

# Non-stationary Harmonic Tracking using Piecewise-overlapped Group-harmonic Algorithm

Hsiung Cheng Lin, Guo-Shing Huang, Liang Yih Liu

**Abstract**—The Discrete Fourier Transform (DFT) is still a widely-used tool for analyzing and measuring both stationary and transient signals in power systems. However, the misapplications of DFT can lead to incorrect results caused by some problems such as aliasing effect, spectral leakage and picket-fence effect. A strategy of Piecewise-overlapped Group-harmonic Energy Restoring (PGER) algorithm is developed for both stationary and non-stationary harmonic evaluation in power systems. The proposed algorithm can restore dispersing spectral leakage energy caused by the DFT, and thus retrieve the respective real harmonic value. Based on the piecewise-overlapped method, especially every time-varying harmonic can be extracted fast and accurately. The numerical examples are presented to verify the proposed algorithm in term of robust, fast and high-precision performance.

## I. INTRODUCTION

With increasing use of power electronic systems and time-variant non-linear loads in industry, the generated power harmonics and interharmonics have resulted in serious power line pollution. Power supply quality is therefore aggravated. Traditional harmonics may cause negative effects such as signal interference, overvoltage, data loss, equipment malfunction, equipment heating and damage, etc. The noise on data transmission line is also related with harmonics. At some special systems, harmonic current components may cause effect of carrier signals, and thus interfere other carrier signals. As a result, some facilities may be affected. Once harmonics source enter computer instruments, the data stored in the computer may be lost. Moreover, harmonics may also cause transformer and capacitor over heating, thus reducing their working life. The resulting rotor heating and pulsating output torque will decrease the driver's efficiency [1-8].

The presence of power system interharmonics has not only brought many problems as harmonics but produced additional problems. For instance, there are thermal effects, low frequency oscillation of mechanical system, light and CRT flicker, interference of control and protection signals, high

frequency overload of passive parallel filter, telecommunication interference, acoustic disturbance, saturation of current transformer, subsynchronous oscillations, voltage fluctuations, malfunctioning of remote control system, erroneous firing of thyristor apparatus, and the loss of useful life of induction motors, etc. These phenomenons may even happen under low amplitude [5,9-12].

Conventionally, Discrete Fourier transform (DFT) method is efficient for signal spectrum evaluation because of the simplicity and easy implementation. An improper use of DFT based algorithms can, however, lead to multiple interpretations of spectrum [12-14]. For example, if the periodicity of DFT data set does not match the periodicity of signal waveforms, the spectral leakage and picket-fence effect will occur. Since the power system frequency is subject to small random deviations, some degree of spectral leakage can not be avoided. A number of algorithms, e.g. short time Fourier Transform [15], least-square approach [16-18], Kalman filtering [19-20], artificial neural networks [14,21], other methods [22-24] have been proposed to extract harmonics. The approaches may either suffer from low solution accuracy or less computational efficiency. None is reported to perform well in interharmonic identification under system frequency variations though each demonstrates its specific advantages.

## II. BACKGROUND OF SYSTEM HARMONIC/INTERHARMONIC MEASUREMENT

In practice, only major harmonics including interharmonics are concerned to affect power systems. Consequently,  $i_s(t)$  can be simply expressed as a certain series of sinusoidal harmonics, and the response to each harmonic can be thus determined by the following equation.

$$i_s(t) = \sum_{m=1}^M A_m \cos(\omega_m t + \phi_m) \quad (1)$$

where  $\omega_m = 2\pi f_m$ ,  $\phi_m = \tan^{-1}(-b_m/a_m)$  and

$$A_m = (a_m^2 + b_m^2)^{1/2}.$$

With the sampling interval  $\Delta t$  to acquire either voltage or current waveform data in power systems, the equ. (1) is given in a discrete form as follows.

$$i_s[n] = \sum_{m=1}^M A_m \cos(\omega_m n \Delta t + \phi_m) \quad (2)$$

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where  $n$  is the time step in the discrete sample sequence and  $t = n\Delta t$ .

The  $m^{\text{th}}$  major harmonic amplitude located at the discrete frequency  $f_k$  is expressed as

$$A_m[f_k] = \sqrt{P[f_k]} = \sqrt{2I_s[k]} \quad (3)$$

where  $m=1,2,\dots,M$ .

The power of the  $m^{\text{th}}$  harmonic at  $f_k$  may disperse over a frequency band around the  $f_k$  due to the spectral leakage. Hence, the total power of harmonics within the adjacent frequencies around  $f_k$  can be restored into a "group power" [13]. Each "group power", i.e.,  $P_m^*[f_k]$ , can be collected between  $f_{k-\Delta k}$  and  $f_{k+\Delta k}$  as follows.

$$P_m^*[f_k] = \sum_{\Delta k=-\tau}^{+\tau} (A_m[f_{k+\Delta k}])^2 \quad (4)$$

where  $\tau$  is an integer number and denotes the group bandwidth.

Each harmonic amplitude can be estimated as

$$A_m^*[f_k] = \sqrt{P_m^*[f_k]} \quad (5)$$

### III. THE GROUP-HARMONIC ENERGY DISTRIBUTION ALGORITHM

#### III.1 THE RELATION BETWEEN SAMPLING POINT AND HARMONIC VALUES

The power line waveform  $s(t)$  (voltage/current) is sampled using the sampling rate  $f_s (= 1/T_s)$ , which has the fundamental frequency  $f_d$  and its respective amplitude  $A_d$ , as follows.

$$s(n) = s(t) \Big|_{t=nT_s}, \quad n = 0,1,2,\dots,N-1 \quad (6)$$

where  $N$  is the sampled point of Fourier fundamental period  $T_f$ .

In general, the distorted signal can be composed of three parts, as follows.

$$s(n) = s_d(n) + s_h(n) + s_i(n) \quad (7)$$

where  $s_d(n)$  is the fundamental component,  $s_h(n)$  is the harmonic components, and  $s_i(n)$  represents the interharmonic components.

Due to possible fundamental frequency drift, it may determine the new  $\Delta f' = \frac{f_s}{N'}$  to find the correct fundamental frequency  $f_d'$  and its respective amplitude  $A_d'$ . Accordingly, the fundamental frequency signal  $s_d'(n)$  and its harmonic signals  $s_h'(n)$  can be obtained, as follows.

$$s'(n) = s(t) \Big|_{t=\frac{n}{f_s}} = s_d'(n) + s_h'(n) + s_i'(n), \quad n = 0,1,2,3,\dots,N'-1 \quad (8)$$

Similarly, the same concept as above can be applied to the interharmonics evaluation in the distorted signal  $s(n)$ , assuming the major interharmonic component as the fundamental one and neglecting  $s_d(n) + s_h(n)$ , shown in equ. (9). Therefore, the individual major interharmonic frequency  $f_{ij}'$  and its respective amplitude  $A_{ij}'$  can be found.

$$s''(n) = s(n) - [s_d(n) + s_h(n)] = s_i(n) = \sum_{j=1}^m s_{ij} \quad (9)$$

where  $m$  denotes the number of major interharmonics.

This proposed PGER method extends the "group" concept that has been mentioned by IEC 61000-4-7 and some papers [22-25].

#### III.2 THE PROPOSED PGER ALGORITHM

The total dispersed energy, i.e.,  $P_m^{**}[f_k]$ , around the dominant frequency is defined as

$$P_m^{**}[f_k] = \sum_{\Delta k=-\tau}^{+\tau} (A_m[f_{k+\Delta k}])^2 - (A_m[f_k])^2 \quad (10)$$

where it denotes the dispersed bandwidth energy, excluding the dominant component.

Based on the concept of group-harmonic power as above, once the total dispersed energy is collected into the dominant frequency, the harmonic amplitude  $A_m^*[f_k]$  can be thus recovered to the actual one.

$$A_m^*[f_k] = \sqrt{P_m^{**}[f_k] + (A_m[f_k])^2} \quad (11)$$

In order to track time-varying harmonics, the waveform is sampled using the proposed piecewise-overlapped method. The depict of the sequent waveform sampling mechanism with this method is shown in Fig. 1. The sampling period is set as  $T_f$ , where the number  $n(=1,2,3,4,\dots)$  denotes the sampling sequence with a  $T_f$  time period, and next number, i.e.,  $n+1$ , has a overlapped period ( $T_o$ ) with the current number  $n$ .

The overlapped percentage  $\eta$  is defined as the ratio of overlapped period to the sampling sequence as follows.

$$\eta = \frac{\text{overlapped period } (T_o)}{\text{period of } T_f} \quad (\%) \quad (12)$$

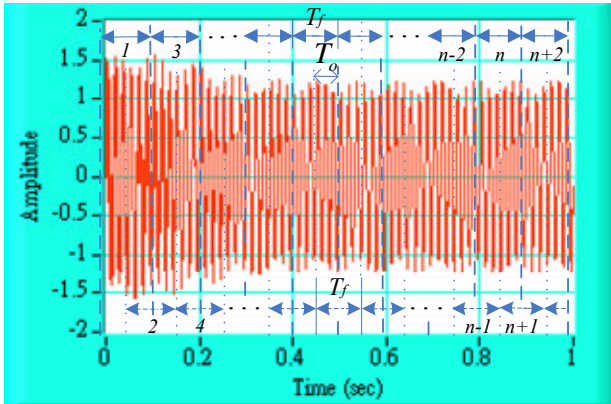


Fig. 1 Depict of piecewise-overlapped sampling period

To find each harmonic/interharmonic amplitude, the flowchart of the scheme procedure is given in Fig. 2, briefly introduced as follows.

- (1) Set  $f_s = 5\text{kHz}$ ,  $N=1000$ ,  $\tau = 3$ ,  $\eta = 50\%$ ,  $P_{min} = 0.0001$ .
- (2) Sampling the line signal  $i_s(t)$ .
- (3) Implement DFT.
- (4) Determine the number ( $M$ ) of major harmonics/interharmonics, and set  $m=1$ .
- (5) Implement DFT.
- (6) If  $A_m[f_{k+1}] > A_m[f_{k-1}]$ ,  $N=N-1$ . Otherwise, go to next step.
- (7) If  $A_m[f_{k+1}] < A_m[f_{k-1}]$ ,  $N=N+1$ . Otherwise, go to next step.

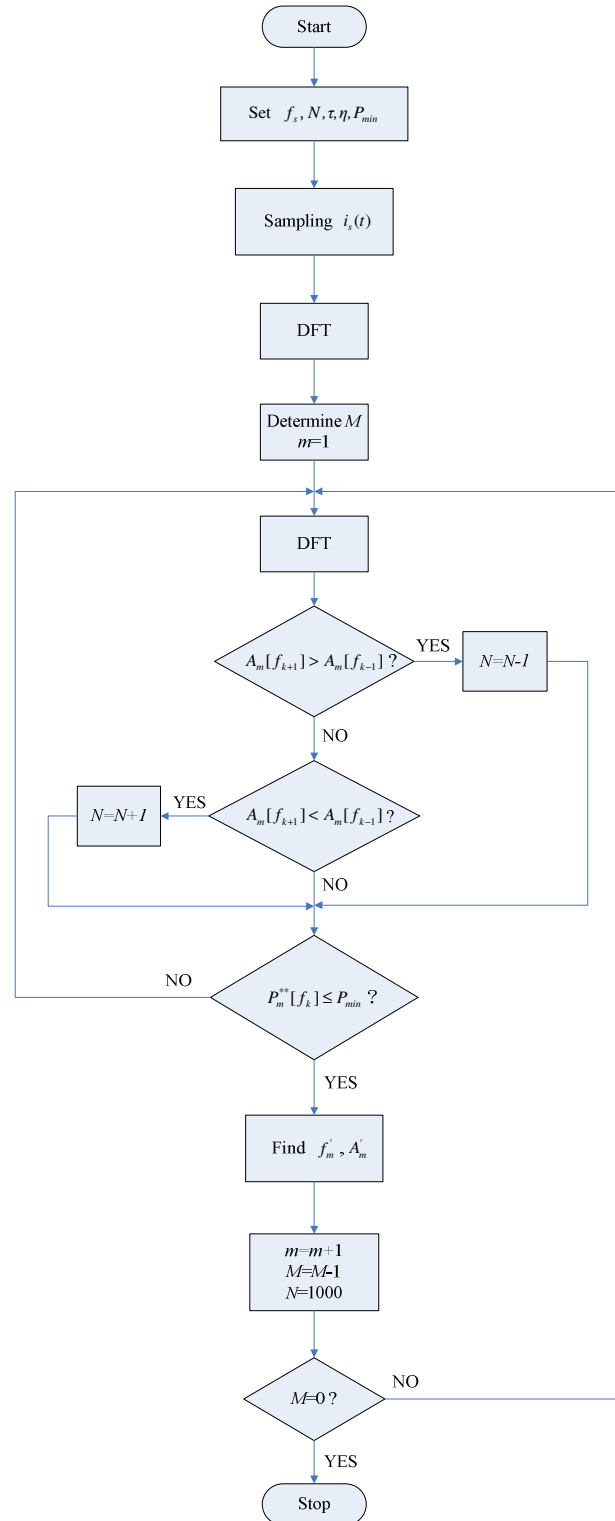


Fig. 2 The flowchart of the proposed PGER Algorithm

- (8) Check if  $P_m^{**}[f_k] \leq P_{min}$ . If yes, the iteration loop stops and determine the updated  $N$ , i.e.,  $N'$ . The major

harmonic/interharmonic frequency  $f'_m$  and amplitude  $A'_m$  including the fundamental one can be thus obtained. Otherwise, go back to Step (5) to repeat the procedure until  $P_m^{**}[f_k] \leq P_{min}$ .

- (9) Let  $m=m+1$ , and  $M=M-1$ , and  $N=1000$ .
- (10) Check if  $M=0$ . If yes, the iteration loop stops. Otherwise, go back to Step (5). Note that this iteration loop will continue until each major harmonic/interharmonic frequency  $f'_m$  and amplitude  $A'_m$  is found.

#### IV. MODEL VALIDATION WITH NUMERICAL EXAMPLES

##### IV.1 ANALYSIS OF STATIONARY WAVEFORM

The proposed PGER algorithm has been tested by the synthesized line signal (voltage/current) to verify the effectiveness of harmonic/interharmonic analysis. The following example is used to illustrate the harmonic analysis of a distorted waveform, where similar examples were applied for verification on several previous publications such as [22-24].

$$s(t) = \sin(2\pi f_d t + 52^\circ) + 0.25 \sin(2\pi \cdot 3 \cdot f_d \cdot t - 41^\circ) + 0.2 \sin(2\pi \cdot 5 \cdot f_d \cdot t + 32^\circ) + 0.22 \sin(2\pi \cdot 129 \cdot t - 54^\circ) + 0.1 \sin(2\pi \cdot 457 \cdot t + 89^\circ) \quad (13)$$

where  $f_d = 59.48$  Hz is the fundamental frequency.

Generally, the system frequency drift is a concern in energy systems because it may vary slightly from time to time due to the change of system loads. This effect indeed influences the traditional DFT spectrum analysis. As above, the line signal has a fundamental frequency, i.e., 59.48 Hz, with 0.52 Hz drift and a scaled amplitude of 1V. The 3<sup>rd</sup> and 5<sup>th</sup> harmonic components are included in the synthesized waveform to present a possible distorted waveform situation. Non-integer components, i.e., interharmonics, such as 129 Hz and 457 Hz are considered, reflecting a possible polluted line case.

Step (a): Measurement of fundamental and integer harmonics with a 0.52 Hz frequency drift

In this case, the fundamental frequency component including 3<sup>rd</sup> harmonic and 5<sup>th</sup> harmonic is considered to have a 0.52 Hz variation. The dispersed power of the harmonics over around the frequency band is significantly reduced from 0.023 to 0.0002 within only 9 iteration loops, shown in Fig. 3. Fig. 4 indicates that each harmonic is approaching toward its true amplitude step by step. The amplitudes of fundamental, third and fifth component are thus obtained as 1.0, 0.22 and 0.14 at the sixth iteration loop from 0.99, 0.22 and 0.14 at the first iteration loop, respectively. Also, the fundamental frequency is found as 59.48 Hz, matching the true one.

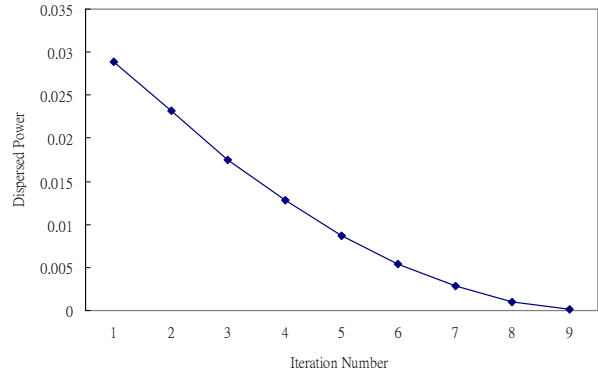


Fig. 3 Convergent curve of the dispersed power at the harmonic components

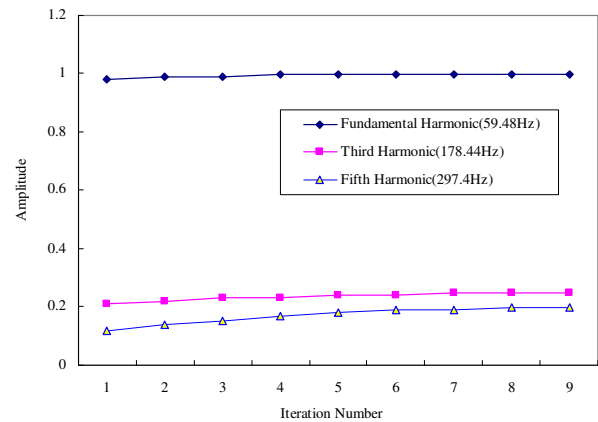


Fig. 4 Amplitude tracking curve of the harmonic components

Step (b): Measurement of the interharmonic at 129 Hz

In this stage, the dispersed power of the supposed fundamental band (interharmonic at 129 Hz) is approaching toward to almost zero from 0.007 within 8 iteration loops, shown in Fig. 5. Accordingly, its amplitude is obtained as 0.22 from 0.19 and the 129 Hz component is therefore confirmed, shown in Fig. 6.

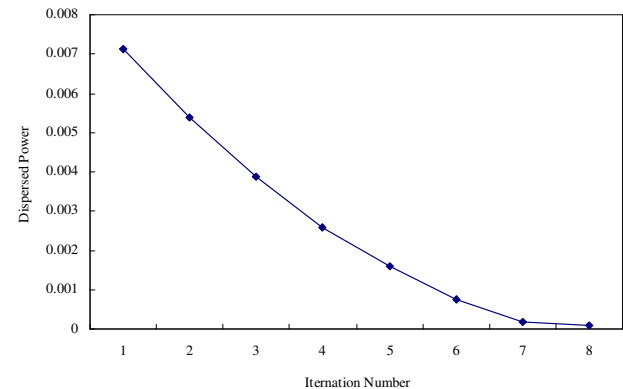


Fig. 5 Convergent curve of the dispersed power at the 129 Hz interharmonic

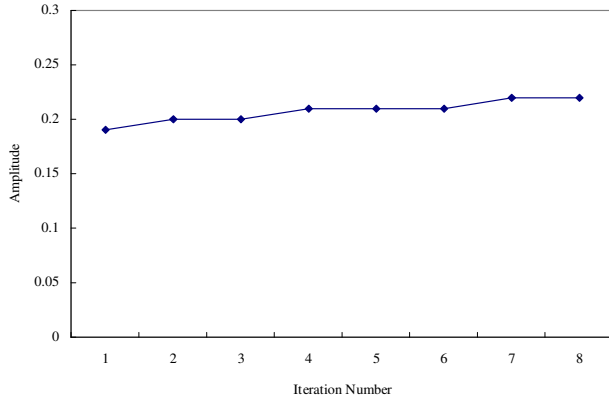


Fig. 6 Amplitude tracking curve of the 129 Hz interharmonic

Step (c): Measurement of the interharmonic at 457 Hz

The dispersed power of the supposed fundamental band (interharmonic at 457 Hz) is going down quickly to almost zero from 0.0065 within only 7 iteration loops, shown in Fig. 7. As a result, its amplitude is obtained as 0.1 from 0.054 and the 457 Hz component is thus confirmed, shown in Fig. 8.

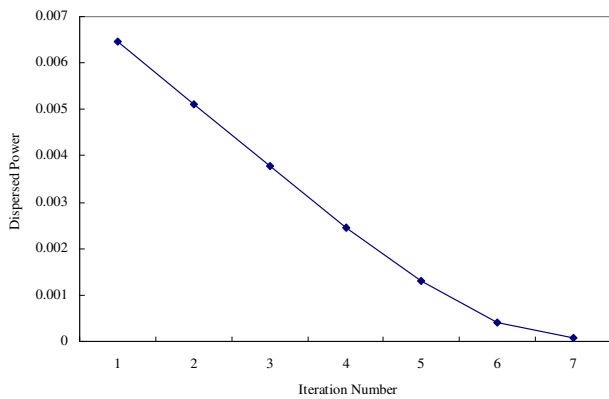


Fig. 7 Convergent curve of the dispersed power at the 457 Hz interharmonic

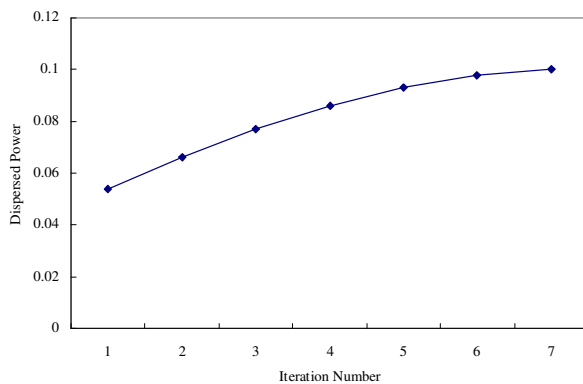


Fig. 8 Amplitude tracking curve of the 457 Hz interharmonic

IV.2 ANALYSIS OF NON-STATIONARY WAVEFORM

The non-stationary waveform that generates time-varying

harmonics is considered in this study. The example waveform of the equ. (14) contains the 5<sup>th</sup> and 7<sup>th</sup> harmonics with different time-decaying constants.

$$s(t) = \sin(2\pi f_1 t) e^{-t/T_{c1}} + 0.5 \sin(2\pi f_2 t) e^{-t/T_{c2}} \quad (14)$$

where  $f_1=300$  Hz,  $T_{c1}=0.2$  sec,  $f_2=420$  Hz,  $T_{c2}=0.1$  sec.

The frequency resolution  $\Delta f$  is the reciprocal of the truncation interval, and proportional to the sampling rate. Fig. 9 indicates the group-harmonic variation against time with appropriate frequency resolution, i.e., 10 Hz. If  $\Delta f = 10$  Hz is chosen, the proposed method are exactly tracking both 5<sup>th</sup> and 7<sup>th</sup> harmonics in a reasonable time. In contrast, a bigger or smaller  $\Delta f$ , e.g. 10 Hz or 50 Hz, may cause the problem from the fact that the adjacent leakages move far away from their central frequencies and involves into other groups of specified harmonics.

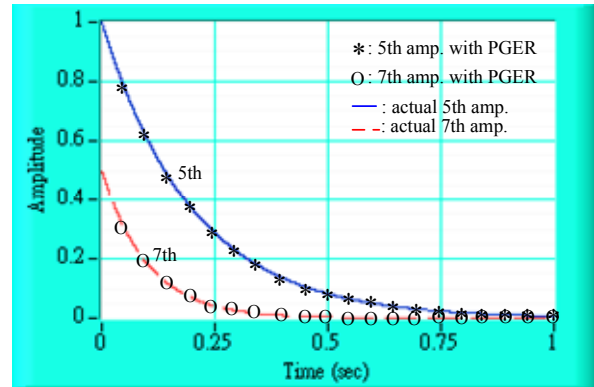


Fig. 9 Non-stationary harmonics tracking with  $\Delta f = 10$ Hz

V. CONCLUSIONS

Although the DFT has certain limitations in the harmonic analysis, it is still widely used in industry today. Based on the concept of group-harmonic energy with suitable sampling window length, most dispersing energy caused by the DFT can be almost restored so that harmonics/interharmonics can be analyzed accurately. Especially, the piecewise-overlapped method integrated with the PGER promises that every time-varying harmonic/interharmonic can be tracked fast. There is no theoretical restriction in the locations of interharmonic components while the group bandwidth ( $\tau$ ) of each harmonic/interharmonic and overlapped percentage ( $\eta$ ) should be chosen appropriately. Moreover, the PGER methodology has been implemented by a LabVIEW programming successfully so that it can be easily extended to other software packages like microprocessor for on-line measurement. Additionally, the proposed PGER can provide an advanced improvement for most measurement devices with some inherent errors because of the spectrum leakages caused by harmonics/interharmonics.

## REFERENCES

- [1] M.B. Rifai, T.H. Ortmeier, and W.J. McQuillan, "Evaluation of current interharmonics from Ac drives", *IEEE Trans. Power Del.*, Vol. 15, No. 3, pp. 1094-1098, Jul. 2000.
- [2] Hao Qian, Rongxiang Ahaio, and Tong Chen, "Interharmonics Analysis Based on Interpolating Windowed FFT Algorithm", *IEEE Trans. Del.*, Vol. 22, No. 2, pp. 1064-1069, April 2007.
- [3] Quanming Zhang, Huijin Liu, Hongkun Chen, Qionglin Li, and Zhenhuan Zhang, "A Precise and Adaptive Algorithm for Interharmonics Measurement Based on Iterative DFT", *IEEE Trans. Del.*, Vol. 23, No.4, pp. 1728-1735, Oct. 2008.
- [4] G.W. Chang, C.Y. Chen, and M.C. Wu, "A Modified Algorithm for Harmonics and Interharmonics Measurement", 2007 IEEE Power Engineering Society General Meeting, pp. 1-5, 24-28 June 2007.
- [5] Testa, A.; Akram, M.F.; Burch, R.; Carpinelli, G.; Chang, G.; Dinavahi, V.; Hatziadoniu, C.; Grady, W.M.; Gunther, E.; Halpin, M.; Lehn, P.; Liu, Y.; Langella, R.; Lowenstein, M.; Medina, A.; Ortmeier, T.; Ranade, S.; Ribeiro, P.; Watson, N.; Wikston, J.; Xu, W., "Interharmonics: Theory and Modeling", *IEEE Trans. Del.*, Vol. 22, No.4, pp. 2335-2348, Oct. 2007.
- [6] D.E. Steeper and R.P. Stratford, "Reactive Compensation and Harmonic Suppression for Industrial Power Systems Using Thyristor Converters", *IEEE Trans. on Industry Applications*, Vol. IA-12(3), pp.232-254, May/June 1976.
- [7] K.H. Sueker, S.D. Hummel, and R.D. Argent, "Power Factor Correction and Harmonic Mitigation in a Thyristor Controlled Glass Melter", *IEEE Trans. on Industry Applications*, Vol. 25(6), pp.972-975, Nov./Dec. 1989.
- [8] H.C. Lin, "Fast Tracking of Time-varying Power System frequency and Harmonics using Iterative-loop Approaching Algorithm", *IEEE Trans. on Industrial Electronics*, Vol. 54, No. 2, pp.974-983, April 2007.
- [9] D. Gallo, R. Langella, and A. Testa, "Interharmonics, Part 1: Aspects Related to Modeling and Simulation", Sixth International Workshop on Power Definitions and Measurements under Non-Sinusoidal Conditions, Milano, 13-15, pp.168-173, Oct. 2003.
- [10] D. Gallo, R. Langella, and A. Testa, "Interharmonics, Part 2: Aspects Related to Measurement and Limits", Sixth International Workshop on Power Definitions and Measurements under Non-Sinusoidal Conditions, Milano, 13-15, pp.174-181, Oct. 2003.
- [11] Barros, J., Prez, E., Pigazo, A., Diego, R.I., "Simultaneous measurement of harmonics, interharmonics and flicker in a power system for power quality analysis", 2002. Fifth International Conference on Power System Management and Control, pp.100-105, 17-19 April 2002.
- [12] Masoud Karimi-Ghartemani and M. Reza Iravani, "Measurement of harmonics/inter-harmonics of time-varying frequency", *IEEE Trans. on Power Delivery*, Vol. 20, No. 1, pp. 23-31, Jan. 2005.
- [13] C.S. Moo and Y.N. Chang, "Group-harmonic identification in power systems with nonstationary waveforms", *IEE Proc.-Gener. Transm. Distrib.*, Vol. 142, No. 5, pp. 517-522, Sep. 1995.
- [14] H.C. Lin, "Intelligent neural network based adaptive power line conditioner for real-time harmonics filtering", *IEE Proc.-Gener. Transm. Distrib.*, Vol. 151, No. 5, pp.561-567, Sep. 2004.
- [15] Kwok, H.K.; Jones, D.L. "Improved instantaneous frequency estimation using an adaptive short-time Fourier transform", *IEEE Trans. on Signal Processing*, Vol. 48, Issue 10, pp.2964-2972, Oct. 2000.
- [16] Soliman, S.A.; Alammari, R.A.; El-Hawary, M.E.; Mostafa, M.A.; "Effects of harmonic distortion on the active and reactive power measurements in the time dominant: a single phase system", 2001 IEEE Power Tech Proceedings, Vol. 1, Page(s):6, 10-13 Sep. 2001.
- [17] M. Bettayeb and U. Qidwai, "Recursive estimation of power system harmonics", *Elect. Power Syst. Res.*, Vol. 47, pp. 143-152, 1998.
- [18] Al-Kandari, A.; El-Naggar, K.M., "Time dominant modeling and identification of nonlinear loads using discrete time-filtering estimator", 2003 IEEE Transmission and Distribution Conference and Exposition, Vol. 1, pp.126-131, 7-12 Sep. 2003.
- [19] A.A. Girgis, W.B. Chang, and E.B. Makram, "A digital recursive measurement scheme for on-line tracking of power system harmonics", *IEEE Trans. on Power Delivery*, Vol. 6, No. 3, pp. 1153-1160, Jul. 1991.
- [20] Macias, J.A.; Gomez, A.; "Self-Tuning of Kalman Filters for Harmonic Computation", *IEEE Trans. on Power Delivery*, Vol. 21, Issue 1, pp.501-503, Jan. 2006.
- [21] H.C. Lin, "Intelligent Neural Network based Fast Power System Harmonic Detection", *IEEE Trans. on Industrial Electronics*, Vol. 54, No. 1, pp.43-52, Feb. 2007.
- [22] H.C. Lin, "Inter-Harmonic Identification using Group-Harmonic Weighting Approach Based on the FFT", *IEEE Transactions on Power Electronics*, Vol. 23, Issue 3, May 2008, pp. 1309-1319.
- [23] H.C. Lin, "Power Harmonics and Interharmonics Measurement using Recursive Group-harmonic Power Minimizing Algorithm", *IEEE Transactions on Industrial Electronics*, Vol. 59, No. 2, pp. 1184-1193, Feb. 2012.
- [24] H.C. Lin, "DFT-based recursive group-harmonic energy distribution approach for power interharmonic identification", *Computers and Mathematics with Applications*, Vol. 64, No. 5, pp. 1128-1139, Sep. 2012.
- [25] IEC 61000-4-7: "Testing and measurement techniques: Harmonics and interharmonics: General guide on harmonics and interharmonics measurements and instrumentation for power supply systems and equipment connected thereto", 2002.