Preprints of the 10th IFAC International Symposium on Dynamics and Control of Process Systems
The International Federation of Automatic Control
December 18-20, 2013. Mumbai, India

Self-optimising control of sewer systems

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Abstract: Self-optimising control is a useful concept to select optimal controlled variables from a set of candidate measurements in a systematic manner. In this study, use self-optimizing control tools and apply them to the specific features of sewer systems, e.g. the continuously transient dynamics, the availability of a large number of measurements, the stochastic and unforeseeable character of the disturbances (rainfall). Using a subcatchment area in the Copenhagen sewer system as a case study we demonstrate, step by step, the formulation of the self-optimising control problem. The final result is an improved control structure aimed at optimizing the losses for a given control objective, here the minimization of the combined sewer overflows despite rainfall variations.

Keywords: Self-optimising control; control design; sewer system.

1. INTRODUCTION

Sewer systems are essential part of urban water management which collects the sewage (both rain water as well as domestic wastewater) and transport it to centralized wastewater treatment plants for purification, before discharging to surface waters. Control and operation of sewer systems is a challenging problem characterized by stochastic disturbances (rain events) and transient dynamics. In addition, the EU water framework directive (2000) requires a reduction in the combined sewer overflows (CSOs) generated in urban environments. The practical difficulties of undergoing a structural modification of the system itself, due to the high capital costs and organizational problems related to civil works in dense populated areas, has forced to look for improvements of the system operation performance, rejecting major design modifications.

Current research in sewer system control focuses mainly on model development of the sewer systems (Breinholt et al. 2011) and on their application by predictive control and/or offline optimisers (Duchesne et al. 2001, Ocampo-Martinez 2010). In contrast, the design of the regulatory layer in sewer system control has received little or no attention to the best of our knowledge. In this contribution, we adapt and apply the ideas of self-optimising control strategies to the design of a control structure that will keep the system close to the control objective.

Self-optimising control, first formulated by Skogestad and co-workers (Skogestad 2000), is based on the selection of controlled variables (CVs) which, kept with constant setpoints, lead to an acceptable operation given a defined objective. In practice, this is achieved defining a “loss function” that depends on the CVs of the control structure. The self-optimising structure corresponds to the set of CVs that minimise the “loss function”. Further developments of self-optimising control have formulated the CVs as linear combinations of a several measurements, leading to a control structure more robust towards disturbances (Halvorsen et al. 2003). Hence, this method is particularly relevant for sewer systems where the number of candidates for CV (the system measurements) generally exceeds the number of actuators.

Self-optimising control tools can be effectively applied to control design of sewer systems provided that some certain problems specific to sewer systems are addressed, namely: 1) transient dynamics especially lack of steady-state at sewer systems, prevent from defining one or several nominal operating points; 2) key parts of the sewer systems are the storage basins which, as integrators, cannot be evaluated at steady state (or very low frequencies); and 3) the availability of too many measurements as alternative candidate for CVs leading to the combinatorial nature of the resulting optimisation problem.

We illustrate the applicability of self-optimising control to a case study based on a the design of a control for minimisation of CSOs in a subcatchment area in the Copenhagen sewer system.

2. METHODS

The aim of self-optimising control is to minimise the loss of the objective function taking into account disturbances, sensor noise and implementation errors. It can be proven (Halvorsen et al. 2003) that minimising the loss of the objective function is equivalent to minimise the maximum singular value of the matrix M defined as:

$$\boldsymbol{M} = (M_d\ M_n)\ ;\ \boldsymbol{M_d} = J_{uu}^{\frac{1}{2}}\left(J_{uu}{}^{-1}J_{ud} - G^{-1}G_d\right)W_d\ ;$$

$$\boldsymbol{M_n} = J_{uu}{}^{1/2}G^{-1}W_n \tag{1}$$

where $G = HG^y$; $G_d = HG_d^y$ and $W_n = HW_n^y$.

The plant gain matrix (G^y) and the disturbance gain matrix (G_d^y) relate respectively the available measurements with the

MVs and the disturbances; the uncertainty matrix W_d represents the maximum expected magnitude of each disturbance and W_n^y the implementation error for all the available measurements; J_{uu} and J_{ud} are the second derivatives of the cost function with respect to the MVs (u) and the disturbances (d). To define the previous elements a number of tools and methods were used. They are provided here below.

2.1. *System analysis and objective definition.*

We performed an analysis of the variables in the system in order to evaluate the number of manipulated variables (MVs) (i.e. the control degrees of freedom), available measurements (Ys) and disturbances (ds). The user should define objective of the control system and, thereby, the objective function (OF).

Given the large number of variables, it is also convenient to define the number of measurements available for the optimisation (nSubset) and the number of measurements on which the controlled variables would depend (nY).

2.2. *Definition of scenarios and trajectory of actuators.*

The evaluation scenarios are defined setting inputs and disturbances to the system and when the linearisation takes place. Disturbances in a sewers system are basically the dry weather flow and the rain input. In particular, designed storms (e.g. by the Chicago method (Keifer, Chu 1957)) are useful, as one can create a synthetic rain event with the main characteristics of historical rain events for a given return period. This type of synthetic rain events are widely applied as they are used in sewer system design.

For a given set of inputs, the trajectories of the actuators are defined so that the objective function is optimised. Hence, it can be ensured that the controller will tend to drift the system towards its optimal trajectory.

Basins are the key component in sewer systems control. If the in- and outflow of a basin are independent of the degree of filling, as it is often the case, the transfer function relating the levels and the MVs contain pure integrating modes. As steady-state gains cannot be evaluated in this case, a range of frequency must be defined that covers the range of operation of the control system.

Finally, the evaluation scenarios must be defined. In effect, in contrast with the original formulation of self-optimising control, aimed at continuous chemical processes, sewers systems are characterized by transient dynamics lacking a relevant steady state. Furthermore, the relation between the variables in the system is highly nonlinear leading to different behaviours depending on the state of the system. Hence, a scenario based analysis is carried out as follows: the system is evaluated at different "points of operation" or scenarios, i.e. empty basins, first overflow, full basins. For a given input, each scenario is related to a certain state or the variables and evaluation time (t_i) which is subsequently used for linearisation.

2.3 *Linearization and determination of objective function derivatives.*

The plant gain matrix (G^y) and the disturbance gain matrix (G_d^y) must be determined by linearising around a certain operating point and evaluating the transfer function at the defined range of frequency. As the condition number of G^y is subsequently used for measurement screening, G^y must be appropriately scaled (Skogestad, Postlethwaite 2005). The uncertainty matrices W_d and W_n^y are defined as positive diagonal matrices in which each element is related to a certain measurement or disturbance. The cost function derivatives (J_{uu}, J_{ud}) are also determined at this stage.

2.4. *Screening of measurements.*

In large systems, the number of measurements can be very high. A fraction of the measurements in the system can be ruled out without affecting the final solution. Thus, the size of the problem is reduced and the subsequent optimisation becomes more manageable. First, a measurement is discarded when the manipulated variable does not have any influence in the measurement for the evaluation time analysed (reflected as a row of zeros in G^y. After this first screening, combinations of the resulting G^y with the dimensions fixed by nSubset, are generated. The sets of combinations are ranked given a certain condition. In this case, the minimum condition number was used as a controllability criterion (Skogestad, Postlethwaite 2005). Only the measurements that form the set with the minimum condition number are kept for the next steps.

2.5. *Determination of the controlled variables.*

The determination of the controlled variables (CVs) is carried out through a minimisation of the loss function. As previously described, this is equivalent to minimising the maximum singular value of the M matrix (eq.1). Hence, the controlled variables are defined by the measurement combination matrix H obtained by the minimisation defined as:

$$\underset{\mathrm{H}}{\operatorname{argmin}}\, L = \tfrac{1}{2}\overline{\sigma}(M) \qquad (2)$$

This procedure is done for all the possible measurement combinations. The H_{opt} selected is the one that provides the minimum loss value.

3. CASE STUDY

Self-optimising control has been applied to a case study consisting of a subcatchment area of the Copenhagen sewer system (fig. 1). This system is modelled by the virtual tank (VT) approach, which represents subcatchments defined by a certain surface area and pipe volume. Other elements include basins, represented by their level (L), pumps (P) and weirs (Ocampo-Martinez 2010). The model was implemented in Matlab Simulink and linearisations were performed using Matlab Simulink for different scenarios. All the calculations in the self-optimising methodology were also implemented in Matlab.

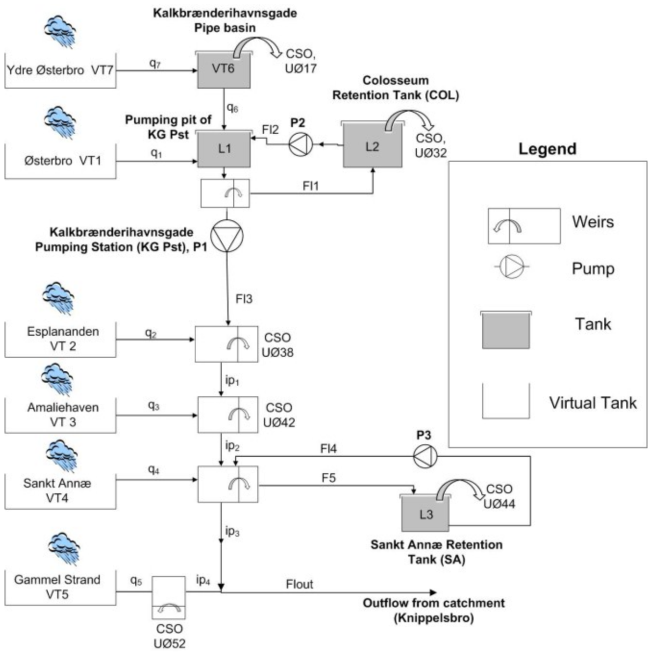

Figure 1. Schematic representation of the virtual tank model diagram for the Copenhagen sewer system

3.1. *System analysis and objective definition.*

The results of the analysis of the available MVs, measurements and disturbances are gathered in table 1. The number of controlled variables is 3 corresponding to the 3 available degrees of freedom. To reduce the problem size, the number of measurements available for optimisation were set as nSubset=6 but this number can be varied depending on the needs or preference of the user. In general it represents a trade-off between the complexity of the controller (including sensor maintenance) and its performance.

The objective of the sewer systems in this case study is to minimise the combined sewers overflow (CSO). Since eq. 1-2 require that the objective be formulated as a minimisation, the cost function is written as follows:

$$J = \int_0^T \left(\sum_{i=1}^{nO} UØ_i \right) dt \qquad (3)$$

Table 1. Variable analysis of the system, displaying the number of measurements (Ys), manipulated variables (MVs) and disturbances (ds).

	Overflows	Flows	Levels
Ys (n=24)	UØ17,UØ32, UØ38,UØ42, UØ44	$q_1,q_2,q_3,q_4,q_5,q_6,q_7$ ip_1,ip_2,ip_3 F_5 $Fl_1,Fl_2,Fl_3,Fl_4,Fl_{out}$	L_1, L_2,L_3
MVs (n=3)	Pump flow (vP1, vP2, vP3)		
ds (n=1)	Rainfall intensity input		

3.2. *Definition of scenarios and trajectory of actuators.*

The rain event chosen as disturbance for this system is a synthetic rain event generated by applying the Chicago method to real rain data from the Copenhagen area (fig. 2). In this method the rain event characterizes global rain behaviour in a certain area in terms of intensity, duration and frequency. Hence, for every real rain event that takes place a maximum average rain intensity value is assigned to each of them. With this procedure, a discretization for every real rain events is acquired. With these data stored, the Chicago method is able to provide a characteristic synthetic rain event for a certain return period, duration and average rain intensity in the area analysed. For the rain event used in this case study the chose values were of 5 years, 240 min and 630 mm/year respectively. The pump action during the rain event was set as constant throughout the whole rain event. The simulation of the rain event can be checked in fig. 3.

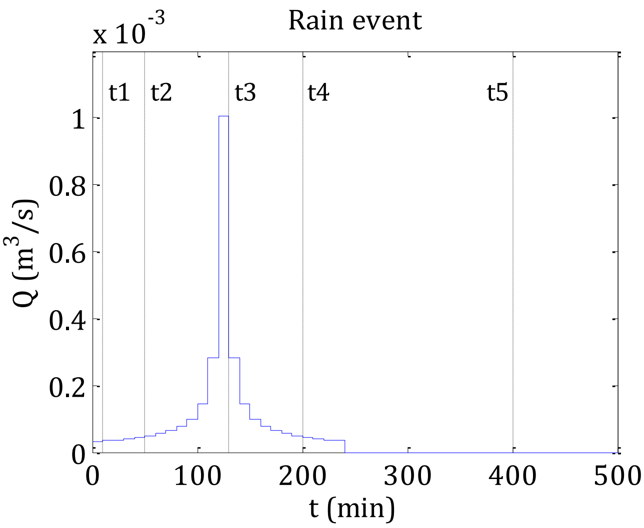

Figure 2. Synthetic box rain (return period 5 years) used as an input in this case study

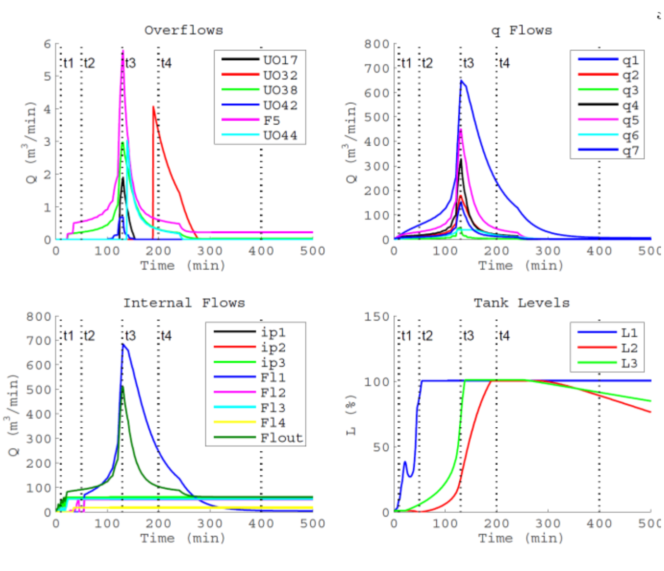

Figure 3. Evolution of the main variables in the subcatchment area during simulation of the rain event

The range of frequency that limits the expected control action was estimated conservatively as follows:

1) The lower bound of the range was taken as the inverse of the expected period of a rain event. To determine this, the series of two years of rain in the Copenhagen area were decomposed with spectral analysis. The main mode corresponded to a frequency $\omega_2 = 0.046$ rad min^{-1} (equivalent to 136.5 min)

2) The highest bound corresponds to the fastest variations in the disturbances (rainfall) that will be propagated. Since the virtual tanks are first-order systems they act as low pass filters depending on their time constant. Hence, the highest bound is taken as the frequency that provides an amplitude ratio $AR = 1 / \sqrt{2}$ for the virtual tank with the lowest time constant, in this case VT 2. This frequency is equal to $\omega_1 = 0.59$ rad min^{-1}

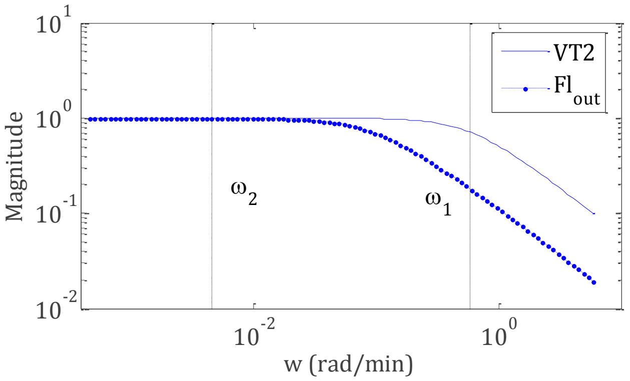

Figure 4. Bode plot for G_d^y for a fast output (VT 2 level) and a slow output (Fl_{out})

To check that the range of frequency covered the main dynamics of the system, fig. 4 shows the AR of a fast and a slow variable with the rainfall as an input variable. As fast variable, the level of VT2 was chosen as it is the virtual tank with the lowest time constant. As a slow variable, Fl_{out} was chosen as a slow variable given that, at open loop, it depends of all the virtual tanks. As it can be seen, the range of frequency covers the low frequency range until the decrease of the AR is significant. For simplicity in the rest of the analysis, all the transfer functions were evaluated at the lowest bound of the frequency range $\omega_2 = 0.046$ rad min^{-1}.

From the inspection of the system evolution during the simulation of the rain event, five scenarios or points of operation were selected (table 2). These were used subsequently for linearisation.

3.3 Linearization and determination of the objective function derivatives.

$G^y$and G_d^y were generated using the linearization tool of Matlab Simulink and evaluated at $\omega_2 = 0.046$ rad min^{-1}. The elements in the uncertainty matrix W_n^y were defined as the measurement error expected for each class of sensor. W_d consists of a single element, the maximum value of the rainfall intensity, equal for all the VT. J_{uu} and J_{ud} were determined numerically by applying a second order finite-difference scheme.

Table 2. Scenarios selected according to the state of the system

Scenario	State of the system	Evaluation time
1	Dry weather	$t_1 = 10$ min
2	Filling	$t_2 = 50$ min
3	Overflow from structures with little or no volume	$t_3 = 130$ min
4	Overflow from structures with large volumes	$t_4 = 200$ min
5	Emptying	$t_5 = 400$ min

3.4. Screening of measurements.

The dimensions of $G^y{}_{24\,x3}$were reduced discarding the rows related to measurements not affected by the MVs. Combinations of 6 row matrices (nSubset value fixed by the user) were generated from the resulting matrix $G^{y\prime}{}_{10x3}$. The condition numbers (γ) of all of them were calculated. The matrix with the minimum condition number [$\gamma\ (G^{y\prime\prime}{}_{6x3}) = 3.82$] was selected, giving the 6 measurements further investigated for the optimisation (in this case Fl_2, L_2, Fl_3, Fl_1, L_3 and ip_2). This screening procedure reduced the optimisation from 2024 to 20 subsets.

3.5. Determination of the controlled variables.

The controlled variables were obtained through the minimisation of the loss function defined in eq. 1 and 2. Table 3 shows the value of *H* if the number of measurements considered vary from 2 to 6 for scenario 3 (see table 2 for a definition). The results show a drastic reduction of the loss function from when comparing 2, 3 and 4 measurements. The use of more measurements to form the CVs only slightly reduces the loss function. Furthermore, it can be seen how the optimum for 5 measurements is the same as for 6 measurements, indicating that for the conditions considered, the addition of a variable does not provide more information to the controller. The cost of a sensor and the complexity of the monitoring is not tackled in this paper but it looks reasonable that 4 measurements would be a good trade-off between performance and simplicity, and would therefore be the selected structure

This analysis, which has been shown for the scenario defined in t_3, was also carried out for the other scenarios. The outcome is five control structures that would depend upon the state of the system. The application of these five controllers can be done in different ways depending on the complexity of the controller and the application. It is indeed possible to switch between the relevant control structures, triggering the switch by the phenomena described in table 2. A simpler solution would be to use only one of the scenarios during the rainfall selecting the one where most CSO is expected to take place.

Table 3. Results of the measurement combination matrix (H_{opt}) that minimises the loss function (L_{opt}) for different number of measurements.

Measurements	H_{opt}	L_{opt}
2	$\begin{array}{c}\begin{matrix} L_2 & L_3 \end{matrix}\\ \begin{pmatrix} 1.15 & 6.30 \cdot 10^{-2} \\ 0.58 & 2.07 \\ 0.25 & 0.27 \end{pmatrix}\end{array}$	$8.79 \cdot 10^{4}$
3	$\begin{array}{c}\begin{matrix} Fl_2 & Fl_3 & L_3 \end{matrix}\\ \begin{pmatrix} 1.21 & 6.67 \cdot 10^{-2} & 0 \\ 0.63 & 2.26 & 0 \\ 0.26 & 0.29 & 0 \end{pmatrix}\end{array}$	$8.01 \cdot 10^{4}$
4	$\begin{array}{c}\begin{matrix} Fl_2 & Fl_3 & Fl_1 & L_3 \end{matrix}\\ \begin{pmatrix} 19.5 & -2.30 \cdot 10^{-2} & 1.48 \cdot 10^{-3} & -1.27 \cdot 10^{-4} \\ 0.30 & 1.94 \cdot 10^{-2} & 1.91 \cdot 10^{-4} & -6.35 \cdot 10^{-5} \\ 0.98 & 4.35 \cdot 10^{-2} & -1.08 \cdot 10^{-2} & 1.52 \cdot 10^{-4} \end{pmatrix}\end{array}$	$8.54 \cdot 10^{-4}$
5	$\begin{array}{c}\begin{matrix} Fl_2 & Fl_3 & Fl_1 & L_3 & ip_2 \end{matrix}\\ \begin{pmatrix} 0.21 & -3.29 \cdot 10^{-2} & 9.13 \cdot 10^{-3} & -1.65 \cdot 10^{-4} & 8.87 \cdot 10^{-4} \\ -0.30 & 4.24 \cdot 10^{-2} & 6.45 \cdot 10^{-4} & -1.40 \cdot 10^{-4} & 7.85 \cdot 10^{-6} \\ 6.01 & 5.37 \cdot 10^{-2} & -7.91 \cdot 10^{-3} & 6.13 \cdot 10^{-5} & -1.05 \cdot 10^{-3} \end{pmatrix}\end{array}$	$6.17 \cdot 10^{-6}$
6	$\begin{array}{c}\begin{matrix} Fl_2 & Fl_3 & Fl_1 & L_3 & ip_2 & L_2 \end{matrix}\\ \begin{pmatrix} 0.21 & -3.29 \cdot 10^{-2} & 9.13 \cdot 10^{-3} & -1.65 \cdot 10^{-4} & 8.87 \cdot 10^{-4} & 0 \\ -0.30 & 4.24 \cdot 10^{-2} & 6.45 \cdot 10^{-4} & -1.40 \cdot 10^{-4} & 7.85 \cdot 10^{-6} & 0 \\ 6.01 & 5.37 \cdot 10^{-2} & -7.91 \cdot 10^{-3} & 6.13 \cdot 10^{-5} & -1.05 \cdot 10^{-3} & 0 \end{pmatrix}\end{array}$	$6.17 \cdot 10^{-6}$

4. CONCLUSIONS

For the first time, to our best knowledge, self-optimising control principles were applied in order to design an optimal control structure for a given set of sensors and actuators in sewers systems. As a result, a set of controlled variables was defined as linear combination of measurements that minimise the deviation from a previously defined optimum state(s). In this case, the optimum was determined with the aim of minimising the CSO from the sewer system.

The immediate future perspectives of this work is to carry out a simulation based study to validate the control structure obtained and benchmark it with other control strategies. Besides, evaluations for other states of the system (as dry weather and emptying scenarios) or for more complex objective functions may also be relevant to acquire a deeper insight about the available control possibilities.

ACKNOWLEDGEMENT

The work is part of the Storm and Wastewater Informatics (SWI) project partly financed by the Danish Agency for Science, Technology and Innovation

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