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Computationally Efficient Globally Linearizing Control of a CSTR using Nonlinear Black Box Models

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Abstract: The main objective of this work is to develop computationally efficient Global linearization based control (GLC) schemes, which are suitable for control of nonlinear processes with fast dynamics. Models used for the controller synthesis are discrete time block oriented nonlinear black box state space models identi=ed directly from the input-output perturbation data. The chosen model structures facilitate construction of closed form solutions to the unconstrained GLC formulations. The efficacy of the proposed control formulations is evaluated by conducting simulation studies on a benchmark continuously stirred tank reactor (CSTR) system which exhibits input multiplicity behavior. Analysis of the simulation results reveals that the proposed GLC formulations are able to achieve a signi=cant reduction in the average computation time without compromising on the closed loop performance.

Keywords: Global Linearization, Wiener Model, NARX Model, Input Multiplicity, Reactor Control

1. INTRODUCTION

In recent years, nonlinear model based control schemes are increasingly being used in the industrial applications. Nonlinear MPC is the most popular member of this class. NMPC is an optimization-based control methodology which, in general, does not lead to a closed form control law. Incorporation of nonlinear dynamic models into the MPC formulation requires that a nonconvex nonlinear programming problem be solved at each sampling instant. This can prove to be a formidable task while controlling systems with fast dynamics.

In the process control literature, global linearization has emerged as another effective tool for design of nonlinear model based controllers. From the viewpoint of controlling systems with fast dynamics, the methods based on global input-output linearization arising from geometric considerations appear to be attractive. If the model has input affine structure and constraints on the manipulated inputs are ignored, then a closed form control law can be derived under the GLC framework. Thus, the computational load associated with a GLC scheme is relatively less. Henson and Seborg (1997) provide an excellent review of the feedback linearizing control for continuous time systems. In practice, however, model based controllers are implemented through microprocessors or digital computers. Since the discrete time systems require different considerations at the design stage, Soroush and Kravaris (1996) have developed multivariable nonlinear control schemes based on the feedback linearization of nonlinear discrete time models. They demonstrate applicability of their GLC schemes using discrete time representation of mechanistic models. In the discrete domain, however, the discrete GLC control problem often has to be solved on-line numerically using iterative methods. This is because, even when a continuous time mechanistic model has input affine structure, the sampled data representation of the mechanistic model is, in general, not affine in the manipulated inputs.

Thus, to enhance the computational bene=ts of GLC, it is necessary to develop efficient approaches for solving the discrete GLC problem. Also, in practice, a reliable mechanistic dynamic model may not be available for the system under consideration. This difficulty can be alleviated if a nonlinear black box model is developed from the input-output perturbation data and the model structures is chosen such that it is amenable to the analytical treatment.

The main objective of this work is to develop computationally efficient GLC schemes, which are suitable for control of nonlinear processes with fast dynamics. Among the various model forms available in the literature, we choose to work with discrete time block oriented nonlinear state space models developed by Srinivasrao et al. (2005) and Srinivasrao et al. (2007) for the following reasons: a)The dynamic component of these models is parameterized using generalized orthonormal basis =lters (OBF), which results in signi=cant reduction in the number of parameters to be estimated and in relatively low dimensional state space representations b) the static nonlinear part is parametrized using quadratic polynomials. As a consequence, the resulting model structure facilitates construction of closed form solutions to the unconstrained GLC formulation. Moreover, these models are capable of capturing dynamics of systems exhibiting both input multiplicities as well as output multiplicities (Srinivasrao et al. (2005, 2007)). The effectiveness of the proposed control formulations is evaluated by conducting simulation studies on a benchmark continuously stirred tank reactor (CSTR) system in which a reversible exothermic reaction is carried out (Li and Biegler (1988)). This system is highly nonlinear and exhibit input multiplicity in the neighborhood of the desired optimum (singular) operating point.

This paper is organized in =ve sections. The next section introduces block oriented NOE and NARX models, which are later used in the GLC controller synthesis. The proposed GLC formulations based on these black box models are developed in Section 3. The simulation results are presented in section 4. The last section presents the major conclusions reached from the analysis of simulation studies.

2. NONLINEAR BLACK BOX STATE SPACE MODELS

Nonlinear black box state space models with NOE (Srinivasrao et al. (2007)) and NARX (Srinivasrao et al. (2005)) structures are brie>y described in this section.

2.1 OBF-NOE Model with Wiener Structure

Consider a Wiener type MISO state space model of the form Srinivasrao et al. (2007)

$$\mathbf{x}^{(i)}(k+1) = \mathbf{\Phi}^{(i)}\mathbf{x}^{(i)}(k) + \mathbf{\Gamma}^{(i)}\mathbf{u}(k) \tag{1}$$

$$\mathbf{y}_i(k) = \mathbf{\Omega}^{(i)}\left[\mathbf{x}^{(i)}(k)\right] + \mathbf{v}_i(k) \tag{2}$$

for the iꞪh output. Here, $\mathbf{x}^{(i)}(k) \in R^{n_i}$ represents the associated state vector, $\mathbf{u}(k) \in R^m$ represents vector of manipulated inputs, $\mathbf{y}_i(k)$ represents iꞪh component of the vector of measured / controlled outputs $\mathbf{y}(k) \in R^r$ and $\mathbf{\Omega}^{(i)}[.]$ represents some nonlinear static map relating states with the outputs for the i^{th} MISO model. It may be noted that $\mathbf{u}(k)$ and $\mathbf{y}(k)$ are de=ned as perturbation variables in the neighborhood of some desired steady state operating point. Matrices $(\mathbf{\Phi}^{(i)}, \mathbf{\Gamma}^{(i)})$ are parameterized using OBF (Patwardhan and Shah (2005)), which represent an orthonormal basis for the set of strictly proper stable transfer functions (denoted as $\mathcal{H}_2$). Ninness and Gustafson Ninness and Gustafsson (1997) have shown that a complete orthogonal set in $\mathcal{H}_2$ can be constructed as follows

$$F_k(z,\xi) = \frac{\sqrt{(1-|\xi_k|^2)}}{(z-\xi_k)} \prod_{i=1}^{k-1} \frac{(1-\xi_i^* z)}{(z-\xi_i)} \tag{3}$$

where $\{\xi_k : k = 1, 2, ...\}$ is an arbitrary sequence of poles inside the unit circle appearing in complex conjugate pairs. The nonlinear state output map $\mathbf{\Omega}^{(i)}[.] : R^{n_i} \to R$ is chosen to be simple multi-dimensional quadratic polynomial functions of the form

$$\mathbf{\Omega}_i[.] = \mathbf{C}^{(i)}\mathbf{x}^{(i)}(k) + \left(\mathbf{x}^{(i)}(k)\right)^T \mathbf{D}^{(i)} \left(\mathbf{x}^{(i)}(k)\right) \tag{4}$$

Here, $\mathbf{C}^{(i)}$ represents a $(1 \times n_i)$ vector and $\mathbf{D}^{(i)}$ represents a $n_i \times n_i$ symmetric matrix. The model (equations 1-2) can be looked upon as a truncated second order Volterra series model. The estimation of OBF poles and the parameters of state-output map can be carried out using a nested optimization approach as proposed by Srinivasrao et al. (2007).

Such r MISO models are stacked to formulate a MIMO OBF-Wiener model as follows

$$\mathbf{x}(k+1) = \mathbf{\Phi}\,\mathbf{x}(k) + \mathbf{\Gamma}\,\mathbf{u}(k) \tag{5}$$

$$\mathbf{y}(k) = \Omega\,[\mathbf{x}(k)] + \mathbf{v}(k) \tag{6}$$

$$\Omega\,[\mathbf{x}(k)] = \mathbf{C}\,\mathbf{x}(k) + \begin{bmatrix} \mathbf{x}^{(1)}(k)^T\,\mathbf{D}^{(1)}\,\mathbf{x}^{(1)}(k) \\ \mathbf{x}^{(2)}(k)^T\mathbf{D}^{(2)}\,\mathbf{x}^{(2)}(k) \\ .. \\ \mathbf{x}^{(r)}(k)^T\,\mathbf{D}^{(r)}\,\mathbf{x}^{(r)}(k) \end{bmatrix} \tag{7}$$

$$= \mathbf{C}\,\mathbf{x}(k) + \{\,\mathbf{D}\,\}\,(\mathbf{x}(k), \mathbf{x}(k)) \tag{8}$$

where

$$\mathbf{x}(k) = \left[\,\left(\mathbf{x}^{(1)}(k)\right)^T\,\left(\mathbf{x}^{(2)}(k)\right)^T\,...\,\left(\mathbf{x}^{(r)}(k)\right)^T\,\right]^T \tag{9}$$

$$\mathbf{\Phi} = block\ diag\left[\,\mathbf{\Phi}^{(1)}\;\mathbf{\Phi}^{(2)}\;...\;\mathbf{\Phi}^{(r)}\,\right]_{n\times n} \tag{10}$$

$$\mathbf{\Gamma} = \left[\mathbf{\Gamma}^{(1)^T}\;\mathbf{\Gamma}^{(2)^T}\;...\;\mathbf{\Gamma}^{(r)^T}\right]^T_{n\times m} \tag{11}$$

$$\mathbf{C} = block\ diag\left[\,\mathbf{C}^{(1)}\;\mathbf{C}^{(2)}\;...\;\mathbf{C}^{(r)}\,\right]_{r\times n} \tag{12}$$

$$\{\mathbf{D}\} = \begin{bmatrix} \left[\,\mathbf{D}^{(1)}\;[0]\;...\;[0]\,\right] \\ \hline .... \\ \hline \left[\,[0]\;[0]\;...\;\mathbf{D}^{(r)}\,\right] \end{bmatrix}_{r\times n\times n} \tag{13}$$

and $n = \sum_{i=1}^{r} n_i$. Note that $\{\,\mathbf{D}\,\}$ is a $(r \times n \times n)$ *bilinear matrix representation* of a three dimensional array of the form (see Appendix A for details of the bilinear matrix representation).

2.2 OBF-NARX Model

A model belonging to this class can be constructed by modifying equation (1) as proposed by Srinivasrao et al. (2005). Thus, a MISO OBF-NARX type observer can be represented as follows

$$\mathbf{x}^{(i)}(k+1) = \mathbf{\Phi}^{(i)}\mathbf{x}^{(i)}(k) + \mathbf{\Gamma}^{(i)}\mathbf{u}(k) + \mathbf{L}^{(i)}\mathbf{y}_i(k) \tag{14}$$

$$\mathbf{y}_i(k) = \Omega^{(i)}\left[\mathbf{x}^{(i)}(k)\right] + \mathbf{e}_i(k) \tag{15}$$

where $\mathbf{x}^{(i)}(k) \in R^{n_i}$ represents the associated state vector. Here, $\{\mathbf{e}_i(k) : k = 1, 2, ...\}$ represents a zero mean white noise sequence. Similar to the OBF-NOE case, the matrices $(\mathbf{\Phi}^{(i)}, \mathbf{\Gamma}^{(i)}, \mathbf{L}^{(i)})$ appearing in the state dynamics are parameterized using OBF. The nonlinear state output map $\mathbf{\Omega}^{(i)}[.] : R^{n_i} \to R$ is chosen to be simple multi-dimensional quadratic polynomial functions of the form given by equation (4). The estimation of OBF poles and the parameters of state-output map can be carried out using a nested optimization approach as proposed by Srinivasrao et al. (2005).

Similar to the NOE case, r MISO OBF-NARX models can be stacked to form a MIMO OBF-NARX model as follows

$$\mathbf{x}(k+1) = \mathbf{\Phi}\,\mathbf{x}(k) + \mathbf{\Gamma}\,\mathbf{u}(k) + \mathbf{L}\mathbf{y}(k) \tag{16}$$

$$\mathbf{y}(k) = \Omega\left[\mathbf{x}(k)\right] + \mathbf{e}(k) \tag{17}$$

$$\Omega\left[\mathbf{x}(k)\right] = \mathbf{C}\,\mathbf{x}(k) + \{\,\mathbf{D}\,\}\left(\mathbf{x}(k), \mathbf{x}(k)\right) \tag{18}$$

where $\mathbf{x}(k), \mathbf{\Phi}, \mathbf{\Gamma}$ and$\{\mathbf{D}\}$ are constructed in a similar manner to equations (10)-(13) presented in the previous sub-section and

$$\mathbf{L} = \left[\, \mathbf{L}^{(1)^T} \;\; \mathbf{L}^{(2)^T} \;\; ... \;\; \mathbf{L}^{(r)^T} \,\right]^T_{n \times m}$$

At the model identi=cation stage, the representation given by equations (16) has an advantage that the state dynamics is linear function of $\mathbf{u}(k)$ and $\mathbf{y}(k)$. However, the representation given by equations (16-17) is not convenient when it is desired to carry out GLC controller design. Thus, the OBF-NARX model is further rearranged as follows

$$\mathbf{x}(k+1) = \digamma\left[\mathbf{x}(k)\right] + \mathbf{\Gamma}\,\mathbf{u}(k) + \mathbf{L}\mathbf{e}(k) \tag{19}$$

$$\mathbf{y}(k) = \Omega\left[\mathbf{x}(k)\right] + \mathbf{e}(k) \tag{20}$$

$$\digamma\left[\mathbf{x}(k)\right] = \mathbf{\Phi}\,\mathbf{x}(k) + \mathbf{L}\Omega\left[\mathbf{x}(k)\right] \tag{21}$$

The following observations can be made based on this rearranged form:

- Unlike the NOE model with Weiner structure, the state dynamics (19) is a nonlinear function of $\mathbf{x}(k)$
- The state dynamics is driven by manipulated inputs and the zero mean white noise sequence $\{\mathbf{e}(k)\}$, which facilitates modeling of the effects of unmeasured disturbances on the measured outputs.

3. CONTROLLER DEVELOPMENT UNDER GLC FRAMEWORK

In this section, closed form control laws are developed using NOE and NARX models under the globally linearizing discrete control framework developed by Soroush and Kravaris (1996). It is assumed that following assumptions:

- The plant to be controlled is internally asymptotically stable in the neighborhood of the desired operating point.
- System under consideration is square, i.e. number of inputs are equal to the number of outputs (i.e. $m = r$).

3.1 Globally Linearizing Control Law

Consider a discrete nonlinear model of the form

$$\mathbf{x}(k+1) = \chi\left[\mathbf{x}(k), \mathbf{u}(k)\right] \tag{22}$$

$$\mathbf{y}(k) = \Omega\left[\mathbf{x}(k)\right] \tag{23}$$

Let us further assume that the manipulated input $\mathbf{u}(k)$ directly affects the output $\mathbf{y}(k+1)$ (Soroush and Kravaris (1996))

$$\mathbf{y}(k+1) = \Omega\left[\chi\left[\mathbf{x}(k), \mathbf{u}(k)\right]\right] \tag{24}$$

and the characteristic matrix satis=es the following rank condition

$$rank\left[\left[\frac{\partial \Omega}{\partial \chi}\right]\left[\frac{\partial \chi}{\partial \mathbf{u}}\right]\right] = m$$

i.e. the discrete system of equation (22)-(23) has *relative order* one. Let $\mathbf{r}(k) \in R^r$ denote the desired setpoint and let $\boldsymbol{\varepsilon}(k) \in R^r$ denote a signal that contains information about the model plant mismatch and / or unmodelled disturbances. Then, following Soroush and Kravaris (1996), a GLC law, that achieves a servo response equivalent to a =rst order reference model of the form

$$\mathbf{y}(k+1) = \mathcal{A}\mathbf{y}(k) + (\mathbf{I}{-}\mathcal{A})\left[\mathbf{r}(k) - \boldsymbol{\varepsilon}(k)\right] \tag{25}$$

between the controlled output and the mismatch adjusted setpoint $[\mathbf{r}(k) - \boldsymbol{\varepsilon}(k)]$ can be derived by =nding manipulated input, $\mathbf{u}(k)$, that solves the following set of nonlinear equations at each sampling time

$$\Omega\left[\chi\left[\mathbf{x}(k), \mathbf{u}(k)\right]\right] = \mathcal{A}\mathbf{y}(k) + (\mathbf{I}{-}\mathcal{A})\left[\mathbf{r}(k) - \boldsymbol{\varepsilon}(k)\right] \tag{26}$$

The unity gain reference model is selected such that eigen values of $\mathcal{A}$ are at desired location inside the unit circle. A simple way to parameterize matrix $\mathcal{A}$ is to choose it to be diagonal, i.e.

$$\mathcal{A} = \mathbf{diag}\left[\,\alpha_1 \;\; \alpha_2 \;\; ... \;\; \alpha_m\,\right];\ \text{with } 0 \le \alpha_i < 1$$

If additional degree of freedom are introduced by using =ltered residuals and =ltered setpoint, then the control law is further modi=ed as follows

$$\Omega\left[\chi\left[\mathbf{x}(k), \mathbf{u}(k)\right]\right] = \mathcal{A}\mathbf{y}(k) + (\mathbf{I}{-}\mathcal{A})\left[\mathbf{r}_f(k) - \boldsymbol{\varepsilon}_f(k)\right] \tag{27}$$

where =ltered mismatch, $\boldsymbol{\varepsilon}_f(k)$, and =ltered setpoint are computed using a unity gain =lter as follows

$$\boldsymbol{\varepsilon}_f(k) = \mathcal{A}_e\boldsymbol{\varepsilon}_f(k-1) + \left[\mathbf{I} - \mathcal{A}_e\right]\boldsymbol{\varepsilon}(k) \tag{28}$$

$$\mathbf{r}_f(k) = \mathcal{A}_r\mathbf{r}_f(k-1) + \left[\mathbf{I} - \mathcal{A}_r\right]\mathbf{r}(k) \tag{29}$$

Here, $\mathcal{A}_e$ and $\mathcal{A}_r$ are diagonal matrices similar to $\mathcal{A}$ and are treated as tuning parameters.

3.2 OBF-NOE Model

Since the OBF NOE model has a fading memory, this model can be used to construct an *open loop state observer* of the form

$$\widehat{\mathbf{x}}(k) = \mathbf{\Phi}\widehat{\mathbf{x}}(k-1) + \mathbf{\Gamma}\mathbf{u}(k-1)$$

The state estimate, $\widehat{\mathbf{x}}(k)$, can be used to compute the model residual signal as follows

$$\boldsymbol{\varepsilon}(k) = \mathbf{y}(k) - \;\mathbf{\Omega}\left[\widehat{\mathbf{x}}(k)\right]$$

The residual signal contains information on the model-plant mismatch and the unmeasured disturbances affecting the plant. The estimate $\widehat{\mathbf{x}}(k)$ can be further used for computing the one step output prediction as follows

$$\widehat{\mathbf{y}}(k+1) = \;\Omega\left[\widehat{\mathbf{x}}(k+1)\right] = \;\Omega\left[\mathbf{\Phi}\widehat{\mathbf{x}}(k) + \mathbf{\Gamma}\mathbf{u}(k)\right] \tag{30}$$

De=ning bilinear matrix $\{\mathbf{\Psi}\}$, matrix $\mathbf{\Lambda}(k)$ and vector $\overline{\mathbf{y}}(k)$ as

$$\{\mathbf{\Psi}\} = \{\{\mathbf{D}\} \circ \mathbf{\Gamma} \bullet \mathbf{\Gamma}\}$$

$$\mathbf{\Lambda}(k) = \mathbf{C}\mathbf{\Gamma} + 2\left\{\{\mathbf{D}\} \circ \mathbf{\Phi} \bullet \mathbf{\Gamma}\right\}\widehat{\mathbf{x}}(k)$$

$$\overline{\mathbf{y}}(k) = \mathbf{C}\mathbf{\Phi}\widehat{\mathbf{x}}(k) + \{\{\mathbf{D}\} \circ \mathbf{\Phi} \bullet \mathbf{\Phi}\}\left(\widehat{\mathbf{x}}(k), \widehat{\mathbf{x}}(k)\right)$$

equation (30) can be rearranged as follows

$$\mathbf{\Omega}\left[\chi\left[\widehat{\mathbf{x}}(k), \mathbf{u}(k)\right]\right] = \{\mathbf{\Psi}\}\left(\mathbf{u}(k), \mathbf{u}(k)\right) + [\mathbf{\Lambda}(k)]\mathbf{u}(k) + \overline{\mathbf{y}}(k) \tag{31}$$

Here, operators $[.] \bullet \{.\}, \{.\} \circ [.]$ and $\{.\} \bullet [.]$ represents left dot product, circle product and right dot product, respectively, between a matrix $[.]$ and a bilinear matrix $\{.\}$. (Refer to Appendix for details).

3.3 OBF-NARX Model

At each sampling instant, the internal NARX MIMO model can be used to estimate the current state, $\widehat{\mathbf{x}}(k)$, and the model residual, $\boldsymbol{\varepsilon}(k)$, as follows

$$\widehat{\mathbf{x}}(k) = \boldsymbol{\Phi}\widehat{\mathbf{x}}(k-1) + \boldsymbol{\Gamma}\,\mathbf{u}(k-1) + \mathbf{L}\mathbf{y}(k-1) \quad (32)$$

$$\boldsymbol{\varepsilon}(k) = \mathbf{y}(k) - \Omega\left[\widehat{\mathbf{x}}(k)\right] \quad (33)$$

To arrive at the control law, consider the one step ahead prediction at instant k

$$\widehat{\mathbf{x}}(k+1) = \mathcal{F}\left[\widehat{\mathbf{x}}(k)\right] + \boldsymbol{\Gamma}\,\mathbf{u}(k) + \mathbf{L}\boldsymbol{\varepsilon}_f(k) \quad (34)$$

$$\mathcal{F}\left[\widehat{\mathbf{x}}(k)\right] = \boldsymbol{\Phi}\ \widehat{\mathbf{x}}(k) + \mathbf{L}\Omega\left[\widehat{\mathbf{x}}(k)\right] \quad (35)$$

$$\widehat{\mathbf{y}}(k+1) = \Omega\left[\widehat{\mathbf{x}}(k+1)\right] \quad (36)$$

where $\boldsymbol{\varepsilon}_f(k)$ represent the =ltered model residuals as given by equation (28). De=ning $\widetilde{\mathbf{y}}(k) = \Omega\left[\widehat{\mathbf{x}}(k|k-1)\right] + \boldsymbol{\varepsilon}_f(k)$, equation (36) can be rearranged in the form given by equation (31) by de=ning the bilinear matrix $\{\boldsymbol{\Psi}\}$ and matrix $\boldsymbol{\Lambda}(k)$ as follows

$$\begin{aligned}\{\boldsymbol{\Psi}\} &= \{\{\mathbf{D}\} \circ \boldsymbol{\Gamma} \bullet \boldsymbol{\Gamma}\} \\ \boldsymbol{\Lambda}(k) &= \mathbf{C}\boldsymbol{\Gamma} + 2\left\{\{\mathbf{D}\} \circ \boldsymbol{\Phi} \bullet \boldsymbol{\Gamma}\right\}\widehat{\mathbf{x}}(k) \\ &\quad +2\left\{\{\mathbf{D}\} \circ \mathbf{L} \bullet \boldsymbol{\Gamma}\right\}\widetilde{\mathbf{y}}(k)\end{aligned}$$

Vector $\overline{\mathbf{y}}(k)$ appearing in equation (31) is de=ned as

$$\begin{aligned}\overline{\mathbf{y}}(k) &= \left\{\{\mathbf{D}\} \circ \mathbf{L} \bullet \mathbf{L}\right\}\left(\widetilde{\mathbf{y}}(k), \widetilde{\mathbf{y}}(k)\right) \\ &\quad + \left[\mathbf{C}\mathbf{L} + 2\left\{\{\mathbf{D}\} \circ \boldsymbol{\Phi} \bullet \mathbf{L}\right\}\left(\widehat{\mathbf{x}}(k)\right)\right]\widetilde{\mathbf{y}}(k) \\ &\quad + \mathbf{C}\boldsymbol{\Phi}\widehat{\mathbf{x}}(k) + \left\{\{\mathbf{D}\} \circ \boldsymbol{\Phi} \bullet \boldsymbol{\Phi}\right\}\left(\widehat{\mathbf{x}}(k), \widehat{\mathbf{x}}(k)\right)\end{aligned}$$

3.4 Closed Form Controller Synthesis

To arrive at a closed form control law, consider one step ahead prediction equation (31) combined with the GLC control law (27). The resulting controller design equation can be further rearranged as follows

$$Q\left[\mathbf{u}(k)\right] = \{\boldsymbol{\Psi}\}\left(\mathbf{u}(k), \mathbf{u}(k)\right) + [\boldsymbol{\Lambda}(k)]\mathbf{u}(k) + \boldsymbol{\rho}(k) = \overline{\mathbf{0}} \quad (37)$$

where

$$\boldsymbol{\rho}_k = \left[\overline{\mathbf{y}}(k) - \left(\mathcal{A}\widehat{\mathbf{y}}(k) + (\mathbf{I} - \mathcal{A})\left[\mathbf{r}_f(k) - \boldsymbol{\varepsilon}_f(k)\right]\right)\right]$$

Here, $Q[.] : R^m \to R^m$ is the multi-dimensional quadratic operator and $\overline{\mathbf{0}}$ represents $m \times 1$ null vector. A multi-dimensional quadratic equation of the form (37) can be solved analytically using method developed by Rall (1961) as follows:

$$\mathbf{u}(k) = \mathbf{u}(k) - \left[\frac{1}{2}\left\{I + \left(\boldsymbol{\Delta}(k)\right)^{\frac{1}{2}}\right\}\right]^{-1}\widetilde{\boldsymbol{\rho}}(k) \quad (38)$$

$$\boldsymbol{\Delta}(k) = \left(\mathbf{I} - 4\left\{\widetilde{\Psi}(k)\right\}\left(\widetilde{\boldsymbol{\rho}}(k)\right)\right)$$

$$\begin{aligned}Q\left[\mathbf{u}(k-1)\right] &= \{\boldsymbol{\Psi}\}\left(\mathbf{u}(k-1), (k-1)\right) \\ &\quad + [\boldsymbol{\Lambda}(k-1)]\mathbf{u}(k-1) + \boldsymbol{\rho}(k) \\ \left\{\widetilde{\Psi}(k)\right\} &= \left(\nabla_U\left[Q\left(\mathbf{u}(k-1)\right)\right]\right)^{-1} \bullet \{\boldsymbol{\Psi}\} \\ \widetilde{\boldsymbol{\rho}}(k) &= \left(\nabla_U\left[Q\left(\mathbf{u}(k-1)\right)\right]\right)^{-1}\left[Q\left(\mathbf{u}(k-1)\right)\right]\end{aligned}$$

In general, a matrix has multiple square roots and consequently different values of $\mathbf{u}(k)$ will be obtained for every choice of the square root of matrix $\Delta(k)$. From the control view point, it is desirable to =nd the set of solutions for which the sensitivity $\| \mathbf{u}(k) - \mathbf{u}(k-1) \| / \| \boldsymbol{\rho}(k) \|$ is minimum. Patwardhan and Madhavan (1998), have shown that this set corresponds to choosing $(\boldsymbol{\Delta}(k))^{1/2}$ such that its eigen values have non-negative real parts. Also, the matrix square root can have complex elements, and, consequently the resulting $\mathbf{u}(k)$ can be complex. Patwardhan and Madhavan (1998), have suggested that real part of complex solution vector can be used for manipulation when the solution vector becomes complex. Thus, incorporating the above suggestions, the *generic quadratic GLC* becomes

$$\mathbf{u}(k) = \mathbf{u}(k-1) - REAL\left\{\left[\frac{1}{2}\left\{\mathbf{I} + \left(\Delta(k)\right)^{\frac{1}{2}}\right\}\right]^{-1}\widetilde{\boldsymbol{\rho}}(k)\right\} \quad (39)$$

The control law (equation 39) is an unconstrained formulation. In the rest of the text, GLC formulation developed using OBF-NOE model is referred to as NOE-GLC and GLC formulation developed using OBF-NARX model is referred to as NARX-GLC.

4. SIMULATION STUDIES

The efficacy of the proposed control schemes is demonstrated by carrying out simulation studies on a CSTR system, which exhibits input multiplicity and change in the sign of the steady state gain in the desired operating region. Moreover, the desired optimum operating point happens to be a singular point where the steady state gain is reduced to zero and the invertibility is lost. Thus, controlling the CSTR at the optimum point poses a challenging control problem.

4.1 Continuous Stirred Tank Reactor

The process under consideration is a benchmark CSTR in which a non-isothermal, reversible =rst order reaction of type $A \underset{K_2}{\overset{K_1}{\Leftrightarrow}} B$ is carried out. The dynamics of this system can be represented by the following set of ODEs (Li and Biegler (1988))

$$\begin{aligned}\frac{dC_A}{dt} &= \frac{F_i(C_{Ai} - C_A)}{h\ A_c} - K_1 C_A + K_2 C_B \\ \frac{dC_B}{dt} &= -\frac{F_i C_B}{h\ A_c} + K_1 C_A - K_2 C_B \\ \frac{dT}{dt} &= \frac{F_i(T_i - T)}{h\ A_c} + \frac{-H_r}{\rho\ C_p}(K_1 C_A - K_2 C_B) \\ \frac{dh}{dt} &= \frac{1}{A_c}(F_i - k\sqrt{h}) \\ K_1 &= k_{01}\ exp\left(\frac{-E_1}{T}\right);\ K_2 = k_{02}\ exp\left(\frac{-E_2}{T}\right)\end{aligned}$$

The reactor system has four state variables, namely concentration of A (C_A), concentration of B (C_B), temperature (T) and level (h). The inlet >ow (F_i) and inlet temperature (T_i) are treated as the manipulated inputs while the inlet concentration (C_{Ai}) is treated as the unmeasured disturbance. The nominal parameters and the operating steady states used in the simulation studies are given in Li and Biegler (1988). Out of the four states, concentration of B, C_B, and the reactor level, h, are assumed to be measured and controlled outputs. Details of the black box model identi=cation exercises can be found in Deshpande (2010).

The MIMO servo control problem is formulated in such a way that it is desired to shift the operating point from a given suboptimal initial steady state ($C_B(0) = 0.4088$, $h(0) = 0.14$) to the optimum operating point ($C_{B_{opt}} = 0.5088$, $h_{opt} = 0.16$). In addition, to investigate the regulatory behavior, 10% step change in the unmeasured disturbance, C_{Ai}, is introduced simultaneously with the setpoint changes. The controller performance is reported in terms of (a) Average computation time required for computing control moves (Intel(R) Core(TM) 2, Duo CPU@ 2 GHz with 2 GB RAM) and (b)Integral Square Error (ISE). In all GLC controllers, the reference model tuning matrix $\mathcal{A}$ and the plant model mismatch =lter matrix $\mathcal{A}_e$ are selected as

$$\mathcal{A} = diag\,[\,0.9\ \ 0.9\,] \text{ and } \mathcal{A}_e = diag\,[\,0.95\ \ 0.98\,]$$

Use of this error =lter for plant model mismatch helps in improving the robustness of the GLC controllers. For the sake of comparison, an Ideal GLC controller is developed directly using the mechanistic model as an open loop observer and by following the design procedure outlined in the beginning of Section 3.1. The tuning parameters used for the Ideal GLC controller are identical to that of the NOE-GLC and NARX-GLC controllers. The Ideal GLC controller is implemented by solving the controller design equation (27) iteratively using NewtonHs method (H*fsolve*H command in MATLAB).

Figures (1) and (2) present comparison of the controlled outputs and manipulated inputs obtained using NOE-GLC and NARX-GLC formulations. It may be noted that NARX-GLC is able to achieve the desired transition at smaller overshoot in C_B and much less settling time. This may be attributed to the fact that NARX model explicitly models the unmeasured disturbances. As a consequence, the resulting GLC formulation is better suited for rejecting unmeasured step disturbance in C_{Ai} introduced simultaneously with the setpoint change. A comparison of performance indices of all the GLC controllers is presented in Table 1. It may be noted that the average computation time for the Ideal GLC formulation is quite large. The computation times for NARX-GLC and NOE-GLC formulations, on the other hand, are signi=cantly small (about 1/150 of Ideal GLC). Moreover, the ISE values obtained using both the formulations are comparable to that of Ideal GLC.

To test the performances of the proposed GLC formulations in the presence of stochastic disturbances, it is assumed that dynamics of $C_{Ai}(k)$ is governed by the following stochastic process

$$C_{Ai}(k) = \overline{C}_{Ai} + \frac{1}{1 - 0.95q^{-1}} w(k)$$

where $w(k)$ is a white noise sequence with standard deviation 0.1.In addition, the concentration and the level measurements are assumed to be corrupted with zero mean normally distributed random variables with standard deviations equal to 0.005 and 0.0025, respectively. The resulting closed loop behavior is presented in Figures (3) and (4). Both the controllers are able to achieve transition to

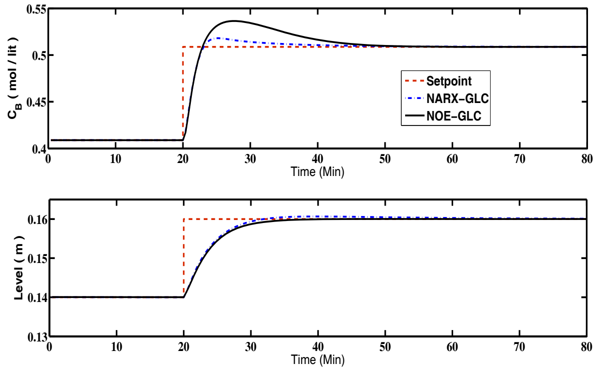

Fig. 1. CSTR System: Comparison of controlled output responses for simultaneousl servo and regulatory changes

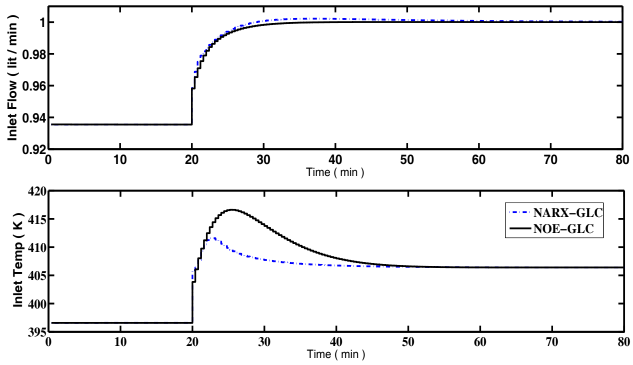

Fig. 2. CSTR System: Comparison of manipulated input pro=les for simultaneousl servo and regulatory changes

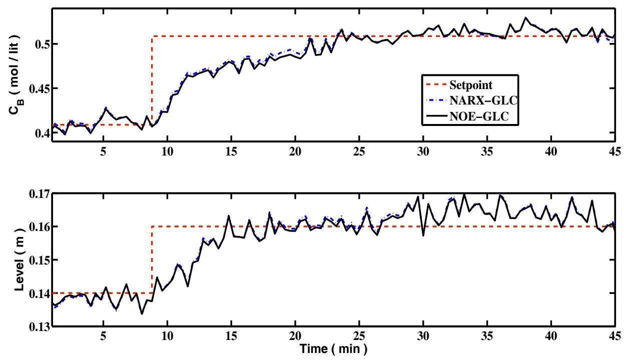

Fig. 3. CSTR System: Comparison of controlled output responses for servo change in presence of stochastic disturbances

the desired setpoints in the face of drifting unmeasured disturbance. As evident from Figure 4, the input pro=le generated using NOE-GLC is relatively smooth. It may be noted that the inlet temperature settles at two different steady states due to the input multiplicity behavior of the CSTR system.

5. CONCLUSIONS

In this work, computationally efficient discrete GLC schemes have been developed using discrete nonlinear

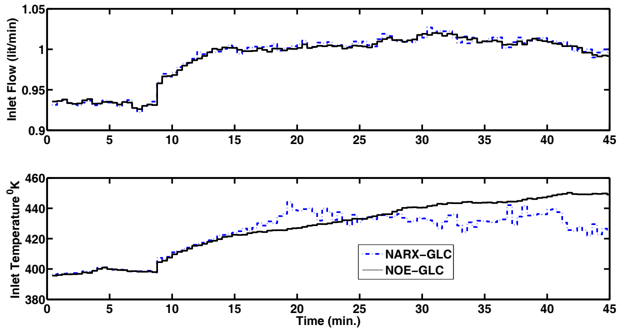

Fig. 4. CSTR System: Comparison of manipulated input pro=les for servo change in presence of stochastic disturbances

Table 1. CSTR: servo Performance Comparison:With stochastic Disturbances

Controller	Avg. Comp.Time	ISE	
	(msec)	$\mathbf{C}_B$	Level
		$(\times 10^{-2})$	$(\times 10^{-3})$
Ideal GLC	187.54	7.65	9.90
NARX-GLC	1.26	5.77	8.49
NOE-GLC	1.15	6.73	8.78

black-box models with NOE and NARX structures. By exploiting structures of state realization of these models and solution method for analytically solving multi-dimensional quadratic equations, closed form control laws are derived. The efficacy of the proposed GLC formulations is evaluated by conducting simulation studies on a benchmark continuously stirred tank reactor (CSTR) system. The simulation studies demonstrate that the proposed GLC formulations are able to achieve a signi=cant reduction in the average computation time without compromising the closed loop performance.

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6. BILINEAR MATRIX OPERATIONS

De=nition A.1 (Bilinear Matrix): A bilinear matrix B of dimension $(r \times n \times m)$ is ordered collection of numbers $b_{\alpha\beta\gamma}, \alpha = 1,2,...r; \beta = 1,2,...n; \gamma = 1,2,...m$. It is highlighted by inclusion in the curly brackets as $\{B\}$ or $\{b_{\alpha\beta\gamma}\}$.

De=nition A.2: A $(r \times n \times m)$ bilinear matrix $\{B\}$ operating on a $(n \times 1)$ vector v is represented as $A = \{B\}(v)$ where A is a $(r \times n \times m)$ matrix with elements

$$a_{\alpha\gamma} = \sum_{\beta=1}^{n} b_{\alpha\beta\gamma} v_\beta$$

De=nition A.3: A $(r \times n \times m)$ bilinear matrix $\{B\}$ operating on a $(n \times 1)$ vector v and a $(m \times 1)$ vector w is represented as $z = \{B\}(v, w)$ where z is a $(r \times 1)$ vector with elements

$$z_\alpha = \sum_{\gamma=1}^{m} \sum_{\beta=1}^{n} b_{\alpha\beta\gamma} v_\beta w_\gamma$$

A $(r \times n \times n)$ bilinear matrix is called symmetric if $\{B\}(v, w) = \{B\}(w, v)$ for every $v, w \in R^n$.

De=nition A.4 (Left Dot Product): The left dot product of a $(r \times n \times m)$ bilinear matrix $\{B\}$ with a $(k \times r)$ matrix A is represented as

$$\{D\} = A \bullet \{B\}$$

where $\{D\}$is a $(k \times n \times m)$ bilinear matrix with elements

$$d_{\alpha\beta\gamma} = \sum_{\eta=1}^{r} a_{\alpha\eta} b_{\eta\beta\gamma}$$

De=nition A.4 (Right Dot Product): The right dot product of a $(r \times n \times m)$ bilinear matrix $\{B\}$ with a $(m \times k)$ matrix A is represented as

$$\{D\} = \{B\} \bullet A$$

where $\{D\}$is a $(r \times n \times k)$ bilinear matrix with elements

$$d_{\alpha\beta\gamma} = \sum_{\eta=1}^{m} b_{\alpha\beta\eta} a_{\eta\gamma}$$

De=nition A.4 (Circle Product): The circle product of a $(r \times n \times m)$ bilinear matrix $\{B\}$ with a $(n \times k)$ matrix A is represented as

$$\{D\} = \{B\} \circ A$$

where $\{D\}$is a $(r \times n \times k)$ bilinear matrix with elements

$$d_{\alpha\beta\gamma} = \sum_{\eta=1}^{n} b_{\alpha\eta\gamma} a_{\eta\beta}$$