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Input-output linearizing control of a thermal cracking furnace described by a coupled PDE-ODE system

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Abstract: This research presents a control scheme for a gas-fired ethylene dichloride (EDC) cracking furnace to handle a cracked gas temperature at the coil outlet. Input-output (I/O) linearizing control scheme is applied to the furnace model of which interaction between a lumped temperature of gas-fire radiating wall and spatially distributed dynamics of cracking coil is considered. In the proposed method, the feedback I/O linearizing controller for coupled PDE-ODE system is applied to force the cracked gas temperature to follow a desired trajectory by manipulation of a fuel gas flow rate. Simulation results showed that the proposed controller successfully forced the controlled output to a desired setpoint without off-set.

Keywords: Coupled PDE-ODE models; Model-based control; Input-output linearization; EDC cracking furnace

1. INTRODUCTION

A gas-fired furnace is an important unit which is basically used in a thermal cracking of hydrocarbons in petrochemical industries. For a practical cracking furnace, the cracking coil behavior which is placed between radiating walls inside gas-fire furnace is varies along the spatial coordinate together with time. The typically proportional-integral-derivative (PID) control applied to the cracking furnace may be insufficient to maintain the controlled output within the desired condition because of interacting dynamics between the cracking coil and radiating wall

There are many research works proposed advance control techniques taking into account the spatial effect of process variables. For example, as pointed out in Shang et al. (2005) and Hoo and Zheng (2001), method of characteristics and Galerkin method were applied to a tubular reactor model governed by a hyperbolic partial differential equation (PDE) system for a simplification of controller formulation. However, the above methods cannot be applied directly to the cracking furnace that the interacting dynamics should be taken into account. For the cracking furnace, the cracking coil dynamics can be described by partial differential equation (PDE) while the furnace wall dynamic can be modeled by ordinary differential equation (ODE). There are a few works mentioned on a controller development for coupled PDE-ODE systems. Moghadam, et al. (2010) presented an application of linear quadratic regulator (LQR) controller for handling a continuous stirred tank reactor-plug flow reactor (CSTR-PFR) system. Panjapornpon et al. (2012) proposed the control method for a thermal ethylene dichloride (EDC) cracking furnace that the interactions between process dynamics were addressed. The lumped dynamics of tube wall and furnace wall are assumed in the developed method. The control objective is to handle both tube wall temperature and mass production rate of vinyl chloride monomer (VCM).

This work proposed an extended study of a control method for a thermal EDC cracking furnace described by coupled PDE-ODE system. The lumped dynamics of furnace wall and the spatially distributed dynamics of cracked gas and tube wall conduction are currently considered. The objective is to control the average cracked gas temperature in the coil by fuel gas flow rate regulation. The proposed controller is developed based on input-output (I/O) linearizing control which is applied to the coupled PDE-ODE system.

The paper is structured as follows. In Section 2, preliminaries of mathematical model and input-output linearization technique are explained. In Section 3, the mathematical model of an EDC cracking furnace is described. Section 4 presented the formulation of the control system and applied to the process model. In Section 5, the simulated closed-loop responses of the process are illustrated.

2. PRELIMINARIES

2.1 Formulation of the problem

Consider the mathematical model of the distributed parameter system described in Eq. (1). The model is a coupling of PDE and ODE with the controlled output (y) and manipulated input (u)

$$\begin{aligned}
\frac{\partial x_p(z,t)}{\partial t} &= A\frac{\partial x_p(z,t)}{\partial z} + B\frac{\partial^2 x_p(z,t)}{\partial z^2} + \\
&\quad M\left(x_p(z,t)\right) + N\left(x_o(t), x_p(z,t)\right) \\
\frac{dx_o(t)}{dt} &= f\left(x_o(t), x_p(z,t), u(t)\right) \\
y &= h\left(x_0(t), x_p(z,t)\right)
\end{aligned} \tag{1}$$

with the following boundary and initial conditions:

$$\begin{aligned}
x_p(0,t) &= x_{p,z=0}(t) \\
x_p(L,t) &= x_{p,z=L}(t) \\
x_p(z,0) &= x_{p,0}(z) \\
x_o(0) &= x_{o,0}
\end{aligned}$$

where $x_p(z,t)$ denote the state variables which depend on the spatial coordinate and time, $x_o(t)$ denote the state variables which depend on the time, y denote the output variable, $z\in[0, L]$ is the spatial coordinate, $t\in[0,\infty]$ is the time, $u(t)$ are the manipulated variables and A, B are constant matrices.

2.2 Input-output linearization technique

The dynamic behavior of $x_p(z,t)$ and $x_o(t)$ are considered in order to investigate the relationship between the controlled output y at the exit position ($z = L$) and the manipulated input u. The compact form of system in Eq. (1) at the considered output can be written as

$$\begin{aligned}
\dot{x} &= f(x, x_z, x_{zz}, u) \\
y_L &= h(x)\big|_{z=L}
\end{aligned} \tag{2}$$

where $x=[x_p\ x_o]^T$ is the vector of state variables, $x_z = \partial x/\partial z$ is the first-order spatial derivatives of x, $x_{zz} = \partial^2 x/\partial z^2$ is the second-order spatial derivatives of x, u is the manipulated input, and $f(x, x_z, x_{zz}, u)$, $h(x)$ are the vectors of nonlinear function. For the nonlinear system in Eq. (2), the relative order (degree) of the controlled output y_L is denoted by r, and it is finite. The notation of the relative order for the partial differential equation system has the expressions given below.

$$\begin{aligned}
y_L &= h\,(x)\big|_{z=L} \\
\frac{dy_L}{dt} &\triangleq \left[\frac{h}{\partial x}\frac{\partial x}{\partial t}\right]_{z=L} = h^1\left(x, x_z, x_{zz}\right)\big|_{z=L} \\
\frac{d^2 y_L}{dt^2} &\triangleq \left[\frac{\partial h^1}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial h^1}{\partial x_z}\frac{\partial x_z}{\partial t} + \frac{\partial h^1}{\partial x_{zz}}\frac{\partial x_{zz}}{\partial t}\right]_{z=L} \\
&= h^2\left(x, x_z, x_{zz}, \frac{\partial x_z}{\partial t}, \frac{\partial x_{zz}}{\partial t}\right)\Bigg|_{z=L} \\
&\vdots \\
\frac{d^{r-1} y_L}{dt^{r-1}} &\triangleq \left[\frac{\partial h^{r-2}}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial h^{r-2}}{\partial x_z}\frac{\partial x_z}{\partial t} + \frac{\partial h^{r-2}}{\partial x_{zz}}\frac{\partial x_{zz}}{\partial t} + \ldots + \frac{\partial^{r-2}}{\partial t^{r-2}}(x_{zz})\right]_{z=L} \\
&= h^{r-1}\left(x, x_z, x_{zz}, \frac{\partial x_z}{\partial t}, \ldots, \frac{\partial^{r-2}}{\partial t^{r-2}}(x_z), \frac{\partial x_{zz}}{\partial t}, \ldots, \frac{\partial^{r-2}}{\partial t^{r-2}}(x_{zz})\right)\Bigg|_{z=L} \\
\frac{d^r y_L}{dt^r} &\triangleq \left[\frac{\partial h^{r-1}}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial h^{r-1}}{\partial x_z}\frac{\partial x_z}{\partial t} + \frac{\partial h^{r-1}}{\partial x_{zz}}\frac{\partial x_{zz}}{\partial t} + \ldots + \frac{\partial^{r-1}}{\partial t^{r-1}}(x_{zz})\right]_{z=L} \\
&= h^r\left(x, x_z, x_{zz}, \frac{\partial x_z}{\partial t}, \ldots, \frac{\partial^{r-1}}{\partial t^{r-1}}(x_z), \frac{\partial x_{zz}}{\partial t}, \ldots, \frac{\partial^{r-1}}{\partial t^{r-1}}(x_{zz}), u\right)\Bigg|_{z=L}
\end{aligned} \tag{3}$$

3. MATHEMATICAL MODEL OF EDC CRACKING FURNACE

For the EDC cracking process, the raw material, EDC, is cracked to mainly produce vinyl chloride monomer (VCM) and hydrogen chloride (HCl) inside the cracking coil. A typical example of the cracking furnace is shown in Fig. 1, the VCM production process overview can be found in Ullmann's Encyclopedia (1997). Since the EDC cracking is an endothermic gas-phase reaction, the heat energy is permitted by gas-fired inside the furnace. The coupled PDE-ODE system is developed under the integrated consideration of gas-fire radiation effect and tube wall conduction. The dynamic of the EDC concentration, cracked gas temperature and tube wall temperature are spatially distributed along the coil length which can be described by PDE models while the lumped furnace wall temperature can be described by ODE model. Additional simplified assumptions are considered in the one-dimensional EDC cracking furnace model:

(1) The gas behavior is ideal gas.
(2) The side reactions are neglected.
(3) The cracked gas in the tube is in a turbulent flow regime and the gas flow pattern is closed to a plug flow pattern.
(4) The properties of the cracked gas are constant.

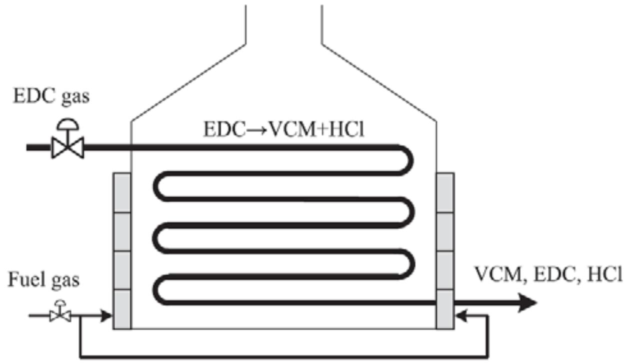

Fig. 1 Scheme of the typically EDC cracking furnace.

The dynamic of EDC concentration, cracked gas temperature and tube wall temperature of the cracking coil:

$$\frac{\partial C}{\partial t} = -v\frac{\partial C}{\partial z} - k_0 C\, e^{-\frac{E}{RT_g}}$$

$$\frac{\partial T_g}{\partial t} = -v\frac{\partial T_g}{\partial z} + \frac{r_A(-\Delta H)}{\rho_g Cp_g} + \frac{1}{V_t \rho_g Cp_g}\left(\frac{2\pi L}{\frac{\ln(R_{to}/R_{ti})}{k_t}+\frac{1}{R_{ti}h_g}}(T_{to}-T_g)\right) \tag{4}$$

$$\frac{\partial T_t}{\partial t} = \frac{k_t}{Cp_t\rho_t}\frac{\partial^2 T_t}{\partial z^2} + \frac{A_W F\sigma\left(T_W^4 - T_t^4\right)}{m_t Cp_t} - \frac{(1/m_t Cp_t)2\pi L}{\left(\ln\left(R_0/R_i\right)/k_t\right)+\left(1/R_i h_g\right)}\left(T_t - T_g\right)$$

with the boundary and initial conditions of

$$C_A(0,t) = C_0$$
$$T_g(0,t) = T_{g,0}$$
$$T_t(0,t) = T_{t,0}$$
$$\left.\frac{\partial T_t(t)}{\partial t}\right|_{z=L} = 0$$
$$C\,(z,0) = C_0$$
$$T_g(z,0) = T_{g,0}$$
$$T_t(z,0) = T_{t,0}$$

The dynamic of furnace wall temperature:

$$\frac{\partial T_W}{\partial t} = \frac{\dot{m}_f H_{comb}}{m_w Cp_w} - \frac{\sigma F A_W\left(T_w^4 - T_t^4\right)}{m_w Cp_w} \tag{5}$$

with the initial conditions of

$$T_w(z,0) = T_{w,0}$$

where $z \in [0,300]$, C denotes the EDC concentration, T_g denotes the cracked gas temperature, T_t denotes the tube wall temperature, T_w denotes the furnace wall temperature. The controlled output chosen in this work is the cracked gas temperature, T_g, at the exit of the cracking coil while the manipulated input is the fuel gas flow rate, $\dot{m}_f$. All process parameters descriptions and values are presented in Table 1. The function h_g can be expressed as

$$h_g = 0.023\frac{k_i}{D}\left(\frac{\rho_g D v}{\mu}\right)^{0.8}\left(\frac{\mu Cp_g}{k_i}\right)^{0.4}$$

Table 1. Parameters of the cracking furnace

Symbol	Quantity	Value
A_w	Radiating area of furnace wall	218 m^2
Cp_t	Heat capacity of cracking coil	444 J/kg K
Cp_g	Average heat capacity of cracked gas	653.76 J/kg K
Cp_w	Heat capacity of furnace wall	1000 J/kg K
D_i	Internal tube diameter	0.19 m
E	Activation energy	1.15×10^5 J/mol
H_{com}	Heat of combustion	4.25×10^7 J/mol
k_0	Kinetic constant	1.15 × 10^7
k	Thermal conductivity of cracked gas	2.655×10^{-2} W/m K
k_t	Thermal conductivity of cracking coil	20.5 W/m K
L	Tube Length	300 m
m_t	Mass of cracking coil	7.783×10^3 kg
m_w	Mass of furnace wall	4.191 × 10^5 kg
R	Gas constant	8.314 J/mol K
R_i	Internal cracking coil radius	0.095 m
R_o	External cracking coil radius	0.1 m
v	Feed velocity	4.855 m/s
V_t	Volume of cracking coil	8.5059 m^3
ΔH	Heat of reaction	7.1 × 10^4 J/mol
ρ_g	Average density of cracked gas	35.43 kg/m^3
ρ_t	Density of cracking coil	8470 kg/m^3
$\dot{m}_f$	Fuel gas flow rate	0-0.6 kg/s
σ	Stefan–Boltzmann constant	5.67×10^{-8} W/m^2 K^4
μ	Viscosity of cracked gas	1.695 × 10^{-5} kg/m s

4. FORMULATION OF THE CONTROLLER SYSTEM

The regulation of the controlled output at the exit of cracking coil is proposed in this work by adjusting the manipulated input based on input-output linearization technique.

4.1 Feedback input-output linearizing controller

The system in Eq. (2) is used to formulate a controller based on I/O linearization technique. The responses of the closed-loop process output, at the exit of tube (y_L), have a linear form:

$$(\beta D+1)^r y_L = y_{sp} \tag{6}$$

where D is the differential operator (i.e. $D \triangleq d/dt$), y_{sp} is the desired output setpoint, and β is the tuning parameter. The feedback controller can be recast as Eq. (7) by substituting Eq. (4) into Eq. (6):

$$u = \psi(x, x_z, y_{sp}, \varepsilon)\Big|_{z=L} \tag{7}$$

where

$$\varepsilon = \phi\left(\frac{\partial x_z}{\partial t}, \ldots, \frac{\partial^{r-1}}{\partial t^{r-1}}(x_z), \frac{\partial x_{zz}}{\partial t}, \ldots, \frac{\partial^{r-1}}{\partial t^{r-1}}(x_{zz})\right) \tag{8}$$

To apply the feedback controller, the nonlinear function of time derivatives of state gradient, ε, is set to be zero.

4.2 Control system for the cracking furnace

The cracked gas temperature evaluated at the exit in closed-loop system is arrange in the linear form

$$(\beta D+1)^r T_{g,L} = T_{g,sp} \tag{9}$$

where $T_{g,L}$ is the gas temperature at position $z = L$, $T_{g,sp}$ is the setpoint and β is tuning parameter. In this system, the relative order of the developed controller equal to 3 (r=3). The compact form of the feedback controller can be written as:

$$u = \psi(C, T_g, T_t, T_w, \frac{\partial C}{\partial z}, \frac{\partial T_g}{\partial z}, \frac{\partial^2 T_t}{\partial z^2}, T_{g,sp})\Big|_{z=L} \tag{10}$$

4.3 Finite-element based state observer design and integrator

In this work, the finite-based, open-loop state observer of the process as shown in Eq. (11) is used to estimate the unmeasurable states, $\tilde{x}$, and the state derivatives.

$$\begin{aligned} \dot{\tilde{x}} &= f(\tilde{x}, \frac{\partial \tilde{x}}{\partial z}, \frac{\partial^2 \tilde{x}}{\partial z^2}, u) \\ y &= h(\tilde{x}) \end{aligned} \tag{11}$$

To compensate the offset because of the model-process mismatch and the error in the parameter estimation from the observer, the following error dynamics are introduced:

$$\begin{aligned} \dot{\varepsilon}_T &= \lambda_T\left(y_{sp} - y_L\right) \\ v_T &= y_{sp} - \varepsilon_T \end{aligned} \tag{12}$$

where ε_T is the error of the temperature, λ_T is a positive parameter, and v_T is a new corrected setpoint.

The combination of the feedback I/O controller in Eq. (7), the finite-based state observer in Eq. (11), and the integrator in Eq. (12), a schematic diagram of the developed controller system can be formulated: see Fig. 2.

5. SIMULATION RESULTS

In order to evaluate the performance of the control system, closed-loop simulations for the EDC cracking furnace are performed. The gas temperature is controlled at the location z = 300 m in the closed-loop system. The initial condition of the cracking coil is $C(0,t)$ = 359.83 mol/l and

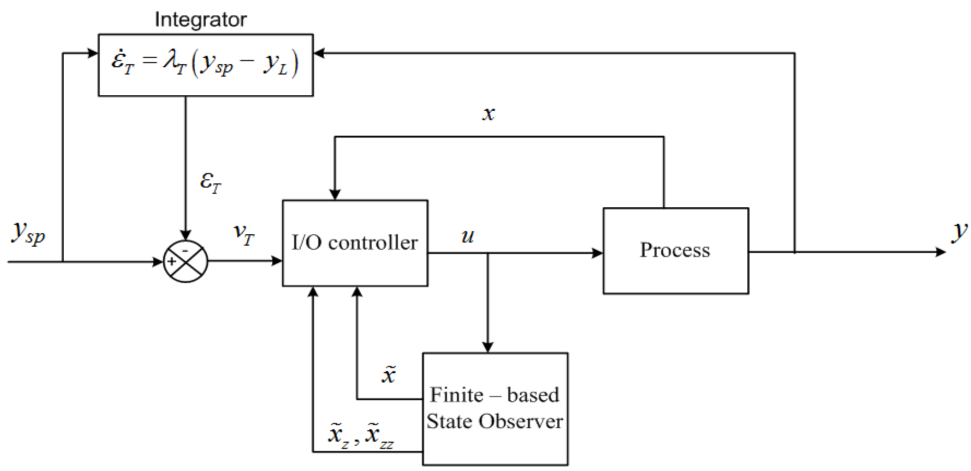

Fig. 2 Schematic diagram of the developed control system

$T_g(0,t)$ = 478 K with the setpoint, $T_{g,sp}$ = 677.12 K and the tuning parameter β = 4. The initial condition of the tube wall temperature is $T_{t,0}$ = 550 K and the furnace wall temperature is $T_{w,0}$ = 713 K. Fig. 3 shows the open-loop profile evolution of the cracked gas temperature inside the cracking coil along the tube length and operating time. The comparison between the cracked gas temperature of the developed coupled PDE-ODE system and a typically lumped model is illustrated in Fig. 4. Fig. 5 is the EDC concentration profile at the exit of cracking coil corresponding to Fig. 3.

The profile of the evolution of the closed-loop response of cracked gas temperature along the tube length and time under the developed controller is shown in Fig. 6. The comparison between the open-loop profile and the closed-loop response is illustrated in Fig. 7. Fig. 8 is the manipulated input, fuel gas flow rate, corresponding to the closed-loop system. The developed controller is also extended to regulate the EDC cracking furnace with other desired condition. Fig. 9 shown the closed-loop response when the setpoint is $T_{g,sp}$ = 695 K. These simulation studies demonstrate that the input-output linearizing controller successfully forced the cracked gas temperature to achieve the desired setpoint asymptotically.

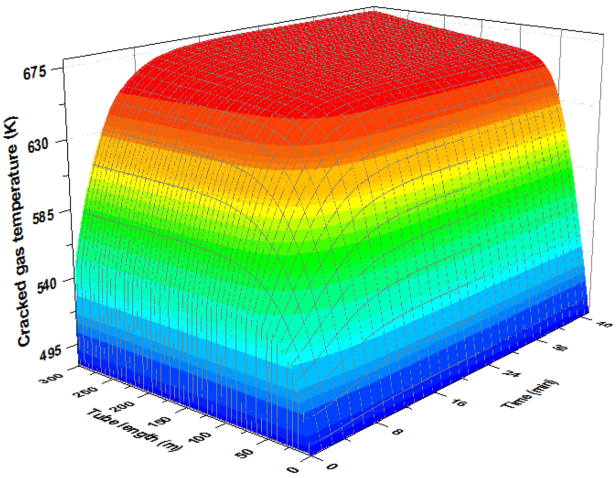

Fig. 3 Evolution of open-loop profile of cracked gas temperature

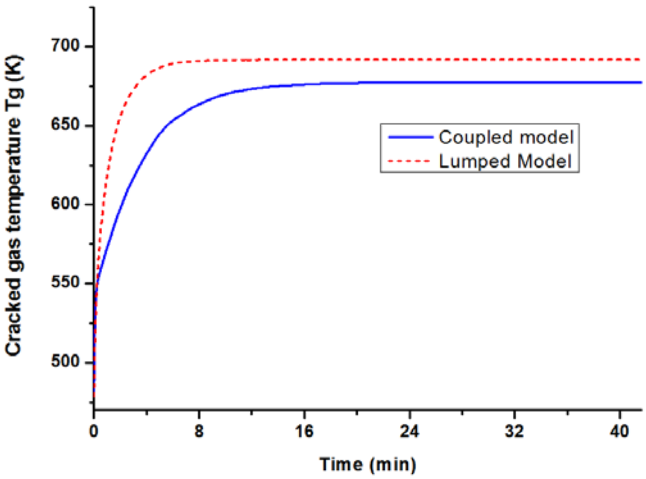

Fig. 4 Cracked gas temperature responses: lumped model and coupled PDE-ODE model.

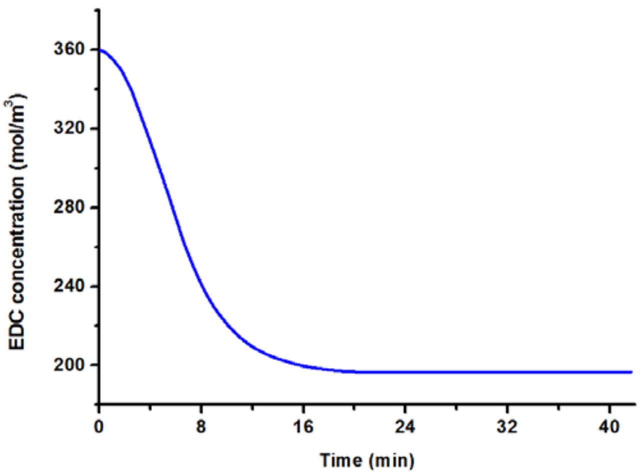

Fig. 5 Open-loop profile of the EDC concentration corresponding to Fig. 3

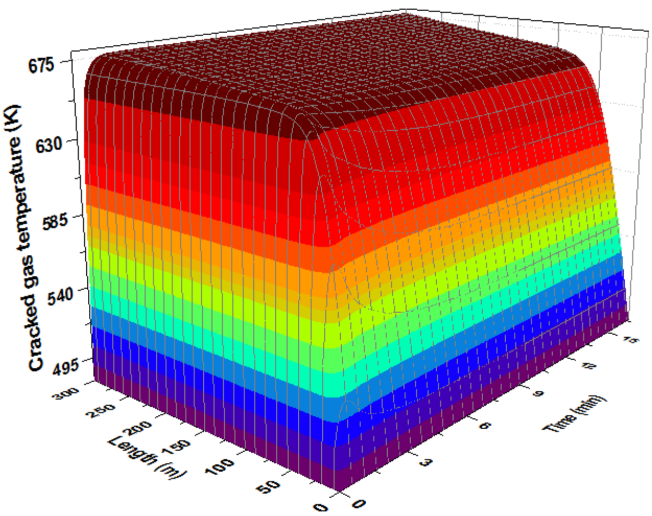

Fig. 6 Evolution of the closed-loop response of the cracked gas temperature

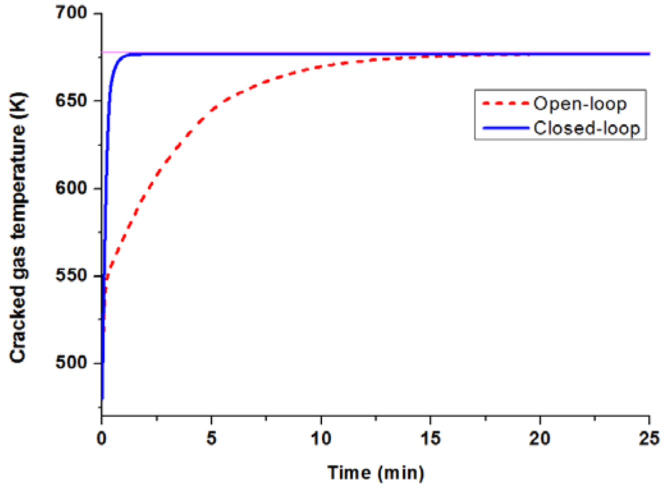

Fig. 7 Cracked gas temperature responses: the open-loop profile and the closed-loop response comparison.

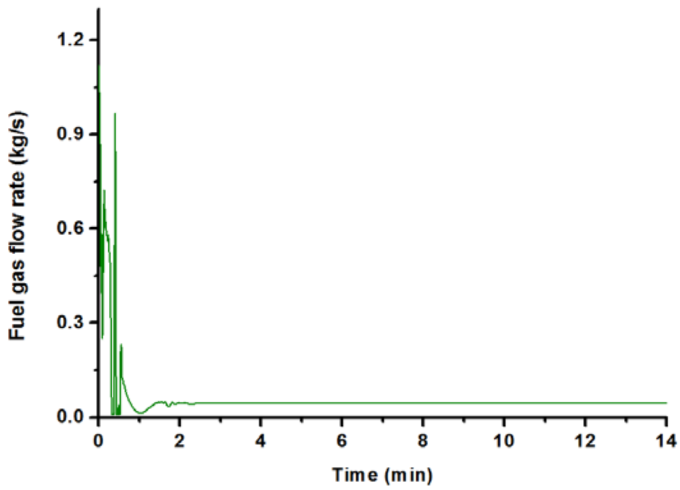

Fig. 8 Fuel gas flow rate corresponding to the closed-loop system

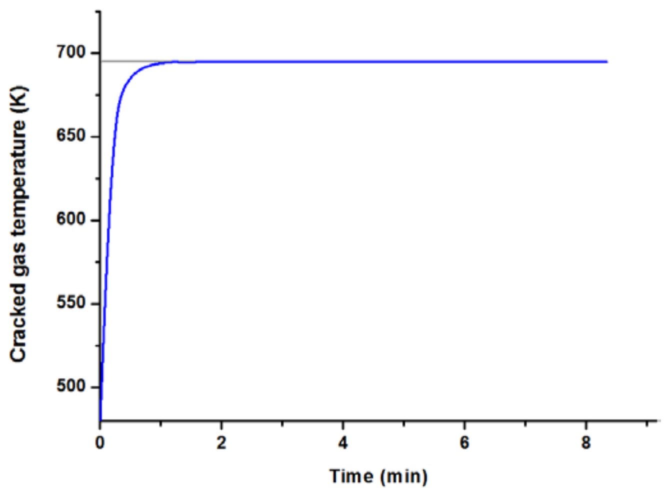

Fig. 9 Closed-loop response of the cracked gas temperature with $T_{g,sp}$ = 695 K

6. CONCLUSIONS

This work proposed the development of a controller system based on the feedback I/O linearizing control technique. The developed controller is applied to the coupled PDE-ODE mathematical model system of an EDC cracking furnace in order to track the gas temperature to follow the desired setpoint. Simulation results showed that the controller successfully forced the cracked gas temperature to achieve the desired setpoint asymptotically. Furthermore, this proposed technique should be extended to control the process under condition affected by outside factors.

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