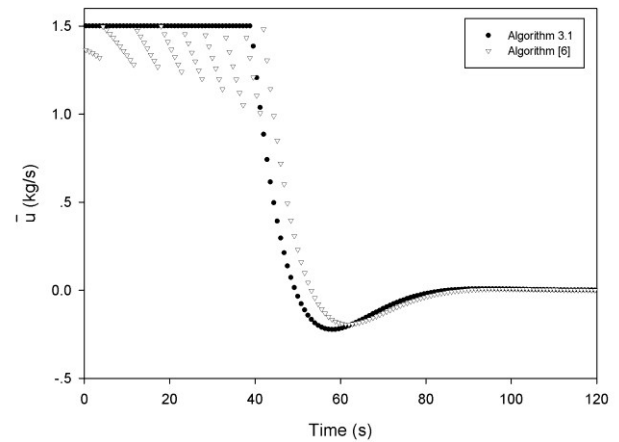
Figure 2(b) shows the control input. The input discontinuities appeared in the response of an off-line robust MPC algorithm [6] are caused by the switching of feedback gains based on the distance between the state and the origin. In comparison, the proposed algorithm which is based on the interpolation can overcome this issue.

Figure 3 shows the cumulative cost. It is seen that the cumulative cost of the proposed algorithm is lower than the cumulative cost of an off-line robust MPC algorithm [6].

The overall off-line computational burdens are shown in table II. For both algorithms, all of the computational burdens are moved off-line so the on-line computation is tractable. However, the proposed algorithm requires less off-line computational time than an off-line robust MPC algorithm [6] because only two polyhedral invariant sets are needed to be constructed off-line. Moreover, the computational complexity of the proposed algorithm required to be solved online is linearly dependent of the number of vertices of the polytope.



(a) Regulated output



(b) Control input

Figure 2. The closed-loop responses of the nonlinear two-tank system

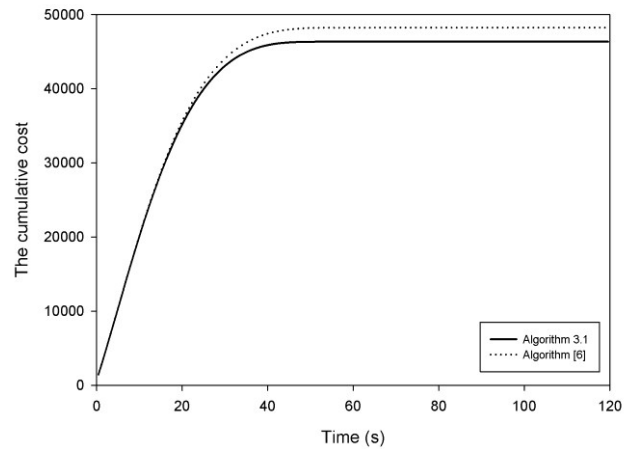


Figure 3. The cumulative cost $\sum_{i=0}^l x(i)^T Qx(i) + u(i)^T Ru(i)$

TABLE II.

THE OVERALL OFF-LINE COMPUTATIONAL BURDENS

Algorithm	Off-line CPU time(s)	On-line CPU time(s)/step
An off-line robust MPC algorithm [6]	22.743	-
Algorithm 3.1	2.214	0.001

V. CONCLUSION

In this paper, we have presented an interpolation-based robust MPC algorithm using polyhedral invariant sets. The proposed algorithm computes off-line only two polyhedral invariant sets. The real-time control law is then calculated by interpolation between the two state feedback gains corresponding to these two polyhedral invariant sets pre-computed. The optimization problem solved at each time step is linear programming so only a computationally low-demanding optimization problem is needed to be solved on-line. An example that illustrates the implementation of the proposed off-line robust MPC algorithm is presented. The proposed algorithm can achieve better control performance without input discontinuities while on-line computations are still tractable and linearly dependent of the number of vertices of the polytope.

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