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# A moving horizon approach to multivariable input design in general linear systems with constraints

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**Abstract:**
The quality of a model determines the closed loop performance of model predictive controllers. However, identification of high quality multivariable models is a time and energy intensive exercise. The industrial model predictive controllers are designed using large dimensional multivariable models and they are often identified using ad-hoc single input bump tests. A novel multivariable input design approach is developed using a modified model predictive control objective function. It is shown that the proposed input design approach is trace optimal with respect to the covariance of model parameters. The approach is shown to work well in closed loop on both well and ill-conditioned processes even under model-plant mismatch while meeting input and output constraints.

*Keywords:* Input design, closed loop systems, model predictive control

## 1. INTRODUCTION

Model predictive control is a popular multivariable controller that is widely used in processes with constrained variables. Standard software and communication protocols allow fast deployment of advanced control applications. However, over the last four decades since the invention of model predictive control, the basic approach to step testing and modelling has largely remained the same. The step tests are done by manipulating one input variable at a time. This approach is 'optimal' only if the model is diagonal and otherwise optimal inputs typically are correlated and have to be obtained by solving a covariance minimization problem (Koung and MacGregor (1994)). There have been some recent advances on multivariable input design and closed loop identification, however, the industrial applications have lagged behind due to: (a) lack of off-the-shelf tools, and (b) the interruption that all input design algorithms cause to an operating process.

The performance of the model predictive controllers depends critically on the quality of the estimated dynamic models. Industrial experience has shown that the most challenging and time-consuming task in an MPC commissioning project is that of step testing and model identification. A traditional approach to step testing would involve a control engineer who spends many shifts in the control room operating the plant in open loop. Additionally, during MPC maintenance phase, the main task is often model re-identification. A traditional model identification test on a refinery unit, such as the crude unit, can take several weeks. The quality of collected data depends primarily on the experience of the control engineer. After the test, it can take significant time to analyze the data and to identify appropriate models. Currently available modeling tools involve significant amounts of trial and error to make the models conform to the industrial data. At the end of the modeling exercise, the control engineer is left with, at best, an intuitive feel for the fidelity of the individual models. Since the models form the heart of any MPC application, it is critical that the project team has confidence in the models before deploying them online.

There is a growing demand for more efficient model identification methods that reduce duration of plant tests, the time needed for model identification and the disturbances to optimal operation of the plant during the test. The quality of models estimated and the efficiency of these identification methods depend on the choice of input during the plant test and whether the test is done under open or closed loop conditions. Multiple technology vendors have started offering closed loop step testing packages as part of their advanced control portfolio. The closed loop approach to step testing and modeling is still in its early stages of adoption in the industry.

There is extensive literature on designing inputs for linear processes Goodwin and Payne (1977); Ljung (1999); Hjalmarsson (2005); Jansson and Hjalmarsson (2005); Qin (2006). While a large portion of the literature focusses on input design under open loop, there have also been some attempts to articulate the need for closed loop identification and its relevance to high performance controllers in general Van Den Hof and Schrama (1995); Gopaluni et al. (2003, 2002); Forssell and Ljung (1999, 2000). These traditional approaches to input design are based on finding an optimal input sequence that minimizes a function of the parameter covariance matrix. Consequently, there are a few common challenges to implementation of these input design algorithms: (a) the optimization problem involved is often nonconvex, (b) the optimal input depends on the "true" process model, and (c) the input and output constraints are not explicitly accounted. To the best knowledge of the authors, Cooley and Lee (2001) and Jansson and Hjalmarsson (2005) are some of the few articles that attempt to formulate a convex input design optimization problem and account for constraints. Bruwer and MacGregor (2006) emphasizes the role of weak gain directions on closed loop performance and propose systematic ways of ensuring sufficient excitation along those directions. It also states that in the closed loop case orthogonal excitation along the setpoints is adeuqate to guarantee excitation along all the relevant gain directions

In this work, we explore a novel approach to the generation of an information rich test signal relevant to MPC applications. The idea of using a model predictive framework based on the current controller model is formulated to calculate a set of moves that maximize the output (controlled variable) variability to the extent allowed by the process constraints. The test moves are then implemented in receding horizon manner, i.e., the first step or move is implemented and the entire sequence recalculated at the next sampling instant. We show that this approach is equivalent to designing a T-optimal (trace optimal) input for autoregressive exogenous (ARX) input models. The application of the model predictive input design approach is demonstrated on two examples - 1) on an illconditioned process and 2) on a process involving model plant mismatch.

This approach has numerous advantages: (a) the input is designed by solving a convex optimization problem, (b) the receding horizon nature of the algorithm ensures that the "true" process model is not needed, (c) the input and output constraints are explicitly included in the input design optimization problem, (d) the plant tests are done in closed loop, this ensures that both the strong and weak gain directions have adequate excitation, and (e) the implementation in off-the-shelf MPC technology is rather straightforward.

## 2. MODEL PREDICTIVE CONTROL

The basic philosophy behind model predictive control is to minimize a weighted sum of control errors and rate of input change over a prediction horizon at every sample instant. This minimization is performed while meeting input and output constraints. The input thus generated is implemented in a receding horizon fashion so as to account for unmodelled dynamics and unmeasured disturbances. Mathematically, the following quadratic objective is minimized at every sample instant to calculate a sequence of future input moves,

$$J_k = (\mathbf{r}_k - \mathbf{y}_k)^T \boldsymbol{\Gamma} (\mathbf{r}_k - \mathbf{y}_k) + \Delta \mathbf{u}_k^T \boldsymbol{\Lambda} \Delta \mathbf{u}_k \tag{1}$$

where

$$\begin{aligned}
\mathbf{r}_k &= \begin{bmatrix} \mathbf{r}_{1,k}^T & \mathbf{r}_{2,k}^T & \cdots & \mathbf{r}_{n_y,k}^T \end{bmatrix}^T \in \mathbb{R}^{Pn_y \times 1} \\
\mathbf{r}_{i,k} &= \begin{bmatrix} r_{i,k+1} & r_{i,k+2} & \cdots & r_{i,k+P} \end{bmatrix}^T \in \mathbb{R}^{P \times 1} \\
\mathbf{y}_k &= \begin{bmatrix} \mathbf{y}_{1,k}^T & \mathbf{y}_{2,k}^T & \cdots & \mathbf{y}_{n_y,k}^T \end{bmatrix}^T \in \mathbb{R}^{Pn_y \times 1} \\
\mathbf{y}_{i,k} &= \begin{bmatrix} y_{i,k+1} & y_{i,k+2} & \cdots & y_{i,k+P} \end{bmatrix}^T \in \mathbb{R}^{P \times 1} \\
\Delta \mathbf{u}_k) &= \begin{bmatrix} \Delta \mathbf{u}_{1,k}^T & \Delta \mathbf{u}_{2,k}^T & \cdots & \Delta \mathbf{u}_{n_u,k}^T \end{bmatrix}^T \in \mathbb{R}^{Mn_u \times 1} \\
\Delta \mathbf{u}_i &= \begin{bmatrix} \Delta u_{i,k+1} & \Delta u_{i,k+2} & \cdots & \Delta u_{i,k+M} \end{bmatrix}^T \in \mathbb{R}^{M \times 1}.
\end{aligned}$$

In the above formulation, $P$ and $M$ are the prediction and control horizons, respectively, and $\boldsymbol{\Gamma}$ and $\boldsymbol{\Lambda}$ are the output and input weighting matrices. $r_{i,k}$, $y_{i,k}$, and $u_{i,k}$ denote the $i$th set points, outputs and inputs at the sampling instant $k$. The number of inputs and outputs are denoted by $n_u$ and $n_y$, respectively. $\Delta$ is the difference operator. Clearly, at a given sample time $k$ the objective function requires future values of output. An estimate of future values of output is obtained using a model. The objective function is often minimized subject to process operating constraints. The following standard optimization problem is solved at every sampling instant,

$$\begin{aligned}
\underset{\Delta \mathbf{u}_k}{\text{minimize}} \quad & J_k \\
\text{subject to} \quad & \mathbf{y}_L \le \mathbf{y}_k \le \mathbf{y}_H \\
& \mathbf{u}_L \le \mathbf{u}_k \le \mathbf{u}_H \\
& \Delta \mathbf{u}_L \le \Delta \mathbf{u}_k \le \Delta \mathbf{u}_H
\end{aligned}$$

where $()_L$ and $()_H$ are lower and upper bounds on the corresponding variables. One of the advantages of MPC is its ability to formulate the input and output constraints in a consistent way and ensure that they are satisfied in any process situation.

The standard industrial practice in building a model for MPC is to perform single input bump tests. Given that the typical MPC models are as large as $100 \times 100$, such an approach will be time and energy intensive. Moreover, it is not possible to guarantee that the inputs and outputs meet the process constraints during the tests. Single input bump tests are also known to not provide sufficient excitation if the process is ill-conditioned. In the next section a novel input design approach is proposed and shown to address some of the pitfalls of the ad hoc industrial approach.

## 3. MOVING HORIZON INPUT DESIGN

### *3.1 The Approach*

The quality of an estimated model depends on the "information content" in the input-output data and therefore it is important to choose the inputs carefully and maximize the information content. There are different matrices to determine the information content and the quality of a model. An optimal input is one that maximizes the information content for a given metric of the model quality. The model quality metric depends not only on the structure of the model but also on the controller being used. For instance, a model that can provide good multi-step ahead predictions is known to provide better closed loop performance with MPC. The input design for a multivariable process is often based on different principles than that for a univariate process, especially when the process is ill-conditioned Koung and MacGregor (1994).

The information content is often measured by the condition number of a particular matrix consisting of inputs and outputs. Shouche et al. (1998) had taken the approach of imposing the condition number of the information matrix as an explicit constraint in the MPC objective function. This approach ensures that the designed input is capable of exciting the process to the extent permitted by the constraints. Rivera et al (..) attempt to use the predictive capability of the existing process model to design an input sequence that generates information without sacrificing the process constraints. The objective in this article is to design an input that is commensurate with the MPC objective function and maximize the information content while maintaining the process constraints.

A reformulation of the MPC optimization problem is proposed below to calculate a sequence of input moves that are capable of generating an information rich data set while maintaining the process constraints. As in MPC, a receding horizon approach is taken to account for model-plant mismatch and unmeasured disturbances. The reformulated optimization problem retains the structure of the original MPC optimization problem with the only difference being the maximization of the MPC objective function rather than minimization. The idea behind this formulation is that by maximizing the MPC objective function one can maximize the variance in the inputs while still keeping the process within the constraints. The inputs are generated by solving the following optimization problem

$$\begin{aligned} \underset{\Delta \mathbf{u}_k}{\text{maximize}} \quad & J_k \\ \text{subject to} \quad & \mathbf{y}_L \leq \mathbf{y}_k \leq \mathbf{y}_H \\ & \mathbf{u}_L \leq \mathbf{u}_k \leq \mathbf{u}_H \\ & \Delta\mathbf{u}_L \leq \Delta\mathbf{u}_k \leq \Delta\mathbf{u}_H. \end{aligned}$$

As in MPC, at each sample instant a sequence of inputs are generated but only the first input is applied to the process and the remaining inputs are discarded. While this intuitive approach to input design is appealing it is shown in the next section that this approach in fact is trace optimal with respect to the parameter covariance matrix.

### 3.2 T-Optimality

There are different metrics for input optimality. The most popular among them is a metric on covariance of the parameter matrix. The intuitive argument presented in the previous section for the new input design method can be justified by showing that the MPC objective $J_k$ is in fact proportional to the trace of inverse of parameter covariance matrix (in other words trace ($T$) of the information matrix) for certain model structures. Therefore the proposed input design method is $T$-optimal.

*ARX Model*

The covariance matrix of the parameters in a linear model is a function of the input-ouput data and the noise covariance matrix. Finite impulse response (FIR) models and Autoregressive Exogenous input (ARX) models are often used in MPC. The special structure of FIR and ARX models lends itself to formulate a least squares problem for parameter estimation. Therefore, it is straightforward to analytically derive an expression for the covariance matrix and hence the information matrix for these model structures. These model structures are not only theoretically appealing but are also often used by practicing engineers. Modern identification methods often rely on initial estimation of high order ARX models followed by model order reduction for compatibility with the intended MPC or PID application Zhu (2001).

Consider a multivariable ARX model of the following form (for the $i$ th output)

$$\begin{aligned} A_i(q)y_{i,k} = B_{i1}(q)\Delta u_{1,k} + B_{i2}(q)\Delta u_{2,k} + \cdots \\ + B_{im}(q)u_{n_u,k} + e_{i,k} \end{aligned}$$

where $A_i(q)$ is a polynomial of order $n_i$ for $i = 1$ to $n_y$ and $B_{ij}$ is a polynomial of order $m_{ij}$ for $j = 1$ to $n_u$. Assume that the noise sequences $e_{i,k}$ are independent. The $A_i(q)$ and $B_{ij}(q)$ polynomials are of the form

$$\begin{aligned} A_i(q) &= a_i^{(0)} + a_i^{(1)}q^{-1} + \cdots + a_i^{(n_i)}q^{-n_i} \\ B_{ij}(q) &= b_{ij}^{(0)} + b_{ij}^{(1)}q^{-1} + \cdots + b_{ij}^{(m_{ij})}q^{-m_{ij}} \end{aligned}$$

where $a_i^{(.)}$ and $b_{ij}^{(.)}$ are coefficients of the respective polynomials. These coefficients can be easily estimated from data by solving a simple least squares problem Ljung (1999). Note that the ARX model uses differenced input. Let us consider a time period from $k$ to $k+N$, where $N$ denotes the number of samples considered. By stacking the outputs during this period and expanding the corresponding ARX models, we can write

$$\mathbf{y}_i = \mathbf{Z}_i\theta_i + \mathbf{e}_i \tag{2}$$

where $\mathbf{y}_i = [y_{i,k} \ \cdots \ y_{i,k+N}]^T$, $\mathbf{Z_i}$ is a corresponding data matrix obtained using the $i$th ARX equation and $\theta_i$ is a vector of corresponding parameters in $A_i(q)$ and $B_{ij}(q)$. Similarly $\mathbf{e}_i = [e_{i,k} \ \cdots \ e_{i,k+N}]^T$. Now stacking together similar linear equations for each output, we can create the following set of equations

$$\mathbf{y} = \mathbf{Z}\theta + \mathbf{e}$$

where $\mathbf{y} = [\mathbf{y_1}^T \ \mathbf{y_2}^T \ \cdots \ \mathbf{y_{n_y}}^T]^T$, $\theta = [\theta_\mathbf{1}^T \ \theta_\mathbf{2}^T \ \cdots \ \theta_\mathbf{n_y}^T]^T$, $\mathbf{Z} = diag(\mathbf{Z_1}, \mathbf{Z_2}, \cdots, \mathbf{Z_{n_y}})$ [1], $\theta = [\theta_\mathbf{1}^T \ \theta_\mathbf{2}^T \ \cdots \ \theta_\mathbf{n_y}^T]^T$ and $\mathbf{e} = [\mathbf{e_1}^T \ \mathbf{e_2}^T \ \cdots \ \mathbf{e_{n_y}}^T]^T$. The least squares solution to the parameter vector, $\theta$ is given by

$$\hat{\theta} = (\mathbf{Z}^T\mathbf{Z})^{-1}\mathbf{Z}^T\mathbf{y} \tag{3}$$

and the corresponding variance of the estimated parameters, $\hat{\theta}$ is proportional to Ljung (1999)

$$cov(\hat{\theta}) \propto (\mathbf{Z}^T\mathbf{Z})^{-1}. \tag{4}$$

The matrix $\mathbf{F} := (\mathbf{Z}^T\mathbf{Z})$ is also called Fisher information matrix and is inversely proportional to the parameter covariance matrix. Inputs for system identification are often

[1] $diag(.)$ is used to denote a matrix obtained by stacking its arguments along the diagonal

designed by minimizing some function of this parameter covariance matrix. For instance, we can minimize the trace (A - optimal design), eigenvalue (E - optimal), or determinant (D - optimal) of the covariance matrix. The *minimization* of the inverse of the covariance matrix often is a nonlinear and complex function of the inputs and therefore not amenable to convex optimization techniques. Instead, *maximization* of a function of the information matrix tends to be convex problem. The following proposition shows that the maximization of the trace of information matrix (also called T - optimal design) is equivalent to maximization of the MPC objective function under some mild technical constraints.

*Proposition 1.* The MPC objective $J_k$ as defined in (1) is proportional to $trace(\mathbf{F})$ for ARX models with the following choice of prediction horizons,

$$P = N - \max_i(n_i) \tag{5}$$

$$M = N - \max_{i,j}(m_{ij}) \tag{6}$$

with appropriate input and output weights.

A proof is not presented due to lack of space but the idea behind the proof is to rearrange the terms in the covariance matrix in (4) in the form of the MPC objective function.

*Box-Jenkins Model*

The arguments developed for showing $T$-optimality of the covariance of an ARX model can be extended to that of a more general Box-Jenkins model. Consider a standard SISO Box-Jenkins model of the following form (for national simplicity a SISO model is considered but extension to MIMO model is rather straightforward),

$$y_k = \frac{B(q,\theta)}{A(q,\theta)}u_k + \frac{C(q,\theta)}{D(q,\theta)}e_k \tag{7}$$

The corresponding one step ahead prediction error is Gopaluni et al. (2004)

$$\varepsilon_k(\theta) = G(q,\theta)u_k + H(q,\theta)y_k \tag{8}$$

where $G(q,\theta) = -\frac{D(q,\theta)B(q,\theta)}{C(q,\theta)A(q,\theta)}$ and $H(q,\theta) = \frac{D(q,\theta)}{C(q,\theta)}$. Assume that $G(q,\theta_1)$ and $H(q,\theta_2)$ are parametrized independently with $\theta_1$ and $\theta_2$ such that $\theta = [\theta_1 \quad \theta_2]$. Then the information matrix of a standard maximum likelihood approach to estimate the parameters is given by

$$\mathbf{F} = \mathbb{E}\left(\frac{\partial}{\partial\theta}\log\mathcal{L}(\theta|\mathbf{y})\right)^2 \tag{9}$$

where $\log\mathcal{L}(\theta|\mathbf{y})$ is the log-likelihood function given by

$$\log\mathcal{L}(\theta|\mathbf{y}) = -\frac{1}{2N}\log 2\pi\sigma - \frac{1}{2\sigma^2}\sum_{k=1}^{N}\varepsilon_k^2(\theta) \tag{10}$$

Assuming ergodicity of the prediction errors the information matrix at time $k$ can be approximated for sufficiently large $N$ by

$$\mathbf{F} \approx \sum_{k=1}^{N}\begin{pmatrix} \left(\frac{\partial}{\partial\theta_1}\varepsilon_k(\theta)\right)\left(\frac{\partial}{\partial\theta_1}\varepsilon_k(\theta)\right)^T & \left(\frac{\partial}{\partial\theta_1}\varepsilon_k(\theta)\right)\left(\frac{\partial}{\partial\theta_2}\varepsilon_k(\theta)\right)^T \\ \left(\frac{\partial}{\partial\theta_2}\varepsilon_k(\theta)\right)\left(\frac{\partial}{\partial\theta_1}\varepsilon_k(\theta)\right)^T & \left(\frac{\partial}{\partial\theta_2}\varepsilon_k(\theta)\right)\left(\frac{\partial}{\partial\theta_2}\varepsilon_k(\theta)\right)^T \end{pmatrix} \tag{11}$$

Therefore the $T$-optimal input design objective function is

$$\begin{aligned} trace(\mathbf{F}) &\approx \sum_{k=1}^{N} trace\left(\frac{\partial}{\partial\theta_1}\varepsilon_k(\theta)\right)\left(\frac{\partial}{\partial\theta_1}\varepsilon_k(\theta)\right)^T \\ &\quad + trace\left(\frac{\partial}{\partial\theta_2}\varepsilon_k(\theta)\right)\left(\frac{\partial}{\partial\theta_2}\varepsilon_k(\theta)\right)^T \qquad (12) \\ &\approx \sum_{k=1}^{N} trace\left(\frac{\partial}{\partial\theta_1}G(q,\theta_1)u_k\right)\left(\frac{\partial}{\partial\theta_1}G(q,\theta_1)u_k\right)^T \\ &\quad + trace\left(\frac{\partial}{\partial\theta_2}H(q,\theta_2)y_k\right)\left(\frac{\partial}{\partial\theta_2}H(q,\theta_2)y_k\right)^T \\ &\approx \sum_{k=1}^{N}\sum_{j=1}^{M}\alpha_{kj}u_k u_j + \sum_{k=1}^{N}\sum_{j=1}^{M}\beta_{kj}y_k y_j \qquad (13) \end{aligned}$$

for some large $M$ and constant coefficients $\alpha_{kj}$ and $\beta_{kj}$. Assuming that the contributions of the cross correlation between inputs and outputs at nonzero lags is negligible (this assumption can be justified by the fact for a stable system the cross correlation is negligible after a sufficiently large lag),

$$trace(\mathbf{F}) \approx \sum_{k=1}^{\hat{N}}\alpha_{kk}u_k^2 + \sum_{k=1}^{\hat{N}}\beta_{kk}y_k^2 \tag{14}$$

for some sufficiently large $\hat{N}$. Thus the MPC objective function $J_k$ can be made proportional to (14) with appropriate choice of weights.

### *3.3 Merits and Demerits*

The proposed input design approach and the MPC have the same objective function. Consequently, the inputs and outputs are given appropriate weights that are equal. Unlike traditional single input bump tests, this approach automatically provides a multivariate input sequence. The input and output prediction horizons used in MPC and in the input design method are the same and therefore the designed input will be control relevant in the sense of generating an input for multistep ahead predictions. In addition, the input-output constraints are automatically satisfied and therefore the experimentation does not push the process into unacceptable operating regions. The inputs are designed at every sample instant in a deciding horizon fashion and therefore any model-plant mismatch and unmeasured disturbances are automatically accounted for. An important practical advantage is that the proposed approach can be implemented in the industry on an existing MPC installation by simply maximizing the objective subject to the constraints of the controller. With the proposed approach, MPC will work in either control mode or testing mode (when the input is being designed). While the main argument behind this approach is based on MPC, one can easily use this approach on any controller.

Despite the clear advantages of the propose method there are some challenges as well. This approach generates closed loop data and therefore special closed loop identification methods may be required to ensure unbiassed estimates. This approach will not work on a controller with no input-output constraints.

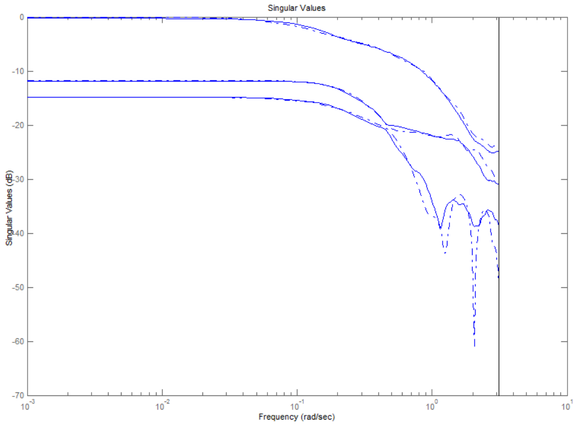


Fig. 1. Comparison of the singular values - estimated model (dash-dotted) with the true system (solid).

## 4. SIMULATION EXAMPLES

### *4.1 Example 1 - Ill Conditioned Process*

The proposed approach is demonstrated here on a high purity distillation column simulation. By its very nature, high purity distillation columns tend to be ill-conditioned from a systems point of view. As such, conventional perturbation methods do not yield expected results for model identification purposes. Moving the process inputs independently leads to inaccurate identification of the weak gain directions - Cooley and Lee (2001); Koung and MacGregor (1994). To design optimal perturbation for these types of systems one has to adopt one of the following approaches: (1) Use a priori knowledge to move inputs in a correlated fashion, the degree of correlation being dependent on the process model, which is often unknown at the identification stage, and (2) conduct the experiment under closed loop conditions and rely on the controller to provide the necessary correlation to identify the strong and weak gain directions accurately. The process model along with its singular values is shown in Figure 1. This is a $2\times 2$ process with the following transfer function,

$$G(s) = \begin{bmatrix} \frac{0.878}{\tau s+1} & -\frac{0.864}{\tau s+1} \\ \frac{1.0819}{\tau s+1} & -\frac{1.0958}{\tau s+1} \end{bmatrix} \quad (15)$$

where $\tau = 194$, and

$$W = \begin{bmatrix} -0.6246 & -0.7809 \\ -0.7809 & 0.6246 \end{bmatrix} \quad V = \begin{bmatrix} -0.7066 & 0.7077 \\ -0.7077 & -0.7066 \end{bmatrix}$$

$$\Sigma = \frac{1}{\sqrt{\tau^2\omega^2+1}} \begin{bmatrix} 1.9721 & 0 \\ 0 & 0.0139 \end{bmatrix}$$

$$\mathrm{cond}(G) = \frac{\sigma_1(\omega)}{\sigma_2(\omega)} = 141.732$$

where $W$ and $V$ are the unitary matrices of singular value decomposition, $\Sigma$ is the matrix of singular values and $\omega$ denotes the frequency. As can be seen from the singular values, the system is poorly conditioned. Conventional open loop step testing approaches can often lead to models which estimate only the strong gain direction. The input space of an open loop PRBS type test is shown in Figure 2. The receding horizon formulation was next used to carry out a closed loop experiment for the system. The following tuning parameters and constraints were used during the experiment: $P = M = 10$, $\mathbf{u}_L = -200$, $\mathbf{u}_H = 200$, $\mathbf{y}_L = -2$, $\mathbf{y}_H = 2$, $\Gamma = -0.01I$, $\Lambda = -I$. White noise of variance 0.1 was added to each output with power of 0.1. The data from the closed loop step test is shown in Figure 3. The data shows significant excitation along the weak gain direction as opposed to the open loop experiment in the Figure 2 where the strong direction was dominant. The models estimated from the closed loop experiment are compared in Figure 4.

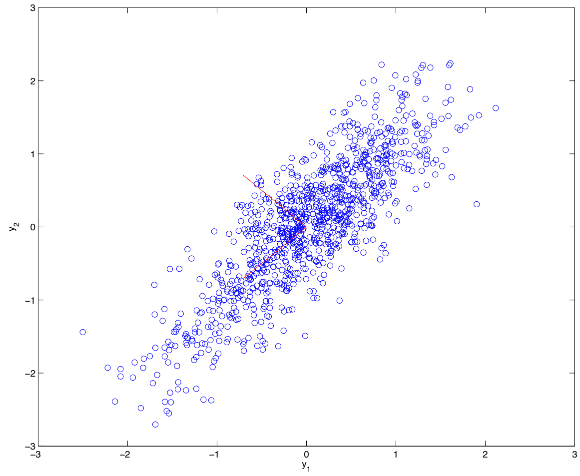


Fig. 2. Open loop excitation shown in the output space (red straight lines denote the directions of the two column vectors in $V$).

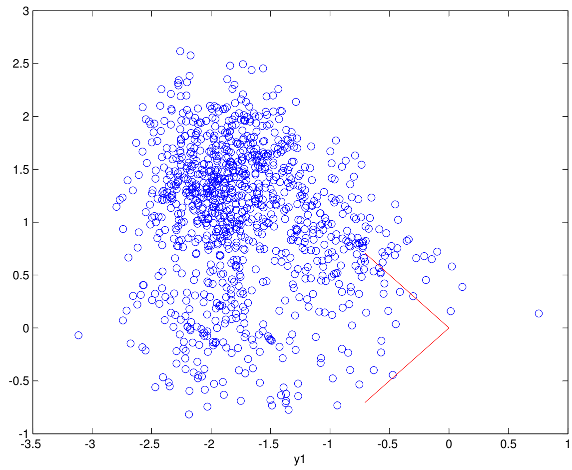


Fig. 3. Output space with the receding horizon approach to experiment design (red straight lines denote the directions of the two column vectors in $V$).

It is even more instructive to look at the gains estimated from the two different approaches and their inverse,

$$K = \begin{bmatrix} 0.8724 & -0.8585 \\ 1.0751 & -1.0890 \end{bmatrix} \quad K^{-1} = \begin{bmatrix} 40.20 & -31.69 \\ 39.68 & -32.20 \end{bmatrix}$$

$$K_1 = \begin{bmatrix} 0.8253 & -0.8114 \\ 1.0279 & -1.0417 \end{bmatrix} \quad K_1^{-1} = \begin{bmatrix} 40.59 & -31.62 \\ 40.05 & -32.16 \end{bmatrix}$$

$$K_2 = \begin{bmatrix} 0.6968 & -0.7424 \\ 0.8940 & -0.9269 \end{bmatrix} \quad K_2^{-1} = \begin{bmatrix} -38.10 & -30.51 \\ -36.75 & 28.35 \end{bmatrix}$$

where $K$ is the gain of the true system, $K_1$ is the gain estimated from the closed loop receding horizon experiment data and $K_2$ is the gain estimated from the open loop

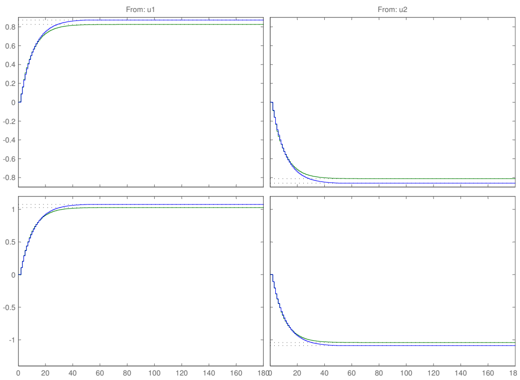


Fig. 4. Comparison of the estimated model with the true system (green - model, blue - true system).

data. Note how different the inverse is for the open loop based model from the true inverse. A controller based on the second model can end up making moves in the wrong direction. This is a direct result of not estimating both the gain directions accurately. Inaccurate estimation of gain directions can have significant impact on the controller performance, especially for the optimization or linear programming layer of the controller.

## 5. CONCLUSIONS

The proposed approach has many advantages: (1) ability to handle constraints in a predictive way during the step test, (2) ability to deal with ill-conditioned processes, (3) ability to account for unmeasured disturbances and mitigating their impact on constraint violations during the step test and (4) ability to switch between control and step testing merely by switching the objectives of the MPC.

On the other hand compared to traditional experiment design approaches, it is not clear how the proposed method will address excitation over different frequency ranges. It is expected that the choice of the prediction horizon will influence the frequency content of the implemented signal. This is a topic that needs further research. One of the advantages of a conventional step testing approach is the transparency of the move plan and complete control over the implemented move sequence. In the case of the proposed approach, an automated move plan is generated, the first move is implemented and the rest discarded. The generated move plan is a function of the: (1) input/output weightings, (2) input/output constraints, (3) prediction horizon, (4) current model plant mismatch, and (5) unmeasured disturbances. More work is needed to establish the relationships between these parameters and the calculated move sequence.

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