Sensor Network Design for Efficient Fault Diagnosis and Signed Digraph Update

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Abstract: An optimally designed sensor network is essential to ensure optimal and safe process operation. Several approaches for designing sensor networks for efficient fault diagnosis have been presented in the literature. Most of these utilize signed digraph (SDG) based process representation and assume that the SDG is accurately known. For a nonlinear system, the signs on the edges in a SDG depend on the operating conditions and can thus change with time or operating mode and hence may not be accurately known. However, such uncertainties in SDG modeling have been largely ignored in sensor network design literature. In this work, we propose a design approach that considers such uncertainties while selecting optimal sensor networks. The resulting network is optimal in the sense that it results in the lowest number (or lowest cost) of sensors that can ensure observability and resolution of faults while simultaneously identifying the signs on the uncertain edges. Similar to faults, the concepts of observability and resolution are defined for such uncertain edges as well and used in the design procedure. The utility of the proposed approach is demonstrated by applying it on a five tank system.

Keywords: Sensor Network Design, Fault Diagnosis, Signed Digraphs, Identification, Five Tank System

1. INTRODUCTION

Over the years, chemical processes have significantly increased in complexity due to tighter heat and mass integration as well as advancements in computer based process control technology. As a result of this increased complexity, it has become essential to have automated monitoring systems that can detect an abnormality in the process as and when it develops as well as identify its root cause. This is the problem of fault diagnosis and in literature several techniques have been proposed for the same. The basic principle in every diagnostic technique is to compare the predictions obtained from a reference model with the actual measurements obtained from the plant to decide the state of the process. This is schematically represented in Figure 1. From this figure it is clear that any diagnostic strategy critically depends on two aspects: (i) the variables being measured in the process and (ii) the correctness of the reference model being used by the diagnostic strategy.

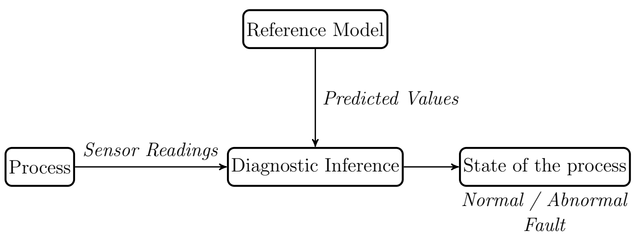

Fig. 1. Fault diagnosis schematic

The problem of selecting variables to be measured in the process is referred to as the sensor network design problem. Several approaches have been presented in literature (Bagajewicz, 2002) for designing sensors networks for a variety of objectives. These approaches in turn are based on several types of process models: both quantitative and qualitative, and have been designed for a variety of process requirements related to control, data reconciliation and fault diagnosis, etc. Amongst the qualitative representations, digraph and signed digraph (SDG) based models are most popular and have been widely used for designing sensor networks for fault diagnosis and are also considered in this work.

In literature, Raghuraj et al. (1999) used a process digraph to perform sensor network design for fault diagnosis while ensuring observability and resolution requirements. They posed the resulting design problems as appropriate set cover optimization problems and presented greedy search heuristics for solving these problems. Bagajewicz et al. (2004) used these ideas and presented the problem as mixed integer linear programming problem. Bhushan and Rengaswamy (2000) extended the work of Raghuraj et al. (1999) for the case when the process model is represented as a signed digraph. Bhushan and Rengaswamy (2002) incorporated sensor failure and fault occurrence probabilities in the sensor network design procedure by using the concept of unreliability of fault. Bhushan et al. (2008) incorporated robustness to the available probability data and the signed digraph representation in the sensor network design procedure. In their work, robustness to mismatch between the process and the underlying signed digraph model was obtained by simply designing a distributed network, i.e. a network where more number of variables were measured. Chen and Chang (2008) utilized the fault evolution sequences in the SDG as an additional piece of information, to obtain the optimal sensor network. Yang et al. (2009) incorporated missed and false alarm rates in the SDG based sensor network design procedure.

In all the above works, it has been assumed that (i) either the underlying signed digraph model is accurate, or (ii) in presence of inaccuracies in the signed digraph model, a distributed network was designed (Bhushan et al., 2008). In practice, some of the signs in a SDG will be typically unknown. Even if they are known accurately to begin with, the signs may change over a period of time as the operating conditions in the process change. However, the issue of designing a sensor network that enables not only efficient fault diagnosis but is also able to identify the uncertain signs in the SDG representation has not been addressed in literature. The focus of the current work is to design sensor networks that can ensure best possible diagnostic observability of faults while simultaneously identifying the unknown/uncertain signs of edges in the signed digraph. Towards this end, we extend known concepts of observability and resolution of faults to cases when some of the signs in the SDG representation are uncertain. We also define observability and resolution of such uncertain edge signs. The sensor network design problem is then posed as a set cover optimization problem that ensures observability and/or resolution of faults alongwith observability and resolution of the uncertain edge signs.

The rest of the paper is structured as follows. In section 2 we present a brief discussion on SDG since SDG is used for process modeling in our work. In section 3, we present the proposed work. The concepts of observability and resolution for fault and edges with uncertain signs are defined here. In section 4, we demonstrate the utility of our sensor network design approach by designing a sensor network for a five tank system before concluding in section 5.

2. SIGNED DIGRAPH (SDG) AS A PROCESS MODEL

In our proposed work, the process knowledge will be represented by a signed digraph. A SDG is a graph G that consists of a set of vertices V and a set of edges E. Each edge $e_k \in E$ is directional and a sign is also assigned to it. The vertices denote process variables and faults while the edges denote cause-effect relationships between the corresponding vertices. To illustrate, if edge e_k is directed from vertex x_j to $x_{j'}$ and has a positive sign, it means that a positive change in x_j from its normal (steady state) value will lead to a positive change in $x_{j'}$ with all other variables being constant. SDG of a simple tank with inlet flow, outlet flow and a leak (fault) is illustrated in figure 2. A positive arc from h to F_0 in this SDG indicates that the exit flow F_0 increases when the height h of liquid in the tank increases. SDGs have been widely used in the area of fault diagnosis as well as sensor network design for fault diagnosis. In this context they have been used to mainly predict the effects of faults on variables (forward modeling) or infer faults from the variables that have been observed to have deviated (inverse propagation).

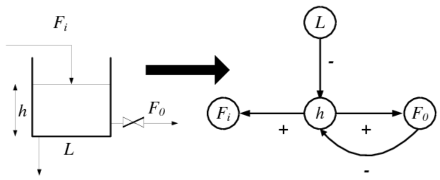

Fig. 2. Schematic of a tank with leak L and the corresponding signed digraph

Some of the reasons for the popularity of SDG are: (i) they are easy to construct (especially at the design stage) as detailed information about the process parameters is not required, (ii) being structural in nature they have the property of completeness which means that if we were to predict the effect of a fault on a variable using a signed digraph, then the predicted effect (positive, negative, zero or indeterminate) would include the sign of actual deviation of the variable, and (iii) they explicitly represent cause-effect relationships in the process thereby enabling explanation capability in case of occurrence of faults. Additionally, since they can be represented as a graph, the rich repertoire of techniques available in graph theory literature, such as depth first and breadth first based search algorithms, etc., can be used for analyzing digraphs. A drawback of SDG based analysis is that they can lead to spurious predictions i.e. effects which may not actually have occurred. For example, from analysis of an SDG, one may conclude that the effect of a fault on a variable can be either positive or negative while in practice it will be only one of the two. The property of completeness will also hold only if the signs of the edges in the SDG correctly represent the signs of the process effects.

Detailed algorithms for constructing SDGs from quantitative process equations and analyzing them are available in literature (Maurya et al., 2003a,b). These approaches incorporate complexities typically encountered in chemical processes such as recycles and feed-back introduced due to control action, and have been applied at the flow sheet level. Based on these algorithms, the SDG can be analyzed to find the effect of a fault on every process variables. We now discuss our proposed sensor network design approach.

3. PROPOSED WORK

In this work we propose design of optimal sensor networks that will enable identification of unknown/uncertain signs of signed digraphs as well as fault detection and diagnosis of process faults. For simplicity, we will restrict ourselves to cases when only one fault can occur at a time (single fault), and only one edge can change sign at a time. These two changes are however allowed to occur simultaneously.

3.1 Observability and Resolution of Faults and Edges

In the literature (Raghuraj et al., 1999), sensor networks have been designed to maximize fault observability and resolution under the single fault assumption. Ensuring observability of a fault ensures that the effect of that fault is felt on atleast one sensor. Similarly, ensuring resolution between a pair of faults ensures that the faults in question can be distinguished from each other based on the effects on the measured variables. We now extend these observability and resolution ideas to deal with cases when signs of some of the edges in the graph can change.

We assume that the process signed digraph, the list of faults F and the set of digraph edges E_U whose sign is assumed to be uncertain is available to us. Let there be m faults in set F and p edges in set E_U. The faults are represented as nodes in the process SDG. Without loss of generality it will be assumed that while the fault node can only become positive i.e. $F_i \in F, \quad i = 1, 2, ..., m$ can be 0 or +1, the edge sign $e_k \in E_U, \; k = 1, 2, ..., p$ can be both positive (+1) or negative (-1). It is further assumed that the default (nominal) values of edge signs e_k are known.

By simulating in the given process SDG, for each scenario of a fault $F_i \in F$ occurring and the edge $e_k \in E_U$ having a sign different from its nominal value, we first find the set of variables affected by this combination alongwith the direction of their effects. This set denoted as $A_{i,k}$ is referred to as faultedge-set and the corresponding scenario of occurrence of i^{th} fault and change in sign of k^{th} edge is called faultedge $\{i, k\}$. While obtaining a faultedge-set it is assumed that other faults have not occurred and the other edges in E_U have their nominal signs. Such faultedge-sets are created for all possible combinations of faults and edges leading to a total of $\tilde{N} = m \times p$ combinations. Faultedge-set for the scenario when a fault has occurred but all the edges have their nominal signs are also identified. These faultedge-set are labeled as $A_{i,0}, i = 1, 2, ..., m$ to indicate that no edge change has occurred. Therefore, the total number of faultedge-sets will be $N = \tilde{N} + m = m \times (p+1)$.

We now define the concepts of observability and resolution of faultedges similar to the observability and resolution concepts for faults defined in literature (Bhushan and Rengaswamy, 2000) .

Observability and Resolution of Faultedges

- *Observability of faultedges*: A faultedge $\{i, k\}$ will be said to be observable if atleast one sensor from the corresponding faultedge-set $A_{i,k}$ is part of the selected sensor network.
- *Resolution of faultedges*: Similar to the idea of Bhushan and Rengaswamy (2000), we propose to create pseudo faultedge-sets corresponding to resolution between the original faultedge-sets. This is explained via the Venn diagram in figure 3 (Raghuraj et al., 1999). The left circle represents the variables affected by faultedge 1, while the right circle represents the variables affected by faultedge 2. To resolve between these two scenarios, we need to place a sensor in the shaded area as a sensor in the intersection area will not enable distinguishing between the two scenarios. The shaded area is the symmetric difference of the two faultedge-sets, i.e. it consists of all those variables that can enable distinguishing between the corresponding faultedges. We then propose to associate a pseudo faultedge $\{i, k, i', k'\}$ with each original pair of faultedges $\{i, k\}$ and $\{i', k'\}$. The faultedge set $A_{(\{i,k\},\{i',k'\})}$ of this pseudo faultedge consists of those variables that can enable distinguishing between faultedges $\{i, k\}$ and $\{i', k'\}$. For N original faultedge scenarios we will have $\binom{N}{2}$ pseudo faultedges. For a given sensor network, a pair of original faultedges will be resolvable if atleast one variable from the corresponding pseudo faultedge set is measured. If a particular pseduo faultedge set is empty, then that means that the corresponding faultedge pair is unresolvable. Selecting sensors from all the pseudo faultedge sets $A_{(\{i,k\},\{i',k'\})}$ alongwith the original faultedge sets $A_{\{i,k\}}$ will ensure observability and resolution of the faultedges.

In the above definition of resolution, we have assumed that at a time only one fault can occur either alone or alongwith change in sign of one edge. Cases where more than one fault can occur simultaneously or more than one edge can change sign simultaneously can also be considered in a similar way. In this work, just for simplicity, we have not considered such cases.

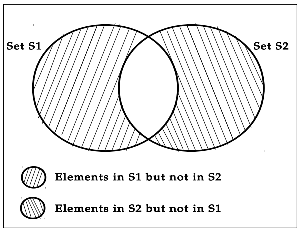

Fig. 3. Venn diagram to illustrate variables that can ensure faultedge resolution (Raghuraj et al., 1999)

While the above definitions were for faultedges which included occurrence of faults alongwith simultaneous changes in edge signs, one may be interested in identifying observability or resolvability of faults separately or edges separately. These concepts are defined next:

Faults

Since faults can occur with or without change in an edge sign, we define two types of observabilities of a given fault:

- *Weak Observability*: For a given sensor network, fault i is said to be weakly observable if it can be observed in absence of any sign change in edges belonging to E_U. In other words, the faultedge $\{i, 0\}$ should be observable for fault i to be weakly observable.
- *Strong Observability*: For a given sensor network, fault i is said to be strongly observable if: (i) it is weakly observable and (ii) it can be observed when it occurs alongwith simultaneous change in sign of any edge in E_U. In other words, all the faultedges $\{i, k\}, k = 0, 1, 2, .., p$ should be observable for fault i to be strongly observable. Here $k = 0$ means no edge sign has changed.

  It can be noted that, strong observability implies weak observability but not vice-versa. However in most cases weak observability would also lead to strong observability, since change in sign of any edge in the SDG will mostly change the sign of effect of fault on one or more variables but it will not make the effect 0. The exception to this case can occur if the sign change leads to totally different behaviour in the SDG, such as conversion of a positive cycle to a negative (compensatory feedback) cycle. Hence the definitions of weak and strong observability are considered separately.

For a pair of faults i, i' we define resolvability as:

- *Weak Resolvability*: For a given sensor network, the pair of faults i, i' is said to be weakly resolvable if the faultedge $\{i, 0\}$ is resolvable from faultedge $\{i', 0\}$.
- *Strong Resolvability*: For a given sensor network, the pair of faults i, i' is strongly resolvable if: (i) the pair is weakly resolvable, and (ii) the faultedge $\{i, k\}$ is resolvable from faultedge $\{i', k\}$ $\forall k = 1, 2, ..., p$. Strong resolvability ensures that the pair of faults can be distinguished from each other irrespective of the edge sign change in the SDG.

Edge Signs

Similar to the case of faults discussed above, for a given sensor network we can define observability and resolution for edge signs as:

- *Weak Observability*: Edge sign e_k is said to be weakly observable if faultedge $\{i, k\}$ is resolvable (distinguishable) from faultedge $\{i, 0\}$ for atleast one $i, i = 1, 2, ..., m$.
- *Strong Observability*: Edge sign e_k is said to be strongly observable if faultedge $\{i, k\}$ is resolvable from faultedge $\{i, 0\}$ $\forall i = 1, 2, ..., m$.
- *Weak Resolvability*: A pair of edges $e_k, e_{k'} \in E_U$ are said to be weakly resolvable if faultedges $\{i, k\}$ and $\{i, k'\}$ are resolvable for atleast one $i, i = 1, 2, ..., m$.
- *Strong Resolvability*: The pair of edges $e_k, e_{k'} \in E_U$ are said to be strongly resolvable if faultedges $\{i, k\}$ and $\{i, k'\}$ are resolvable $\forall i = 1, 2, ..., m$. Strong resolvability implies that the sign changes in edges $e_k, e_{k'}$ can be distinguished from each other irrespective of the fault that has occurred.

Examples: Some examples are presented to explain the above proposed concepts:

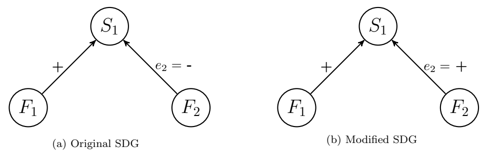

Fig. 4. Signed digraph to illustrate fault resolution

In figure 4a, variable S_1 can resolve between the faults F_1 and F_2. This is because if fault F_1 occurs S_1 will deviate positively from the nominal value when uncertain edge e_2 (from F_2 to S_1) has its nominal sign $(-)$ and if fault F_2 occurs S_1 will deviate negatively. Now the sign of the uncertain edge is changed from $-$ to $+$ as shown in figure 4b. Then faults F_1 and F_2 cannot be resolved through S_1. Therefore, faults are said to be weakly resolvable but not strongly resolvable.

Figure 5a represents a SDG with two faults $\{F_1, F_2\}$, two uncertain edges $\{e_2, e_3\}$ with nominal signs $\{+, +\}$, and two measured variables S_1 and S_2. Both edge signs e_2 and e_3 are observable when F_2 occurs. However only edge sign e_3 is observable when fault F_1 occurs. Thus, edge sign e_2 is weakly observable while edge sign e_3 is strongly observable.

In SDG of figure5b, with measured variables S_1 and S_2, changes in signs e_1 and e_2 from their nominal values $\{+, +\}$ are resolvable when fault F_1 occurs. But they are not resolvable when fault F_2 occurs. Thus, the pair of edge signs e_1, e_2 is weakly resolvable.

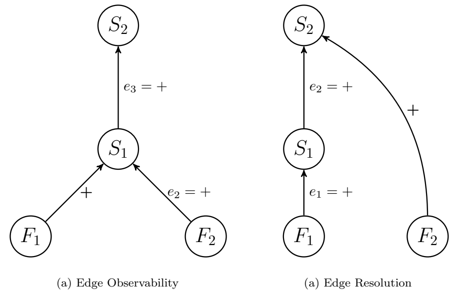

Fig. 5. Signed digraph to illustrate edge observability and resolution

Remark 1: From the above definitions, it can be seen that if for a sensor network all the faults are strongly obervable, then that will ensure that all the edge signs are also strongly observable.

Remark 2: Ensuring resolution of all faultedge pairs is a much more stringent requirement than ensuring strong resolution of either fault pairs or uncertain edge pairs.

3.2 Sensor Network Design

Sensor networks can now be designed to optimize some criteria such as cost or number of chosen sensors, while satisfying various requirements on observability and resolution of faults and/or edge signs. In each case, an integer programming set cover optimization problem needs to be solved. This is briefly discussed next:

Formulation : Minimum Cost Sensor Network Design Problem

$$\min \sum_{j=1}^{n} c_j x_j \tag{1}$$

such that

$$\sum_{j=1}^{n} d_{vj} x_j \geq 1, \quad \forall v \in V \tag{2}$$

$$x_j \in \{0, 1\} \quad j = 1, \cdots, n \tag{3}$$

where, c_j is the cost of putting sensor on variable j. x_j is a binary decision variables such that $x_j = 1$ represents that the j^{th} variable is measured and $x_j = 0$ represents that the j^{th} variable is not measured. d_{vj} is the element from the v^{th} faultedge set such that

$$d_{vj} = \begin{cases} 1 \text{ if variable j} \in \text{faultedge set } v \\ 0 \text{ otherwise} \end{cases}$$

The objective function (Equation 1) in the above formulation is minimization of cost of the selected sensors. The constraint in Equation 2 ensures that faultedge v (from faultedge set V) affects atleast one measured variable, that is product of d_{vj} and x_j is equal to 1 for atleast one j.

V is the faultedge set to be specified by the user. It can correspond to either observability or resolution of faults, edges or faultedges. Therefore, V is specified depending upon the scenario for which sensors have to be designed.

In the next section, sensor network design strategy proposed in this work is applied on five tank system case study and results are discussed.

4. CASE STUDY: FIVE TANK SYSTEM

To illustrate the proposed work, a five tank system (Chang et al. (1993); Bhushan and Rengaswamy (2000)) is considered as shown in figure 6. Tank-3 and Tank-4 are interacting while remaining all tanks are non-interacting in nature. Variable L represents level while q and f represent the flow rates.

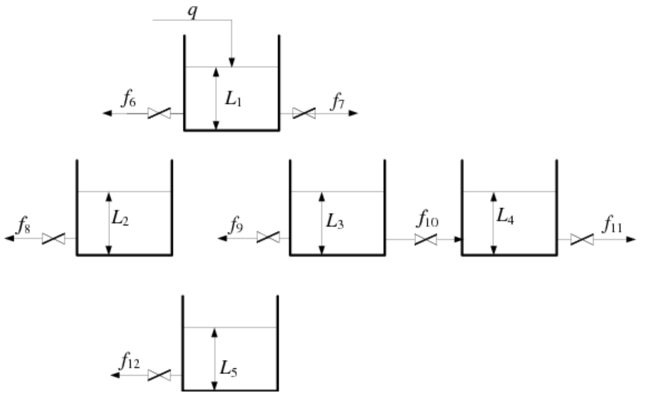

Fig. 6. Five Tank System for Illustration (Bhushan and Rengaswamy, 2000)

We assume that a leak can occur in any of the five tanks. Leak in tank i is denoted as l_i. The process model of the system along with leaks can be written as

$$\left.\begin{aligned} A_1\frac{dL_1}{dt} &= q - f_6 - f_7 - l_1 \\ A_2\frac{dL_2}{dt} &= f_6 - f_8 - l_2 \\ A_3\frac{dL_3}{dt} &= f_7 - f_9 - \alpha f_{10} - l_3 \\ A_4\frac{dL_4}{dt} &= \alpha f_{10} - f_{11} - l_4 \\ A_5\frac{dL_5}{dt} &= f_9 - f_{12} - l_5 \end{aligned}\right\} \quad (4)$$

$$\left.\begin{aligned} \text{Where } f_6 &= k_1\sqrt{L_1} \;;\; f_7 = k_2\sqrt{L_1} \\ f_8 &= k_3\sqrt{L_2} \;;\; f_9 = k_4\sqrt{L_3} \\ f_{10} &= k_5\sqrt{|L_3 - L_4|} \;;\; f_{11} = k_6\sqrt{L_4} \\ f_{12} &= k_7\sqrt{L_5} \end{aligned}\right\} \quad (5)$$

In equation 4, α is a constant with nominal value $+1$ under the assumption that $L_3 > L_4$. However if $L_3 < L_4$ then α would be -1. This uncertainty about α leads to uncertainty in sign of edges from f_{10} to L_3 (edge denoted e_1) and from f_{10} to L_4 (edge denoted e_2). The default sign of these edges is assumed to be $-$ and $+$ respectively. Further, while these edge signs have to be opposite, for illustration we assume that they can change independently. The SDG of the differential algebraic system of equations (DAE) 4 and 5 is generated using the approach given in

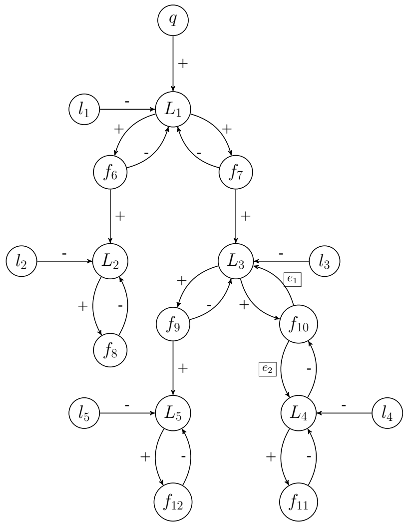

Fig. 7. Signed Digraph of Five Tank System with 2 uncertain edges and 5 leaks as possible faults

Maurya et al. (2003a). For each faultedge case, the corresponding faultedge sets are generated using propagation in the SDG (Maurya et al., 2003a) and is shown in Figure 7. For illustration, sensor network is designed for two cases: (I) considering only two faults (l_1,l_5) and (II) considering all five faults ($l_1 - l_5$). For each of these cases edges e_1, e_2 are assumed to be uncertain.

Case-I : Considering Two Faults

In this case, we are considering two faults i.e., leak in Tank-1 as fault 1 and leak in Tank-5 as fault 5. The corresponding faultedge matrix is given in Equation 6. The rows in the matrix are faultedges affecting variables given in columns. The process consists of 12 variables (5 levels and 7 flowrates) that can potentially be measured. The faultedges are labeled as $F_{i,k}$ corresponding to simultaneous occurrence of i^{th} fault and change in sign of edge e_k.

$$\begin{array}{c} \\ F_{1,0} \\ F_{1,1} \\ F_{1,2} \\ F_{5,0} \\ F_{5,1} \\ F_{5,2} \end{array} \begin{array}{c} \begin{array}{cccccccccccc} L_1 & L_2 & L_3 & L_4 & L_5 & f_6 & f_7 & f_8 & f_9 & f_{10} & f_{11} & f_{12} \end{array} \\ \begin{pmatrix} -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\ -1 & -1 & -1 & 1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & -1 \\ -1 & -1 & -1 & 1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & -1 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix} \end{array} \quad (6)$$

The faultedge sets $A_{i,k}$ for observability and $A_{(\{i,k\},\{i',k'\})}$ for resolution of faultedges are listed in Table 1. The pseudo faultedge sets in this table correspond to those variables that can resolve between the corresponding original faultedge pairs. From this table it is seen that some pairs of faultedges are not resolvable. For example, it is not possible to resolve between occurrence of fault 5 from simultaneous occurrence of fault 5 along with a change in sign of either of the two edges e_1 or e_2. Sensor network is now designed to ensure observability and resolution of the faultedges. The empty faultedge sets in Table 1 are however removed from the set cover problem during the design process. Further, the costs of all the sensors are assumed to be the same, thereby minimizing the number of chosen sensors. The design requirement is to ensure

(i) only observability of faultedges and (ii) resolution of the faultedges listed in equation 6. The sensor networks designed for these cases are listed in rows 2 and 3 (cases 1 and 2 for two faults scenario) in Table 2. From Tables 1 and 2 the following can be noted:

- Measurement of only L_5 is sufficient to ensure observability of both the faults as it is affected by both the faults irrespective of changes in edge signs. In this case, both the original faults are thus strongly observable. It can also be noted (from Table 1) that measuring f_{12} would also have ensured strong observability of both the faults.
- The faults F_1 and F_5 are strongly resolvable as these two can be resolved from each other irrespective of the edge sign change when variables L_4, L_5, f_{10} are measured.
- The edge signs e_1 and e_2 are weakly observable with measurements L_4, L_5 since while occurrence of fault F_1 without any edge sign change can be resolved from occurrence of fault F_1 with change in sign of either of the two edges, this resolution does not hold for occurrence of fault F_5. Also, the edges are only weakly resolvable even after measurement of L_4, L_5 and f_{10}, as changes in signs of these edges cannot be distinguished when fault F_5 occurs.

Table 1. Faultedge sets for case-1

Original Faults Edges	**Variable Sets**
$A_{\{1,0\}} = A_{\{1,1\}} = A_{\{1,2\}}$	L_1, L_2, L_3, L_4, L_5, f_6, f_7, f_8, f_9, f_{10}, f_{11}, f_{12}
$A_{\{5,0\}} = A_{\{5,1\}} = A_{\{5,2\}}$	L_5, f_{12}
Pseudo Faults Edges	**Variable Sets**
$A_{(\{1,0\},\{1,1\})}$	L_4, f_{11}
$A_{(\{1,0\},\{1,2\})}$	L_4, f_{10}, f_{11}
$A_{(\{1,0\},\{5,0\})} = A_{(\{1,0\},\{5,1\})} = A_{(\{1,0\},\{5,2\})} = A_{(\{1,1\},\{5,0\})} = A_{(\{1,1\},\{5,1\})} = A_{(\{1,1\},\{5,2\})} = A_{(\{1,2\},\{5,0\})} = A_{(\{1,2\},\{5,1\})} = A_{(\{1,2\},\{5,2\})}$	L_1, L_2, L_3, L_4, f_6, f_7, f_8, f_9, f_{10}, f_{11}
$A_{(\{1,1\},\{1,2\})}$	f_{10}
$A_{(\{5,0\},\{5,1\})} = A_{(\{5,0\},\{5,2\})} = A_{(\{5,1\},\{5,2\})}$	ϕ

Case-II: Considering Five Faults
In this case, leaks in all the five tanks are considered. Due to lack of space, the faultedge sets for this case are not listed. The optimal sensor network designs obtained for observability and resolution of faultedges is however listed in the last two rows of Table 2. As expected, it can be seen that more number of sensors are selected to ensure observability and resolution in this case than for the two fault case considered earlier. For the sensor networks given in Table 2 it was found that all the faults are strongly observable as well as strongly resolvable. However, both the edge signs are only weakly observable and weakly resolvable.

5. CONCLUSIONS

In the current work, we have proposed a sensor network design approach based on the signed digraph representation of a process. The design enables efficient fault diagnosis as well as simultaneous identification of changes in signs of edges with uncertain signs in the underlying SDG. The design is based on concepts of observability and resolution which have been defined for faultedges corresponding to simultaneous occurrence of a fault and an edge sign change, as well as for faults and edges separately. The sensor network design problem is formulated as an appropriate set cover integer programming problem. The utility of the proposed approach is illustrated on a five tank case study. Extension of the work to deal with varying probabilities of fault occurrence and edge sign changes as well as its implementation on larger case studies involving control loops, is currently under investigation.

Table 2. Optimal Sensor Networks for 5 tank system

Case-I	**Considering two faults**	Sensor Network
1	Observability of faultedges	L_4, L_5
2	Resolution of faultedge	L_4, L_5, f_{10}
Case-II	**Considering five faults**	Sensor Network
3	Observability of faultedges	L_2, L_4, L_5
4	Resolution of faultedge	L_2, L_4, L_5, f_{10}

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