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Adaptive Predictive Control using GOBF-ARX Models: An Experimental Case Study

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Abstract: Industrial applications of model predictive control rely mostly on linear empirical models obtained by employing time series analysis approaches. These models can quickly become obsolete and require maintenance when the operating conditions become significantly different from the design conditions. The need to generate good predictions in the face of changing operating conditions and / or plant characteristics can be fulfilled through updating the linear model parameters online. This work is aimed at the development of adaptive MPC (AMPC) scheme based on ARX models, which are parameterized using generalized orthonormal basis filters (GOBF). The proposed model structure, in addition to capturing the dynamics with respect to the manipulated inputs, facilitates modeling of stationary as well as non-stationary components of the unmeasured disturbances. The feasibility of using the proposed AMPC scheme is established by conducting experimental studies on a benchmark Heater-Mixer setup.

Keywords: Orthonormal Basis Filters, ARX Models, Model Predictive Control, Adaptive Control

1. INTRODUCTION

Model predictive control (MPC) has been used for many years to control the complex process units in chemical process industries. The key component of any MPC scheme is the dynamic model used for carrying out on-line predictions. Development and maintenance of the dynamic model is of paramount importance for achieving good closed loop performance. Qin and Badgwell (2003) in their review paper observed that industrial applications of model predictive control rely mostly on linear empirical models obtained by employing time series analysis approaches. These models can quickly become obsolete and require *maintenance* when the operating conditions become significantly different from the design conditions. The need to generate good predictions in the face of changing operating conditions and / or plant characteristics can be fulfilled through updating the linear model parameters online. This approach, however, has received relatively less attention in the industrial applications of MPC. Qin and Badgwell (2003), in their review of industrial MPC algorithms, point out that only two adaptive MPC (AMPC) algorithms have reached the market place despite strong market incentive for self-tuning MPC. The observed lack of interest in implementing these AMPC on industrial systems may be attributed to the reliability of poor on-line parameter schemes, which forms the core of any AMPC strategy.

In the review of adaptive control presented at CPC-V, Ydsti (1997) identifies the *admissibility problem* as one of the key issues that must be addressed in certainty equivalence control to achieve bounded input bounded output (BIBO) stability and robustness with respect to unmodelled dynamics and disturbances. This implies that the estimated model must be well behaved (controllable or stabilizable). Further, Ydsti (1997) identifies the *instability of the parameter estimator* or the *parameter drift* as another important issue that must be addressed while developing an adaptive control scheme. When the conventional model parameterization is used, the parameter drift can be avoided either by adding a deliberate perturbation to the set point or manipulated inputs or by using parameter projection to ensure that parameters do not go outside a bounded region. An alternate approach suggested by Ydsti (1997) to deal with both the difficulties is to use orthonomal basis filter (OBF) based model parameterization, such as Laguerre or Kautz filter based models, which guarantees that the identified model is well behaved. In the literature, however, OBF based models with output error structure have mostly been employed, which do not model the effect of unmeasured disturbances. Inclusion of the unmeasured disturbance models in model predictions, on the other hand, can significantly improve regulatory performance of an MPC scheme (Patwardhan et al. (2006)). Recently, Muddu et al. (2010) have shown that models with ARX structure can be parameterized using Orthonormal Basis Filters (OBF). These models explicitly capture the dynamics of the unmeasured disturbances and are parsimonious in parameters, which make them ideal candidates for the development of adaptive MPC schemes with better disturbance rejection abilities.

This work is aimed at the development of adaptive MPC scheme based on ARX models, which are parameterized using generalized orthonormal basis filters (GOBF) (Ninness and Gustafsson (1997)). The proposed model structure, in addition to capturing the dynamics with respect to the manipulated inputs, facilitates modeling of stationary as well as non-stationary components of the unmeasured disturbances. The feasibility of using the proposed AMPC scheme is established by conducting experimental studies on a benchmark Heater-Mixer setup (Thornhill et al. (2008)).

The latter part of the paper is organized in the following sequence. Section 2 presents rationale behind the proposed model parameterization and the model parameter estimation schemes. Sections 3 presents the adaptive frame work for the Model Predictive Control. The experimental evaluation of the proposed AMPC schemes are presented in section 4. The main conclusions reached through the analysis of these results are presented in the last section.

2. MODEL IDENTIFICATION

The identification exercise is proposed to be carried out in two phases. The initial model identification is carried out from the offline data obtained from the input-output perturbation plant data. The parameters of the state to output map are later updated on-line. In this work, it is proposed to model an $r \times m$ MIMO system as r MISO OBF-ARX models. However, to keep notations simple, formulation and adaptation of a single MISO GOBF-ARX model is described in this section.

2.1 ARX Model Parameterization using GOBF

To begin with, consider a general form of MISO time series model

$$y(k) = \sum_{k=1}^{m} G_i(z,\theta)\mathbf{u}_i(k) + H(z,\theta)e(k) \tag{1}$$

where $\{e(k)\}$ is a zero mean white noise sequence and $\theta \in R^p$ represents model parameters. For the purpose of parameter estimation, a one step predictor of the form

$$\hat{y}(k|k-1) = \sum_{k=1}^{m} W_{u,i}(z,\theta)\mathbf{u}_i(k) + W_y(z,\theta)y(k) \tag{2}$$

is developed where

$$W_{u,i}(z,\theta) = H(q)^{-1}G_i(z,\theta)$$
$$W_y(z,\theta) = (1 - H(z,\theta)^{-1})y(k)$$

Muddu et al. (2010) have proposed to parameterize $W_{u,i}(z,\theta)$ and $W_y(z,\theta)$ using the generalized orthonormal basis filters (GOBF) Ninness and Gustafsson (1997)

$$F_l(z,\xi) \;=\; \frac{\sqrt{(1-|\xi_l|^2)}}{(z-\xi_l)}\prod_{i=1}^{l-1}\frac{(1-\xi_i^* z)}{(z-\xi_i)}. \tag{3}$$

where $\{\xi_l : l = 1,2,...\}$ is an arbitrary sequence of poles inside the unit circle appearing in complex conjugate pairs. These filters represent an orthonormal basis for the set of strictly proper stable transfer functions. The main advantage of using orthogonal basis filters is that, with a judicious choice of the filter parameters, any strictly proper stable transfer function, say G(z), can be approximated as

$$G(z) \approx \sum_{j=1}^{n} \alpha_j F_u(z,\xi)$$

where ξ represent vector of GOBF poles. Thus, RHS of equation (2) is parameterized using GOBF as follows

$$W_{u,i}(z,\theta) = \sum_{j=1}^{n_i} \alpha_{ij} F_{u,j}(z,\xi_{u_i}) \tag{4}$$

$$W_y(z,\theta) = \sum_{j=1}^{n_y} \beta_j F_{y,j}(z,\xi_y) \tag{5}$$

Following Patwardhan and Shah (2005), a state realization of the MISO OBF-ARX model can be expressed as follows

$$\mathbf{x}(k+1) = \mathbf{\Psi}\mathbf{x}(k) + \mathbf{\Gamma}\mathbf{u}(k) + Ky(k) \tag{6}$$

$$y(k) = \mathbf{C}\mathbf{x}(k) + e(k) \tag{7}$$

where $\mathbf{x}(k) \in R^n$ represents the state vector, $\mathbf{u}(k) \in R^m$ represent the vector of manipulated inputs and $\{e(k)\}$ represents innovation sequence. This model representation, referred to as *predictor form* of state realization in the rest of the text, is convenient to use at the parameter identification stage. For the purpose of controller development, this form can later be transformed into the innovation form as follows

$$\mathbf{x}(k+1) = \mathbf{\Phi}\mathbf{x}(k) + \mathbf{\Gamma}\mathbf{u}(k) + Ke(k) \tag{8}$$

$$y(k) = \mathbf{C}\mathbf{x}(k) + e(k) \tag{9}$$

where $\mathbf{\Phi} = \mathbf{\Psi} + \mathbf{KC}$.

2.2 Off-line Parameter Estimation

The model parameters identified by the prediction error approach, i.e. by minimizing an objective function of the form

$$\mathbf{J} = \frac{1}{N}\sum_{k=1}^{N} e(k)^2 = \frac{1}{N}\sum_{k=1}^{N}[y(k) - \hat{y}(k|k-1)]^2 \tag{10}$$

The key step in the development of OBF based models is the selection of filter poles and the number of basis filters (filter order) necessary to develop a reasonably good approximation of the system dynamics. We choose filter orders *a priori* and perform a search in the set of filter poles as proposed by Patwardhan and Shah (2005). An interesting feature of the predictor form of state space realization is that matrices $(\mathbf{\Psi},\mathbf{\Gamma},K)$ are functions of $\{\xi_{u_i} : i = 1,2,..m\}$ and $\{\xi_y\}$ alone. Also, the C matrix is function of only the series expansion coefficients $\{\alpha_{u_i} : i = 1,2,..m, j = 1,2,...n_i\}$ and $\{\beta_j : j = 1,2,...n_i\}$. This feature is exploited to arrive at a nested optmization formulation of the parameter estimation problem as follows:

$$\left(\widehat{C},\hat{\xi}\right) \;=\; arg\min_{\xi}\frac{1}{N}\sum_{\mathbf{k=1}}^{N}[e(k,\widehat{C}(\xi)]^2, \tag{11}$$

subject to

$$|\xi_i| \leq 1 \quad for\ i = 1,2,...n_i \tag{12}$$

where $\xi = \left[\xi_{u_1}^T \; \xi_{u_2}^T \; ... \; \xi_y^T \right]^T$. Given a guess of pole vector ξ, the parameter vector $\widehat{C}(\xi)$ is estimated by solving another optimization problem.

$$\widehat{C} = arg\min_C \frac{1}{N}\sum_{k=1}^{N}[e(k,C,\xi)]^2. \tag{13}$$

For a given GOBF poles (ξ) the parameter vector, the later problem can be solved analytically using linear regression. This analytical form is as follows:

$$\widehat{C}^T(\xi) = \left(\sum_{k=1}^{N}\mathbf{x}(k)\mathbf{x}(k)^T\right)^{-1}\sum_{k=1}^{N}\mathbf{x}(k)y(k) \tag{14}$$

In the remaining part of the text, the MISO model matrices identified from the offline parameter estimation exercise are referred to either as $(\overline{\mathbf{\Psi}}, \overline{\mathbf{\Gamma}}, \overline{K}, \overline{C})$.

2.3 On-line Model Parameter Estimation

It may be noted that the identified model is valid in the neighborhood of the operating point where initial perturbation data is generated. To accurately predict the plant behavior under changing circumstances, it becomes necessary to update the model parameters on-line. To adapt the model to changing operating conditions, we exploit the fact that the state to output map is linear in parameters and amenable to on-line recursive parameter estimation. It is proposed to model changing operating conditions through modification of the measurement equation as follows

$$y(k) = \overline{y}(k) + C(k)\,\mathbf{x}(k) + e(k) \tag{15}$$

The term $\overline{y}(k)$ is introduced to model the effects of time varying unmeasured disturbances in the output as an additive drift. The parameters of the modified state to output map, i.e.$\overline{y}(k)$ and $C(k)$, are updated online using recursive least square (RLS) estimation approach. Defining parameter vector and regressor vector for the MISO model as

$$\theta(k)^T = \left[\, \overline{y}(k) \;\; (C(k))^T \,\right] \tag{16}$$

$$\varphi(k)^T = \left[\, 1 \;\; (\widehat{\mathbf{x}}(k|k-1))^T \,\right] \tag{17}$$

where $\widehat{\mathbf{x}}(k)$ represents estimate of $\mathbf{x}(k)$, the parameter vector can be updated using the conventional RLS as follows

$$\widehat{\theta}(k) = \widehat{\theta}(k-1) + \mathbf{L}(k)e(k) \tag{18}$$

$$e(k) = y(k) - \;[\varphi(k)]^T\,\widehat{\theta}(k-1) \tag{19}$$

$$\mathbf{L}(k) = \mathbf{P}(k-1)\varphi(k)[\lambda + \varphi(k)^T\mathbf{P}(k-1)\varphi(k)^T]^{-1} \tag{20}$$

$$\mathbf{P}(k) = [\mathbf{I} - \mathbf{L}(k)\varphi(k)^T]\mathbf{P}(k-1)/\lambda \tag{21}$$

where $\mathbf{L}$ represents Kalman gain matrix and $\mathbf{P}$ represents the covariance matrix of the estimation error. The parameter vector is initialized as follows

$$\theta(0)^T = \left[\, 0 \;\; (\overline{C})^T \,\right]^T$$

and $\mathbf{P}(0)$ is initialized using results of the off-line parameter exercise. The λ represents the forgetting factor $0.95 \leq \lambda \leq 1$. Thus, at any instant k, the online parameter estimation involves

- Computation of regressor vector using the predictor form
  $\widehat{\mathbf{x}}(k) = \overline{\mathbf{\Psi}}\widehat{\mathbf{x}}(k-1) + \overline{\mathbf{\Gamma}}\mathbf{u}(k-1) + \overline{K}\,[y(k-1) - \overline{y}(k-1)]$
- Estimation of $(d(k), C(k))$ using the recursive least squares approach

The innovation form representation of the MISO model with the proposed parameter adaptation assumes following form

$$\widehat{\mathbf{x}}(k+1|k) = \Phi(k)\widehat{\mathbf{x}}(k|k-1) + \overline{\mathbf{\Gamma}}\mathbf{u}(k) + \overline{K}e(k) \tag{22}$$

$$e(k) = [y(k) - \overline{y}(k)] - C(k)\widehat{\mathbf{x}}(k|k-1) \tag{23}$$

where $\Phi(k) = \overline{\mathbf{\Psi}} + \overline{K}C(k)$. Thus, the poles of the model used for controller development are varying with time.

3. ADAPTIVE MPC FORMULATION

In this section, adaptive MPC (AMPC) formulation is developed based on GOBF ARX models identified online. Since we have r outputs, r such MISO parameter estimators are used in parallel. Thus, at k'th sampling instant, we have r innovation form of models

$$\mathbf{x}^{(i)}(k+1) = \Phi^{(i)}(k)\mathbf{x}^{(i)}(k) + \overline{\mathbf{\Gamma}}^{(i)}\mathbf{u}(k) + \overline{K}^{(i)}\mathbf{e}_i(k) \tag{24}$$

$$\mathbf{y}_i(k) = \overline{\mathbf{y}_i}(k) + C^{(i)}(k)\,\mathbf{x}^{(i)}(k) + \mathbf{e}_i(k) \tag{25}$$

where $i = 1, 2, ...r$ available for carrying out predictions. For a $r \times m$ multi-input/multi-output (MIMO) process, defining a combined state vector

$$\mathbf{X}(k) = \left[\, \mathbf{x}^{(1)}(k)^T \;\; \mathbf{x}^{(2)}(k)^T \;\; .... \;\; \mathbf{x}^{(r)}(k)^T \,\right]^T \tag{26}$$

r such MISO models can be stacked into one MIMO model as

$$\mathbf{X}(k+1) = \mathbf{\Phi}(k)\mathbf{X}(k) + \overline{\mathbf{\Gamma}}\mathbf{u}(k) + \overline{\mathbf{K}}\mathbf{e}(k) \tag{27}$$

$$\mathbf{y}(k) = \overline{\mathbf{y}}(k) + \mathbf{C}(k)\mathbf{x}(k) + \mathbf{e}(k) \tag{28}$$

where $(\mathbf{\Phi}(k), \overline{\mathbf{\Gamma}}, \overline{\mathbf{K}}, C(k))$ are constructed by appropriately stacking

$$\left\{\Phi^{(i)}(k), \overline{\mathbf{\Gamma}}^{(i)}, \overline{K}^{(i)}, C^{(i)}(k) : i = 1, 2, ., r\right\}$$

3.1 Future Trajectory Predictions

Given a guess of the future manipulated inputs, say $\{\mathbf{u}(k+i|k) : i = 0, 1, 2, ...p-1\}$, the model can be used to generate predictions over future time window $[k+1, k+p]$ as follows

$$\widehat{\mathbf{X}}(k+i+1|k) = \mathbf{\Phi}(k)\widehat{\mathbf{X}}(k+i|k) + \overline{\mathbf{\Gamma}}\mathbf{u}(k+i|k) + \overline{\mathbf{K}}\mathbf{e}_f(k) \tag{29}$$

$$\mathbf{y}(k+i|k) = \overline{\mathbf{y}}(k) + \mathbf{C}(k)\widehat{\mathbf{X}}(k+i|k) + \mathbf{e}_f(k) \tag{30}$$

$$\mathbf{e}_f(k+1) = \alpha\mathbf{e}_f(k) + (1-\alpha)\mathbf{e}(k) \tag{31}$$

$$\mathbf{e}(k) = \mathbf{y}(k) - \left[\overline{\mathbf{y}}(k) + \mathbf{C}(k)\widehat{\mathbf{X}}(k|k-1)\right] \tag{32}$$

where $0 \leq \alpha < 1$ is a tuning parameter used to attenuate the effect of high frequency noise on the future predictions. The initial state at the beginning of the prediction horizon is estimated as follows

$$\widehat{\mathbf{X}}(k|k-1) = \mathbf{\Phi}(k)\widehat{\mathbf{X}}(k-1|k-2) + \overline{\mathbf{\Gamma}}\mathbf{u}(k-1) + \overline{\mathbf{K}}\mathbf{e}(k-1) \tag{33}$$

3.2 AMPC Formulation with Time Varying Terminal Weighting

The model predictive control problem at the sampling instant k is defined as constrained optimization problem where the set of future manipulated input moves,

$$\mathbf{U}_f(k) = \{\mathbf{u}(k|k), \mathbf{u}(k+1|k), ..., \mathbf{u}(k+q-1|k)\}$$

where q is called as control horizon, are computed by minimizing the objective function

$$\begin{aligned} \min_{\mathbf{U}_f^{(k)}} \; & \epsilon(k+p|k)^T \mathbf{W}_\infty(k) \epsilon(k+p|k) \\ & + \sum_{i=1}^{p-1} \mathbf{E}(k+i|k)^T \mathbf{w}_E \mathbf{E}(k+i|k) \\ & + \sum_{i=1}^{q-1} \triangle\mathbf{u}(k+i|k)^T \mathbf{w}_{\triangle U} \mathbf{u}(k+i|k) \end{aligned} \tag{34}$$

subject to the following constraints

$$\mathbf{u}^L \leq \mathbf{u}(k+i|k) \leq \mathbf{u}^H \quad for \;\; i = 0, 1, .., q-1 \tag{35}$$

$$\Delta\mathbf{u}^L \leq \Delta\mathbf{u}(k+i|k) \leq \Delta\mathbf{u}^H \quad for \;\; i = 0, 1, .., q-1 \tag{36}$$

where,

$$\mathbf{E}(k+i|k) = \mathbf{r}(k+i|k) - \hat{\mathbf{y}}(k+i|k) \tag{37}$$

$$\epsilon(k+p|k) = \widehat{\mathbf{X}}(k+p|k) - \overline{\mathbf{X}}_s(k)..... \tag{38}$$

$$\triangle\mathbf{u}(k+i|k) = \mathbf{u}(k+i|k) - \mathbf{u}(k+i-1|k) \tag{39}$$

Here, W_E is a symmetric positive semidefinite error weighting matrix and $W_{\triangle U}$ is symmetric positive definite input weighting matrix. The selection of the above matrix is based on the process economics and relative importance of inputs and/or outputs. Matrix $\mathbf{W}_\infty(k)$ represents time varying terminal weighting matrix, which is evaluated at every sampling instant by solving the discrete Lyapunov equation given as

$$\mathbf{w}_\infty(k) = \mathbf{C}(k)^T \mathbf{w}_E \mathbf{C}(k) + \mathbf{\Phi}(k)^T \mathbf{w}_\infty \mathbf{\Phi}(k) \tag{40}$$

The time dependent *terminal target state*, $\overline{\mathbf{x}}_s(k)$, is estimated as follows

$$\begin{aligned} \overline{\mathbf{X}}_s(k) = & \left[\mathbf{C}(k)(\mathbf{\Phi} - \mathbf{I})^{-1}\overline{\mathbf{\Gamma}}\right]^{-1} \\ & \left[\mathbf{r}(k) - \left(\mathbf{C}(k)(\mathbf{I} - \mathbf{\Phi}(k))^{-1}\overline{\mathbf{K}} + \mathbf{I}\right)\mathbf{e}_f(k)\right] \end{aligned}$$

The above optimization problem is transformed and solved as a quadratic programming (QP) problem and only the first move $\mathbf{u}_{opt}(k|k)$ is implemented on the plant,i.e. $\mathbf{u}(k) = \mathbf{u}_{opt}(k|k)$. The optimization problem is reformulated at the next sampling instant based on the updated information from the plant.

4. EXPERIMENTAL STUDIES ON HEATER-MIXER SETUP

In this section, the proposed adaptive model predict control scheme is demonstrated on a benchmark heater-mixer setup at Automation Lab, Dept. of Chemical Engineering, IIT Bombay and more details found in Thornhill et al. (2008). The heater-mixer setup consists of two stirred

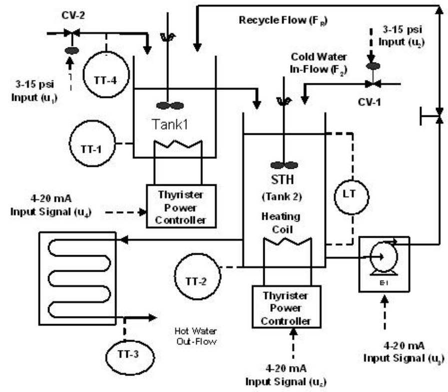

Fig. 1. Schematic Diagram of Heater-Mixer Setup.

tanks in series as shown in Figure (1). The contents in the tanks are well stirred by using variable speed agitators. A cold water stream is introduced in the first tank through CV-2. The contents of the first tank is heated using a 4 kWH heating coil. The hot water that overflows the first tank is mixed with cold water stream entering in to the second tank through CV-1. The contents of the second tank is heated using another 3.5 kwh heating coil. The water from second tank is also recycled back to the first tank. The heat supplied to both the tanks can be varied continuously using thyristor power controllers, which take 4-20 mA as inputs. The cold water inlet flow to both the tanks can be manipulated using pneumatic control valves. The temperatures in the first tank (T_1), in the second tank (T_2) and the liquid level in the second tank (H_2) are measured variables and controlled variable. The heat inputs to the first tank ($\mathbf{u}_1$), to the second tank ($\mathbf{u}_2$) and cold water flow to the second tank ($\mathbf{u}_3$) are treated as manipulated inputs. The cold water flow to the first tank is kept constant. The system is interfaced with a control computer using a data acquisition system (Advantech, ADAM 5000 series hardware) through LABVIEW-8.20 and MATLAB. The steady state operating condition for the process is given in Table (1). The cold water temperature (T_C) changes slowly during the experimentation and acts like a drifting disturbance.

Table 1. Heater Mixer Setup: Operating conditions

Variable	Value
$F1$	10 mA = 79 lph
$\mathbf{u}_1$	12 mA = 2497.1 W
$\mathbf{u}_2$	12mA = 2431.2 W
$\mathbf{u}_3$	10mA = 89 lph
Tc	31.8 ^{0}C
T_a (ambient temperature)	30 ^{0}C
T_1	58 ^{0}C
T_2	52 ^{0}C
H_2	38 cm

For generating data for model identification and validation, simultaneous input perturbations Pseudo Random Binary Signal (PRBS)are introduced into the process. Separate data sets are used for model identification and validation. A sampling time of 5 seconds is used while collecting data. The inputs to the setup were designed using the '*idinput*' function in System Identification toolbox in MATLAB. The details of the construction of these signals are provided in Table 2.

Table 2. Heater Mixer Setup: Input perturbation signal details

Input	Frequency Band	Magnitude of Inputs	
Heat Input 1 ($\mathbf{u_1}$)	[0 0.05]	[-846W	610W]
Heat Input 2 ($\mathbf{u_2}$)	[0 0.05]	[-623W	448W]
Flow Input ($\mathbf{u_3}$)	[0 0.03]	[-18 lph	28 lph]

Out of the total 1600 data points that were collected, the first 900 data points were used for model identification (identification data set) and the remaining 700 data points were used for model validation (validation data set).

4.1 Off-line Model Identification

Three models (two MISO and one SISO) were identified for the above system. The models developed by choosing two poles between each input-output pair and the resulting optimal pole locations of the model are listed in Table 3. It may be noted that, in this table, instead of reporting discrete pole locations (ξ_i), *equivalent continuous time pole location* (a_i) have been reported such that

$$\xi_i = exp(-a_i T) \tag{41}$$

where T represents sampling time.

Table 3. Heater - Mixer Setup: Optimum GOBF Pole Locations for OBF-ARX model.

Input	$\mathbf{MISO-T_1}$	$\mathbf{MISO-T_2}$	$\mathbf{SISO-H_2}$
$\mathbf{u}_1$	[0.197 0.2002]	[0.0674 0.0674]	[-]
$\mathbf{u}_2$	[0.0518 0.0449]	[0.0242 0.0239]	[-]
$\mathbf{u}_3$	[0.0417 0.0319]	[0.0398 0.0398]	[0.4046 0.4046]
$\mathbf{y}_i$	[0.8543 0.0530]	[0.1218 0.0123]	[15.6572]

The dynamic validation (infinite horizon predictions) of the identified model is presented in Figure 2 and corresponding input moves are shown in Figure 3. It is clear from this figure that the models are able to predict initial output variability quite well except for the bias.

4.2 Closed Loop Performance

This section is aimed at demonstrating the efficacy of the proposed AMPC schemes using the benchmark Heater-Mixer experimental setup. The servo experiments involve introducing simultaneous step changes in the temperature and level setpoints and later bringing the system back to the original setpoints. The manipulated input moves are computed subject to the following move constraints

$$-261 \quad W \leq \Delta u_1(k) \leq 234 \ W \tag{42}$$

$$-262 \quad W \leq \Delta u_2(k) \leq 162 \ W \tag{43}$$

$$-13 \quad lph \leq \Delta u_3(k) \leq 17 \,.lph \tag{44}$$

The tuning parameters used for in this study are presented in the Table 4. The variation of controlled variable is shown in Figure 4 and corresponding variations of manipulated variables are shown in Figure 5. It may be noted that

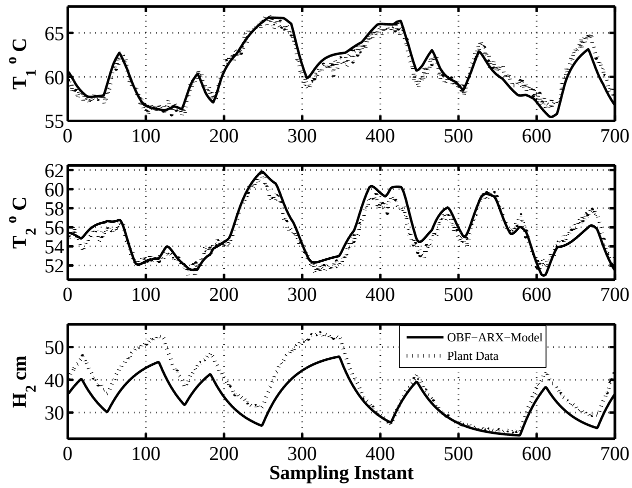

Fig. 2. Heater-Mixer Setup: Dynamic validation model-Variations of Model output and plant data.

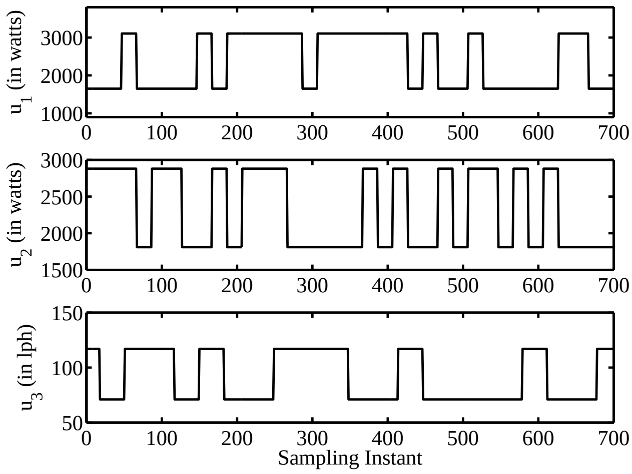

Fig. 3. Heater-Mixer Setup: Dynamic validation model.-Variations of inputs.

Table 4. Heater-Mixer Setup: AMPC Tuning Parameters

Parameters	values
Prediction Horizon	60
Control Horizon	1
Input blocking	[60]
Input weighting matrix	$\mathbf{I}_{3\times3}$
Error weighting matrix	$\mathbf{I}_{3\times3}$
Filter Coefficient	[0.95 0.95 0.9]

the inlet temperature of the cold water entering both the tanks keeps drifting during these servo experiments (see Figure 7) and acts as an unmeasured disturbance. The model adaptation characteristics best explained through variations of model sensitivity and these variations are reported in terms of *model sensitivity matrix (S)*, which is defined as follows

$$\mathbf{S}(k) = \mathbf{C}(k)[\mathbf{I} - \mathbf{\Phi}(k)]^{-1}\mathbf{\Gamma} \tag{45}$$

and these variations of are shown in Figure 6. This figure indicates that there is significant changes in the model as operating conditions changes. The results clearly reflects that the proposed AMPC strategy is able to track the desired set point changes.

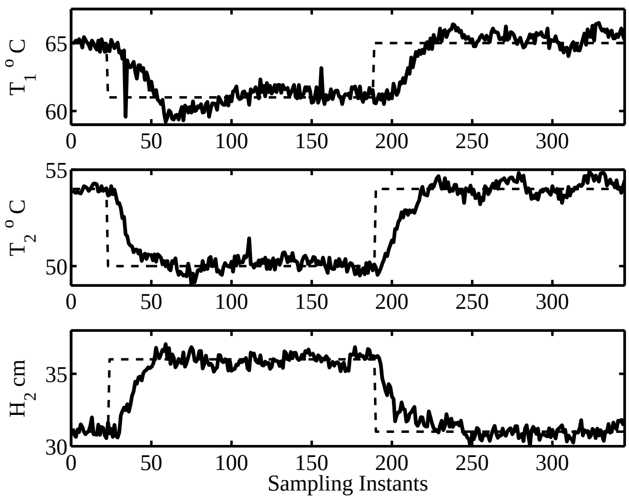

Fig. 4. Heater-Mixer Setup: Variations of controlled variable.

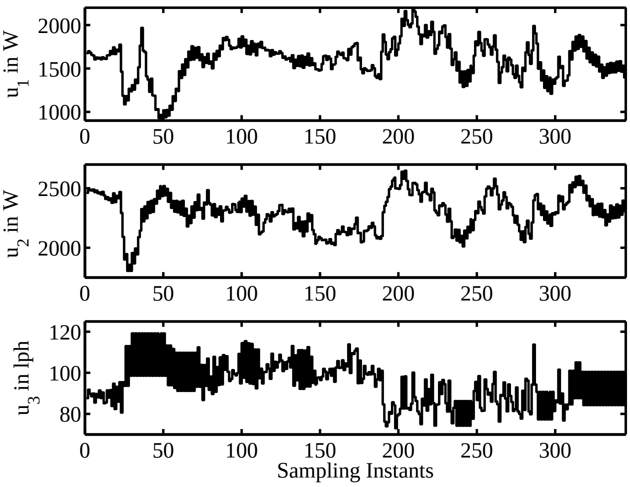

Fig. 5. Heater-Mixer Setup: Variations of manipulated variable.

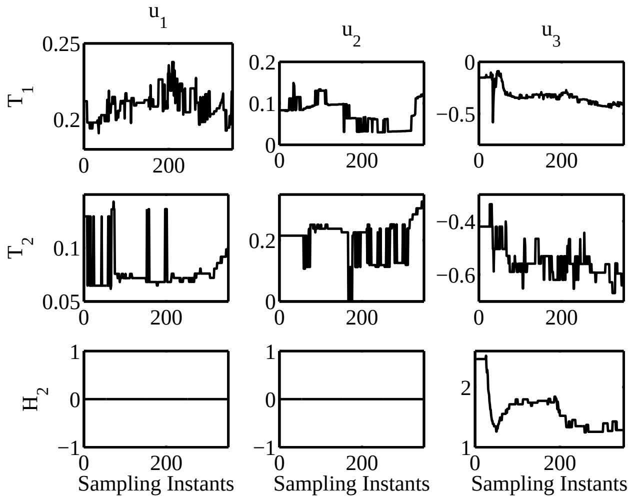

Fig. 6. Heater-Mixer Setup: Variations of model sensitivity.

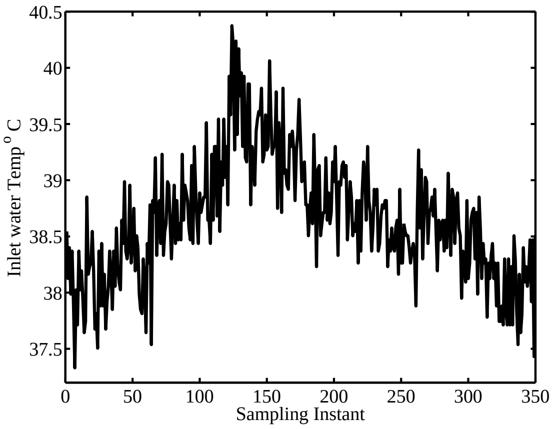

Fig. 7. Heater-Mixer Setup: Variations of unmeasured disturbance (i.e inlet water temperature to Tank-1 and tank-2).

5. CONCLUSION

This work presents an adaptive MPC scheme based on ARX models, which are parameterized using generalized orthonormal basis filters (GOBF). The proposed model structure facilitates modeling of stationary as well as non-stationary components of the unmeasured disturbances. Evaluation of the proposed AMPC scheme using experimental studies on a benchmark experimental Heater-Mixer setup demonstrates that the proposed AMPC strategy is able to track the desired set point changes, while simultaneously rejecting the unmeasured disturbances. The manipulated inputs, however, exhibit undesirable fluctuations and further improvements need to be made to ensure smoother manipulated input profiles.

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