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Model Predictive Control of a Paste Thickener in Coal Handling and Preparation Plants

Ridwan Setiawan*, Chee Keong Tan*, Jie Bao*, Götz Bickert**

**School of Chemical Engineering, The University of New South Wales, Sydney, NSW, 2052 (j.bao@unsw.edu.au)*
***GBL Process Pty Ltd, Federal, NSW, 2480 (gb@gblprocess.com)*

Abstract: The control of paste thickener is important because underflow solids concentration has to be maintained within a certain operating window to maximize water recovery while avoiding operational difficulties, e.g. pump bogging. Paste thickener dynamics is complicated due to the interactions between key process variables, uncertainties in operating conditions and large time constant. In this paper, a dynamic model of the paste thickener which addresses varying coal parameters were developed and a model predictive control (MPC) was formulated based on the model. The simulated MPC results based on actual coal processing plant data show that a significant improvement can be achieved in terms of the ability to control the underflow solids concentration as well as conforming to the constraints imposed by the physical limitations of the process, e.g. pump speed and solids amount inside the thickener.

Keywords: model predictive control, process control, Kalman filter, mining processing.

1. INTRODUCTION

Thickening is a crucial step in mining and wastewater treatment industries. In coal preparation, thickeners are used to achieve the overall water recovery from the tailings, which are important from both cost and environmental point of views. The process involves sedimentation, i.e. the settling of discrete particulates, often enhanced with flocculant to form larger aggregates in a liquid to achieve a higher concentration of solids in the underflow and clarified liquid in the overflow.

The first thickener was invented by Dorr in 1905 rendering the continuous dewatering of solid-liquid mixture possible (Dorr, 1915). These conventional thickeners often occupy large spaces and are unable to achieve higher solids concentration. In the 1970s, paste thickener was introduced (Abbott, et al., 1973), a technology that enables higher solids content in the underflow (> 50% w/w) with smaller equipment size. This is achieved by the combination of high bed (sediment) level and deep cone dimension which allow increased compression of lower layers of sediment resulting in higher dewatering effect. Due to its higher solids concentration, the underflow product is often known as paste as opposed to slurry produced by conventional thickeners.

During thickener operation, it is often desired to control the underflow solids concentration to be within a certain range. In coal industry, tailings should be thickened to high solids content to recover water but not too high so as to damage the thickener or underflow pump. Important variables in thickener operation include underflow rate, feed solids rate and flocculant dosage to the feed stream. Underflow rate will determine the throughput of the thickener affecting the residence time and product quality. Flocculant dosage affects the final settling velocity of particles in hindered settling region as well as the compression of materials in the sediment, thus the underflow solids concentration. These two variables are commonly available to be manipulated by an automatic control system to maintain the underflow solids concentration within a desired operating range. However, the feed solids rate and coal properties which are determined by upstream processes become external disturbances to the paste thickener, which require attenuation.

Although a paste thickener can improve dewatering, its automatic control is still challenging due to the interactions between key process variables, uncertainties in operating conditions (varying feed coal types) and extremely large process time constant. These render existing control strategies used for conventional thickeners ineffective (Segovia, et al., 2011). Furthermore, the knowledge of the dynamic model is still very limited. Previous model development was based on mostly empirical results and it was not until recently that a first principle based modelling was developed using phenomenological theory of sedimentation-consolidation. This theory allows the incorporation of compression/consolidation effects to the thickener modelling which was unavailable previously due to the non-existence/insignificance of bed/sediment inside conventional thickeners. However, this type of models is very difficult to be used directly for control development due to their complexity, especially in partial differential equations representing the physical nature of plug flow inside paste thickeners (Yao, et al., 2012).

This paper proposed an approach to develop a dynamical model suitable for analysis and control purposes based on the extension of the discretized phenomenological model developed by Bürger et al. (Bürger, et al., 2004). Then, a model predictive control (MPC) algorithm is designed and simulated based on the model in conjunction with real coal processing plant data and compared to the existing operating results.

This paper is organized as follows. First a brief explanation is given on the model used and its extension to allow control development. Next, an MPC is designed based on the proposed model. This includes the formulation of a Kalman Filter which allows parameter estimation useful for adaptive model for online application. Simulation results which provide model validation and the comparison among MPC, PID control and existing plant operations are given followed by a discussion on the proposed approach. Finally, a summary, recommendations and on-going study are explained in the conclusion.

2. PASTE THICKENING MODEL

2.1 Model

Previously developed modelling technique for conventional thickener is mostly based on kinematic sedimentation theory by Kynch (1952), which describes the sedimentation of small rigid spheres dispersed in viscous liquid (Kynch, 1952). However, in mineral processing, most particles are not ideal, e.g. coal tailings are flocculated and in a paste thickener, they will form compressible sediments. Therefore, a dynamic sedimentation-consolidation model as an extension to Kynch's sedimentation theory is required to address the compressible sediments by introducing pore pressure and effective solid stress (Bustos, et al., 1990).

In this paper, two regions are considered in the modelling of a paste thickener: 1) hindered settling region, and 2) compression region. As shown in Fig. 1, the fluid flows upward to the overflow producing clarified liquid. In the hindered settling region, the solids volume fraction (ϕ) is below the critical concentration (ϕ_c) (also known as gel point) at which particles start to coalesce to form sediment. Higher than ϕ_c, a compression region is formed.

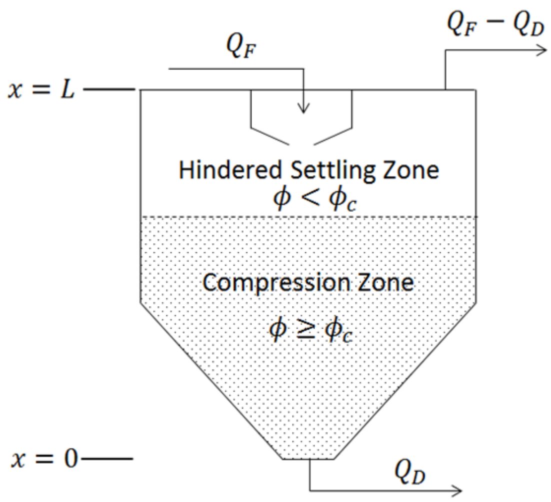

Fig. 1. Continuous paste (deep-cone) thickener.

In continuous thickening, the process can then be modelled by three mechanisms, i.e. convection, sedimentation and consolidation by the following second-order partial differential equation (Bürger, et al., 2004):

$$\frac{\partial\phi(x,t)}{\partial t} + \frac{1}{S(x)}\frac{\partial}{\partial x}(Q_D(t)\phi) + \frac{1}{S(x)}\frac{\partial}{\partial x}\big(S(x)f_{bk}(\phi)\big) = \frac{1}{S(x)}\frac{\partial}{\partial x}\Big(S(x)a(\phi)\frac{\partial\phi}{\partial x}\Big) \quad (1)$$

where $\phi(x,t)$ is the solids volume fraction which is assumed to be constant across each horizontal section, as a function of time (t) and x is the vertical position between underflow discharge level $(x = 0)$ and the thickener height $(x = L)$, $S(x)$ is the cross-sectional area, $Q_D(t) \leq 0$ is the underflow (U/F) volumetric flow rate at discharge level $x = 0$ (note the negative sign), $f_{bk}(\phi)$ is the Kynch's batch sedimentation flux density function and $a(\phi)$ is the consolidation function. The last two terms on the LHS of Eq. (1) describe both the convective and sedimentation mechanisms while the RHS describes the compression mechanism.

There are several equations commonly used to describe $f_{bk}(\phi)$ and $a(\phi)$. In this paper, the Kynch' sedimentation function is given as (Michaels & Bolger, 1962):

$$f_{bk}(\phi) = v_\infty\phi\left(1 - \frac{\phi}{\phi_m}\right)^N, v_\infty < 0, N \geq 1, \quad (2)$$

where v_∞ is the settling velocity of a single particle in pure fluid, ϕ_m is the maximum solids volume fraction and N is a parameter related to particle property.

The consolidation function is described as follows:

$$a(\phi) := -\frac{f_{bk}(\phi)\sigma_e'(\phi)}{\Delta\rho g\phi}, A(\phi) := \int_0^\phi a(s)ds, \quad (3)$$

where $\Delta\rho$ is the solid-fluid density difference, g is the gravitational acceleration and σ_e' is the derivative of effective solid stress function as follows (Tiller & Leu, 1980):

$$\sigma_e(\phi) = \begin{cases} 0 & \text{for } \phi \leq \phi_c, \\ \sigma_0\left[\left(\frac{\phi}{\phi_c}\right)^k - 1\right] & \text{for } \phi > \phi_c, \end{cases} \quad (4)$$

$$\sigma_e'(\phi) = \begin{cases} 0 & \text{for } \phi \leq \phi_c, \\ \sigma_0 k\left(\frac{\phi^{k-1}}{\phi_c^k}\right) & \text{for } \phi > \phi_c, \end{cases} \quad (5)$$

The boundary conditions of Eq. (1) are given as follows:

1. $\phi(x,0) = \phi_0(x), \quad 0 \leq x \leq L, t = 0,$ (6)
2. $(Q_D(t)\phi + S(x)\left[f_{bk}(\phi) - \frac{\partial A(\phi)}{\partial x}\right] = Q_F(t)\phi_F(t), \quad 0 \leq t \leq T, x = L,$ (7)
3. $f_{bk}(\phi) - \frac{\partial A(\phi)}{\partial x} = 0, \quad 0 \leq t \leq T, x = 0.$ (8)

The first condition describes the initial concentration profile, while the second and third conditions describe the feed and discharge of the thickener. It is assumed that the feed has a flow rate of $Q_F(t) \leq 0$ (note the negative sign) with solids volume fraction of $\phi_F(t)$ and at the discharge level, Eq. (1) reduces to only its convective part.

2.2 Discretization

The development of control algorithm based on distributed parameter system as shown in the PDE (Eq. (1)) is complicated. Therefore, in order to enable model-based control design, the PDE is discretized such that the entire thickener is modelled as a number of vertical sections (cells) with the height of $\Delta x := L/J$ and $\Delta t := T/N$, where J and N are integers representing the number of cells and number of time-steps respectively. Let ϕ_j^n denote the approximate value of ϕ at (x_j, t_n), where $x_j := j\Delta x$, $t_n = n\Delta t$. With the knowledge of initial conditions ϕ_j^0 for $j = 0, \dots, \mathrm{J}$, the discretized Eq. (1) is given as follows:

$$\phi_j^{n+1} = \phi_j^n - \frac{1}{S_j}\left\{\frac{\Delta t}{\Delta x}\left[Q_D(t_n)\left(\phi_{j+1}^n - \phi_j^n\right) + S_{j+\frac{1}{2}} f_{bk}^{EO}\left(\phi_j^n, \phi_{j+1}^n\right) - S_{j-\frac{1}{2}} f_{bk}^{EO}\left(\phi_{j-1}^n, \phi_j^n\right)\right] - \frac{\Delta t}{\Delta x^2}\begin{bmatrix} S_{j+\frac{1}{2}}\left(A\left(\phi_{j+1}^n\right) - A\left(\phi_j^n\right)\right) \\ -S_{j-\frac{1}{2}}\left(A\left(\phi_j^n\right) - A\left(\phi_{j-1}^n\right)\right)\end{bmatrix}\right\}, \tag{9}$$

where $\mathrm{f}_{\mathrm{bk}}^{\mathrm{EO}}$ is the Engquist-Osher scheme of f_{bk} given as (Engquist & Osher, 1980):

$$f_{bk}^{EO}\left(\phi_j, \phi_{j+1}\right) := f_{bk}(0) + \int_0^{\phi_j} max\{f_{bk}'(s), 0\}\, ds + \int_0^{\phi_{j+1}} min\{f_{bk}'(s), 0\}\, ds\,. \tag{10}$$

To ensure the convergence of the resulting scheme, the following stability criterion must be satisfied:

$$\frac{1}{S_{min}}\begin{bmatrix}\frac{\Delta t}{\Delta x}\left(max|Q_D(t)| + S_{max}\, max|f_{bk}'(\phi)|\right) \\ + \frac{2\, max\, a(\phi)\Delta t}{\Delta x^2}\end{bmatrix} \leq 1. \tag{11}$$

One means to ensure the condition in Eq. (11) is satisfied is by choosing very small Δt. When $\Delta t \to 0$, the discretized Eq. (9) can be written as an ordinary differential equation (ODE) as follows:

$$\frac{d\phi_j}{dt} = -\frac{1}{S_j}\left\{\frac{1}{\Delta x}\left[Q_D(t)\left(\phi_{j+1}(t) - \phi_j(t)\right) + S_{j+\frac{1}{2}} f_{bk}^{EO}\left(\phi_j(t), \phi_{j+1}(t)\right) - S_{j-\frac{1}{2}} f_{bk}^{EO}\left(\phi_{j-1}(t), \phi_j(t)\right)\right] - \frac{1}{\Delta x^2}\left[S_{j+\frac{1}{2}}\left(A\left(\phi_{j+1}(t)\right) - A\left(\phi_j(t)\right)\right) - S_{j-\frac{1}{2}}\left(A\left(\phi_j(t)\right) - A\left(\phi_{j-1}(t)\right)\right)\right]\right\} \tag{12}$$

Incorporating the approximated boundary conditions given in Eqs. (7) and (8), the process descriptions in the feed $(j = J)$ and discharge $(j = 0)$ are given as follows:

$$\frac{d\phi_J}{dt} = -\frac{1}{S_J}\left\{\frac{1}{\Delta x}\left[Q_F(t)\phi_F(t) - Q_D(t)\phi_J(t) - S_{J-\frac{1}{2}} f_{bk}^{EO}\left(\phi_{J-1}(t), \phi_J(t)\right)\right] - \frac{1}{\Delta x^2}\left[-S_{J-\frac{1}{2}}\left(A\left(\phi_J(t)\right) - A\left(\phi_{J-1}(t)\right)\right)\right]\right\}, \tag{13}$$

$$\frac{d\phi_j}{dt} = -\frac{1}{S_0}\left\{\frac{1}{\Delta x}\left[Q_D(t)\left(\phi_1(t) - \phi_0(t)\right) + S_{\frac{1}{2}} f_{bk}^{EO}\left(\phi_0(t), \phi_1(t)\right)\right] - \frac{1}{\Delta x^2}\left[S_{\frac{1}{2}}\left(A\left(\phi_1(t)\right) - A\left(\phi_0(t)\right)\right)\right]\right\}. \tag{14}$$

3. CONTROL DEVELOPMENT AND STRUCTURE

3.1 Kalman Filter

In order to properly develop a model-based control algorithm for a paste thickener, actual plant data needs to be incorporated with the first principle model elaborated in Section 2 (See Eqs (12), (13) and (14)). This is carried out in two stages: 1) obtaining steady-state operating conditions required to identify the parameters related to different coal types; 2) using a Kalman Filter to predict the bed profile based on the measured U/F solids concentration and bed height. Additionally, the Kalman Filter is also used to allow the process model to incorporate the adaptive coal parameters.

In this work, the Filter gain L is calculated based on the linearized system of Eqs (12), (13) and (14) about a desired operating condition (Ramirez, 1994). To simplify the approach, a steady-state filter gain is applied rather than a time-varying one. This assumption is based on the fact that the observer is designed to operate continuously throughout the thickener operation which can be up to several months. The extended Kalman Filter for the thickener is then written in the form of:

$$\frac{d\hat{\phi}_j}{dt} = f(\hat{\phi}_{j-1}(\mathrm{t}), \hat{\phi}_j(\mathrm{t}), \hat{\phi}_{j+1}(\mathrm{t})) + L_{ss}\left(y(t) - \hat{y}(t)\right), \tag{15}$$

where L_{ss} is the steady-state Kalman filter gain, f is the nonlinear function in Eqs (12), (13) and (14), $y(t)$ is the measured outputs (U/F solids concentration and thickener bed volume) and $\hat{y}(t)$ is the predicted outputs from the observer.

One of the challenges in the control of paste thickener is the time-varying coal type in the feed. The Kalman Filter is also utilized here to estimate the model parameters of the thickener. In the dynamics model, there are five parameters that classify a coal type $(v_\infty, N, k, \phi_c, \sigma_0)$, however to simplify the approach, only k was estimated in the adaptive model due to its high sensitivity on the model. Since the residence time (time constant) of the process is very large, the effect of coal type change and hence its parameter is relatively slow and by using a Kalman Filter, the predicted $\hat{k}$ can be calculated as follows (Ramirez, 1994):

$$\frac{d\hat{k}}{dt} = L_{ss}\left(y(t) - \hat{y}(t)\right). \tag{16}$$

Again the measured outputs $y(t)$ are U/F solids concentration and bed volume.

3.2 Model Predictive Control

In the general case of paste thickeners, the cost function commonly includes the penalty on underflow solids concentration, bed height and discharge flow rate. The main benefit of utilizing MPC lies in its ability to treat operating constraints explicitly. For paste thickeners in this study, constraints include maximum/minimum U/F pump speed, bed level and U/F solids concentration. In some coal processing plant operation, the U/F rate of the thickened tailings is also subjected to the available coarse reject. The detailed cost function and constraints used in this paper are given in the following section.

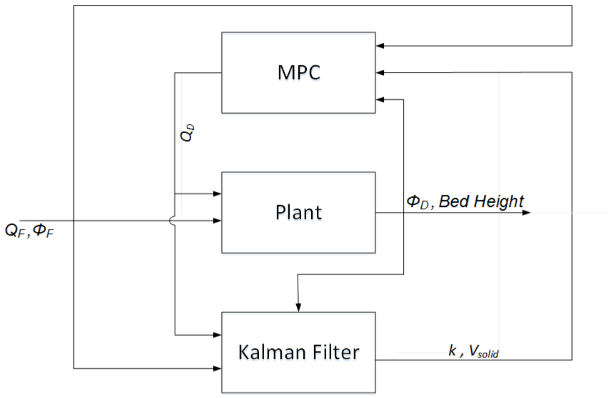

Fig. 2. Control structure for paste thickener with parameter estimation.

3.3 Control Implementation

In this paper, an MPC implementation approach is proposed which allows model parameters (related to the properties of different coal types) to be identified recursively online using the nonlinear process model, as shown in Fig. 2. This is accomplished by utilizing the real plant data in the extended Kalman Filter based on the nonlinear model to predict the coal type parameters inside the thickener. This has the advantage of allowing the implementation of a linear MPC to reduce the complexity on the control algorithm.

The paste thickener is fed with slurry from upstream with certain flow rate (Q_F) and solids volume fraction (ϕ_F) which are regarded as external disturbances to the plant. The actual measurements from the process include U/F solids concentration (ϕ_D) and bed level height. Based on these two measurements, the extended Kalman filter predicts the bed profile (ϕ_j at different discretized cells) and coal parameter k. Based on the predicted concentration profile, the Kalman Filter also calculates the amount of solids inventory (V_{solid}) in the thickener which is required to be within a certain operating range.

The MPC then uses Q_F, ϕ_F as measured disturbances and ϕ_D, V_{solid} as measured outputs to find the optimal underflow rate as a manipulated variable (MV) which regulates the U/F solids concentration to a set-point while maintaining the solids inventory within an allowed range subject to the linearized process model about a desired operating condition.

The MPC control law can be mathematically expressed as follows:

$$\min_{Q_D(t_n)} J = \sum_{t_n=t_k}^{t_k+N_c} (\phi_D(t_n) - \phi_D^*)^2 \tag{17}$$

Subject to:

$$\phi_{D,min} \leq \phi_D(t_n) \leq \phi_{D,max}$$
$$V_{solid,min} \leq V_{solid}(t_n) \leq V_{solid,max}$$
$$Q_{D,min} \leq Q_D(t_n) \leq Q_{D,max}$$

and the discrete process model in Eq. (9) and conditions in Eqs. (6), (7) and (8), where t_k is the current time, N_c is the prediction horizon (which is set to be looking at 60 steps ahead in this case) and ϕ_D^* is the set-point of ϕ_D. On the other hand, the control horizon is chosen to be 10 moves.

In some operations of paste thickener, the discharge is mixed with coarse reject prior to transport. In this case, the MV becomes the ratio (r) between the U/F rate and coarse reject rate (CR) as follows:

$$Q_D(t) = r(t)CR(t). \tag{18}$$

Then, the coarse reject rate becomes a measured disturbance. If this measurement is available several time steps ahead, then the MPC performance is expected to improve.

4. RESULTS AND DISCUSSION

The simulations are based on the actual plant disturbance obtained from Xstrata Bulga (in terms of Q_F and ϕ_F). Additionally, this plant also manipulates the U/F rate subject to the available coarse reject treated as additional disturbance. However, only the results based on using the U/F rate as MV are presented due to space limitation.

The discretized nonlinear model used has 13 states representing the solids concentration (ϕ_j) of 12 volume cells and 1 coal parameter k. The complete coal parameters used in the model are given in Table 1.

Table 1. Coal parameters

v_∞	-10^{-3} m/s
N	5
ϕ_c	0.15 v/v
k	7
σ_0	50 Pa

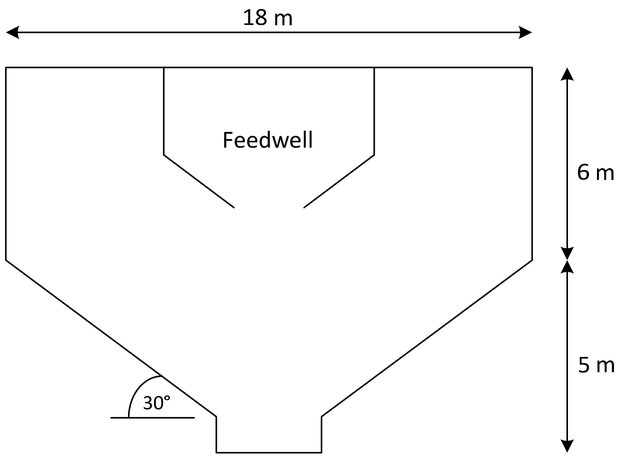

Fig. 3. Paste thickener dimensions.

In this study, the dimensions of the paste thickener are illustrated in Fig. 3. Note that it consists of one cylindrical and one conical sections. The desired operating conditions and set-point of the paste thickener are listed in Table 2.

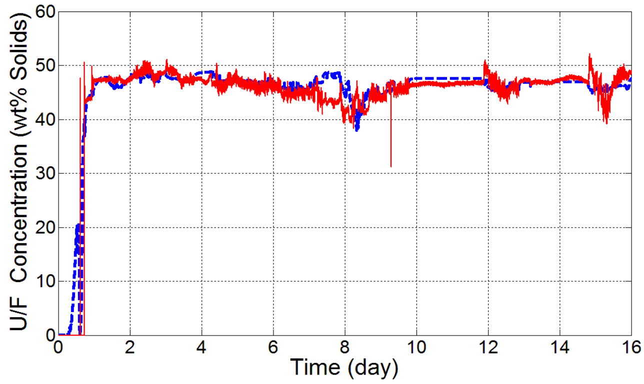

Fig. 4. Model validation with plant data (red (—) = plant data, blue (--) = model prediction).

Table 2. Operating conditions and constraints

ϕ_D^*	0.3206 v/v = 46% w/w
V_{solid}	150 m^3 $\leq V_{solid} \leq 450\ m^3$
Q_D	0 m^3/hr $\leq Q_D \leq$ 60 m^3/hr
$\emptyset_D$	$43.6\ wt\% \leq \emptyset_D \leq 58\ wt\%$

Fig. 4. shows the open-loop simulation of the developed process model indicating that the dynamics of the paste thickener can be modelled reasonably accurate. However, with the application of Kalman Filter as an observer, the predictions of the outlet solids concentration and bed level as shown in Figs. 5 and 6 have gained significant increase in accuracy (note the predicted U/F concentration almost overlaps the plant data). Furthermore, the Kalman Filter also allows the coal parameter k to be predicted which shows to fluctuate around the value of 7 in Fig. 5.

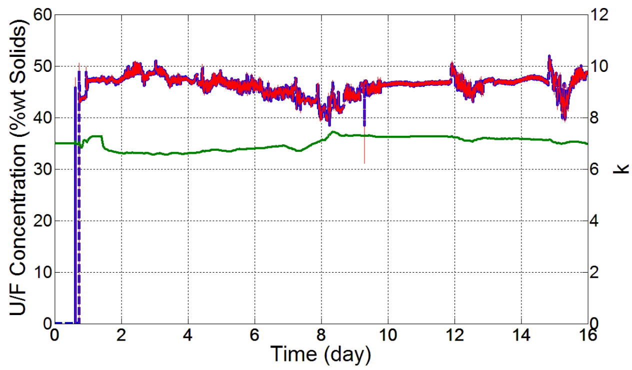

Fig. 5. Model prediction with Kalman Filter (red, (—) = plant data, blue (--) = model prediction, green (—) = k).

The green (-.) curves which indicate the feed solids in Figs. 7 and 8 show a significant level of disturbance entering the paste thickener. With only PI controller, the system cannot be controlled well as shown in Fig. 7. This is due to the inability of PI controller to treat constraints properly. Another important observation is that between day 10 to 12, when there is no feed to the paste thickener, the PI control responds too late while the MPC can correct this problem much faster. One major improvement of the PI controller is the U/F rate has much less fluctuation in comparison to original plant's MV as shown in Fig. 8.

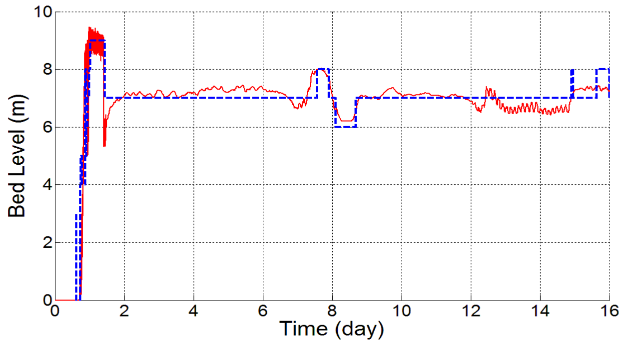

Fig. 6. Bed level prediction with Kalman Filter (red (—) = plant data, blue (--) = model prediction).

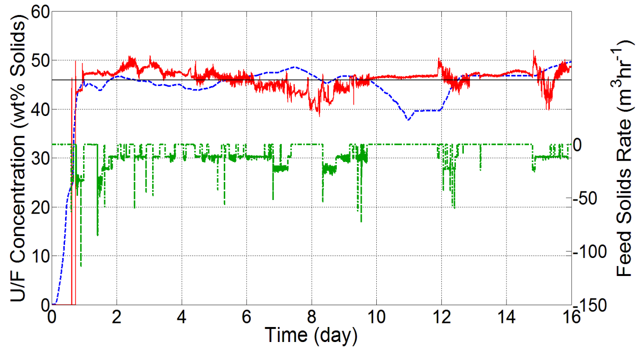

Fig. 7. Comparison of plant and PI controlled U/F solids concentration (red (—)= plant, blue (--) = PI, green (-.) = disturbance).

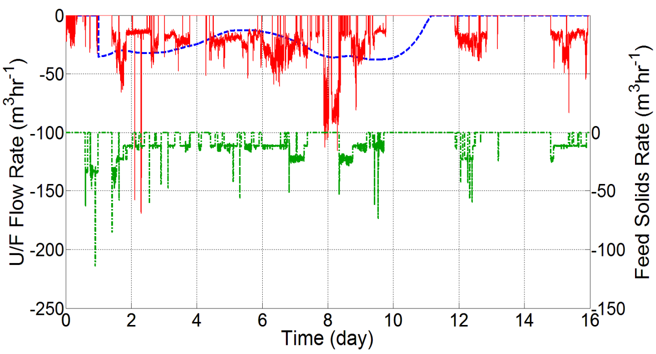

Fig. 8. Plant and PI controlled U/F and feed flow rates (red (—) = plant, blue (--) = PI, green (-.) = disturbance).

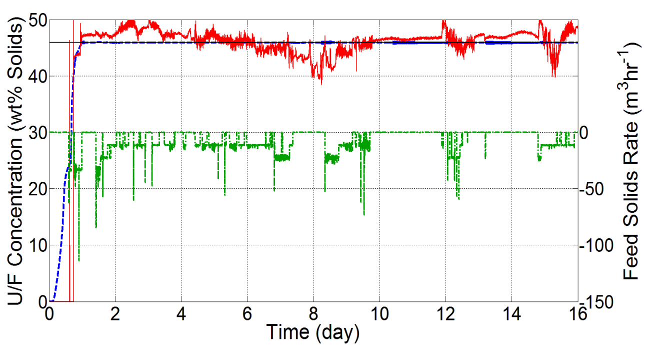

Fig. 9. Comparison of plant and MPC controlled U/F solids concentration (red (—) = plant, blue (--) = PI, green (-.) = disturbance).

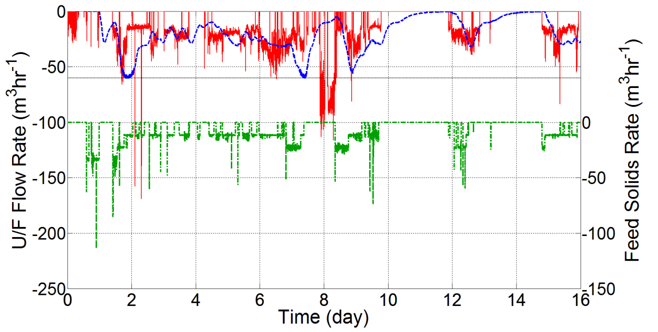

Fig. 10. Plant and MPC controlled U/F and feed flow rates (red (—) = plant, blue (--) = PI, green (-.) = disturbance).

Figs. 9 and 10 illustrate the response of the process model controlled using the MPC. In addition to the ability of MPC to control the process to set-point very well, the capacity of MPC to handle constraints explicitly can also be observed in Fig. 10 where the blue curve representing the U/F rate (as MV) always stays inside the 0 – 60 m^3/hr range. Furthermore, the V_{solid} constraint is also complied as shown in Fig. 11, except for the first day during filling-up. The bed level is also shown to be within 5-8 m which is desired in the paste thickener operation. This is one major advantage of using MPC over PI controller.

Fig. 9 shows the significant potential improvement provided by MPC as the U/F solids concentration always operates very close to the set-point in comparison to the existing control system as well as the result obtained using the PI controller. MPC's capability in utilising the process model in its control formulation has shown to improve the performance significantly. Additionally, by taking into account the plant disturbance (in this case the feed solids), the MPC is able to anticipate and reject the disturbance very effectively.

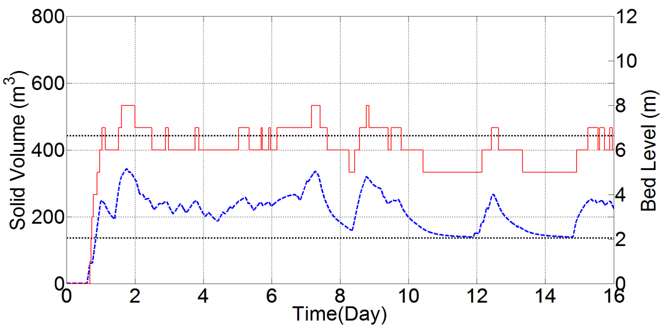

Fig. 11. The constrained solid volume and bed level response using MPC (red (—) = bed level, blue (--) = solid volume).

5. CONCLUSIONS

This paper proposed a dynamic model for paste thickeners based on first principle that can be used for the purpose of control design. Together with a Kalman Filter and online plant data, the model is able to predict the concentration profile and estimate the parameters of different coal types which vary continuously in a coal preparation plant.

An MPC algorithm was also formulated based on the above process model. The simulation results show the capability of MPC to improve the operation of the paste thickener in coal tailings treatment which translates to higher potential of water savings. The ability of MPC to treat process constraints explicitly also suits the paste thickener operation to enable the U/F solids concentration and bed height to be controlled within a certain window as well as to account for the physical restrictions of underflow pump speed. In summary, for paste thickener control, the MPC is able to deliver excellent performance because it utilizes the process model, take into account the effect of external disturbance as well as handle process constraints explicitly.

6. ACKNOWLEDGEMENTS

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REFERENCES

Abbott, J. et al., 1973. *Coal preparation plant effluent disposal by means of deep cone thickeners.* Paris, 6th International coal preparations congress.

Bürger, R., Damasceno, J. J. R. & Karlsen, K. H., 2004. A mathematical model for batch and continuous thickening of flocculated suspensions in vessels with varying cross-section. *International Journal of Mineral Processing,* Volume 73, pp. 183-208.

Bustos, M. C., Concha, F. & Wendland, W., 1990. Global weak solutions to the problem of continuous sedimentation of an ideal suspension. *Mathematical Methods in the Applied Sciences,* Volume 13, pp. 1-22.

Dorr, J. V., 1915. The use of hydrometallurgical apparatus in chemical engineering. *Journal of Industrial and Engineering Chemistry,* Volume 7, pp. 119-130.

Engquist, B. & Osher, S., 1980. Stable and entropy satisfying approximations for transonic flow calculations. *Mathematics of Computation,* Volume 34, pp. 45-75.

Kynch, G. J., 1952. A theory of sedimentation. *Transactions of the Faraday Society,* Volume 48, pp. 166-176.

Michaels, A. & Bolger, J., 1962. Settling rates and sediment volumes of flocculated Kaolin suspensions. *Industrial & Engineering Chemistry Fundamentals,* Volume 1, pp. 24-33.

Ramirez, W. F., 1994. *Process control and identification.* San Diego: Academic Press.

Segovia, J. P., Concha, F. & Sbarbaro, D., 2011. *On the control of sludge level and underflow concentration in industrial thickeners.* Milano (Italy), Preprints of the 18th IFAC World Congress.

Tiller, F. M. & Leu, W.-F., 1980. Basic data fitting in filtration. *Journal of The Chinese Institute of Chemical Engineers,* Volume 11, pp. 61-70.

Yao, Y., Tippett, M. J., Bao, J. & Bickert, G., 2012. *Dynamic modeling of industrial thickeners for control design.* Wellington, CHEMECA 2012.