Preprints of the 10th IFAC International Symposium on Dynamics and Control of Process Systems
The International Federation of Automatic Control
December 18-20, 2013. Mumbai, India

Design of Inner and Outer Gray-Box Models to Predict Molten Steel Temperature in Tundish

Iftikhar Ahmad * **Manabu Kano** ** **Shinji Hasebe** ***
Hiroshi Kitada **** **Noboru Murata** †

* *Dept. of Chemical Engineering, Kyoto University, Nishikyo-ku, Kyoto, 615-8510, Japan (ahmad@cheme.kyoto-u.ac.jp)*
** *Dept. of Systems Science, Kyoto University, Yoshida-Honmachi, Sakyo-ku, Kyoto, 606-8501, Japan (manabu@human.sys.i.kyoto-u.ac.jp)*
*** *Dept. of Chemical Engineering, Kyoto University, Nishikyo-ku, Kyoto, 615-8510, Japan (hasebe@cheme.kyoto-u.ac.jp)*
**** *Nippon Steel and Sumitomo Metal Corporation, Ltd., 1-8 Fusocho, Amagasaki-shi, Hyogo, 660-0891, Japan (kitada.bx8.hiroshi@jp.nssmc.com)*
† *Dept. of Electrical Engineering and Bioscience, Waseda University, 3-4-1 Okubo, Shinjuku-ku, Tokyo, 169-8555, Japan (noboru.murata@eb.waseda.ac.jp)*

Abstract: In order to realize stable production in the steel industry, it is important to control molten steel temperature in a continuous casting process. The present work aims to develop a gray-box model that predicts the molten steel temperature in the tundish (TD temp). In the proposed approach, two parameters in the first-principle model, i.e., overall heat transfer coefficients of ladle and tundish, are optimized for each past batch separately, then the relationship between the two parameters and measured process variables is modeled through random forests (RF). In this inner gray-box model, the statistical models update the physical parameters according to the operating condition. To enhance the accuracy of TD temp estimation, another RF model is developed which compensates errors of the inner gray-box. The proposed approach was validated through its application to real operation data at a steel work.

1. INTRODUCTION

The steel industry faces a stiff competition in the global market, and each steel company has to realize stable and efficient operation and produce high quality products satisfying various customer demand. In the steel making process, whose process diagram is shown in Fig. 1, molten steel temperature in the tundish (TD temp) is one of the key factors to realize stable operation. The tundish is a vessel used for delivering molten steel from a ladle to a mold in the continuous casting process. For example, if TD temp is too high, breakouts may occur and cause tremendous increase in maintenance cost and productivity loss. However, no effective manipulated variable is available in the continuous casting process to control TD temp. To realize the target TD temp, therefore, it is necessary to adjust the molten steel temperature in the Ruhrstahl-Heraeus degassing process (RH degasser) at the end of its operation (RH temp). To control TD temp by manipulating RH temp, a model relating TD temp and RH temp needs to be constructed. In the past, various models such as first-principle models, see for example Austin et al. (1992), Xia et al. (2001), Zabadal et al. (2004), Jormalainen et al. (2006) and Belkovskii et al. (2009), statistical models (Sonoda et al. (2012)), and gray-box models, see for example Gupta et al. (2004) and Okura et al. (2012), have been proposed. The gray-box models which integrate a first-principle model and a statistical model have been the most efficient modeling approaches; they are more accurate than one dimensional first-principle models, more intuitive than the statistical models, and faster and less complicated than the computational fluid dynamics (CFD) models. However, first-principle models do not necessarily describe phenomena of target processes with sufficient accuracy. For instance our previously developed gray-box model (Okura et al. (2012)) is good at predicting the TD temp but it cannot accurately model heat loss during the transportation and the continuous casting process because parameters of the first-principle model are fixed for all batches.

The present work aims to overcome such deficiency. A first-principle model is used to estimate TD temp. Two parameters, i.e., overall heat transfer coefficients of ladle and tundish, of the first-principle model are optimized for each batch separately. The optimized parameters are related to measured process variables through a statistical modeling method, i.e., random forests (RF). As a result, the physical parameters of the the first-principle model can be updated according to the operating condition. To enhance the estimation accuracy of TD temp, another RF model is developed to compensate errors of the gray-box

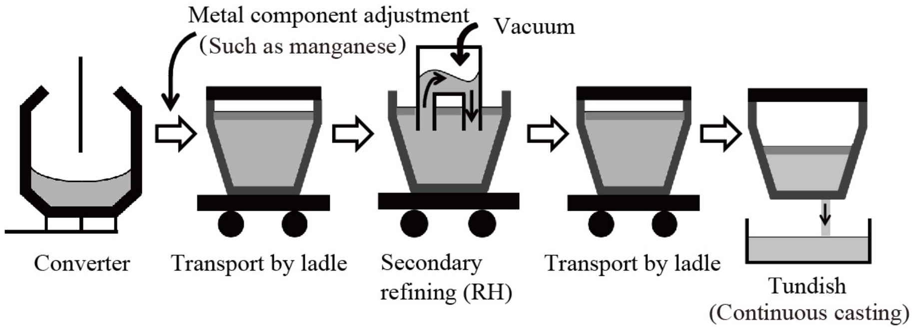

Fig. 1. Process flow diagram of the steel making process

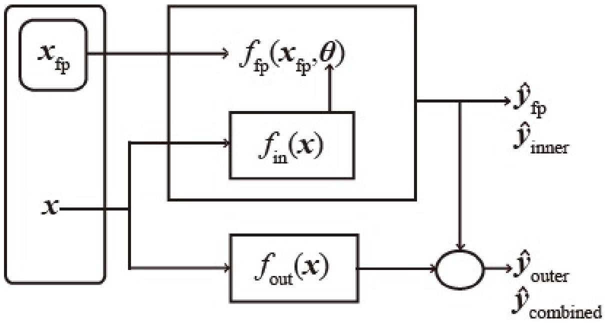

Fig. 2. Generalized framework of gray-box modeling

model. The statistical modeling method, i.e., RF, was used on the basis of its best performance in our preliminary study and previous work (Okura et al. (2012)).

2. GENERALIZED FRAMEWORK FOR THE GRAY-BOX MODELING

A generalized description of the gray-box models discussed in this paper is shown in Fig. 2. The first-principle model f_{fp} is used to estimate TD temp using some process variables $\boldsymbol{x}_{\text{fp}}$, which are a part of available process variables $\boldsymbol{x}$. $\boldsymbol{\theta}$ denotes physical parameters. Two types of statistical models are shown in Fig. 2; f_{in} is used to update the parameters $\boldsymbol{\theta}$ of f_{fp} for each batch while f_{out} is used to compensate errors of f_{fp}. Based on various combinations of f_{fp} and statistical models, three types of gray-box models are developed; 1) the outer gray-box model: a sum of f_{fp} and f_{out}, 2) the inner gray-box model: an integration of f_{fp} and f_{in}, and 3) the combined gray-box model: a sum of inner gray-box model and f_{out}. $\hat{y}_{\text{fp}}$, $\hat{y}_{\text{outer}}$, $\hat{y}_{\text{inner}}$ and $\hat{y}_{\text{combined}}$ represent the predicted TD temp by the first-principle model, the outer gray-box model, the inner gray-box model and the combined gray-box model, respectively.

3. FIRST-PRINCIPLE MODEL

In this section, the first-principle model to predict TD temp is explained. This first-principle model consists of two parts; the first one models phenomena during the transportation period from the secondary refining to the continuous casting, and the second one models phenomena during the casting period. In this model, the degradation of ladles is taken into account.

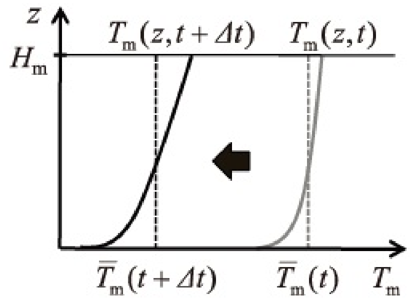

Fig. 3. Update of molten steel temperature in ladle T_m

3.1 First-Principle Model for Transportation Period

Molten Steel in Ladle It is assumed that the ladle is a cylinder of radius R_i. On the basis of the CFD simulation results, indicating that thermal stratification is formed vertically in the standing ladle due to natural convection (Austin et al. (1992)), the molten steel temperature is modeled as a function of time t and position z from the bottom of the ladle.

$$T_m(z,t) = \overline{T}_m(t) + k(t)\left(\sqrt{\frac{z}{H_m}} - \frac{2}{3}\right) \tag{1}$$

where T_m is the molten steel temperature, $\overline{T}_m$ is the average molten steel temperature, k denotes the difference between the molten steel temperature at the top and the bottom of the ladle, and H_m is the depth of the molten steel in the ladle.

The results of CFD simulations have shown that the temperature difference is a function of time (Austin et al. (1992)), thus it is modeled with parameter a.

$$k(t) = at \tag{2}$$

The method of calculating the time evolution of the molten steel temperature T_m is shown in Fig. 3. First, the average molten steel temperature $\overline{T}_m(t+\Delta t)$ is calculated through the heat balance equation.

$$\rho_m c_m \pi R_i^2 H_m \frac{d\overline{T}_m(t)}{dt} = -2\pi R_i \int_0^{H_m} U_w(T_m(z,t) - T_{am})dz - \pi R_i^2 U_b(T_m(0,t) - T_{am}) - \pi R_i^2 h_1(T_m(H_m,t) - T_{sl}(t)) \tag{3}$$

where ρ_m and c_m are the density and the heat capacity of the molten steel, respectively. U_b and U_w are the overall heat transfer coefficients of the ladle bottom and the ladle wall, respectively. T_{am} and T_{sl} are the ambient temperature and the slag temperature, respectively. In addition, h_1 denotes the heat transfer coefficient between the molten steel and the slag. The left side of "(3)"represents the time change of the molten steel enthalpy. The first, second, and third terms of the right side represent the heat conduction from the molten steel to the ladle wall, to the ladle bottom, and to the slag, respectively. The initial molten steel temperature is assumed to be homogeneous and the same as RH temp because the molten steel in the ladle is properly stirred. The temperature difference between the top and the bottom is calculated through "(2)"

Ladle Degradation Due to the repeated use of the ladle, the walls of the ladle gradually degrade. The effect of ladle degradation on the heat conduction flux from the molten steel to the external environment has been discussed in the literature. One study (Fredman (2002)) describes the factors which cause degradation of ladle while another study (Tripathi (2012)) develops a CFD model to relate the heat losses from ladle with the reduction in ladle walls and bottom thickness. To avoid computational complexity and build a simple model, it is assumed that the overall heat transfer coefficients gradually increase with the number of repeated usage, n. Furthermore, the ratio of increase of the overall heat transfer coefficient of the ladle wall is the same as that of the ladle bottom. In addition, it is assumed that the temperature difference between the top and the bottom of ladle increases with the increase of n. The relations are expressed by

$$U_w(n) = \eta U_b(n) \tag{4}$$

$$U_b(n) = U_{b0} + \alpha\sqrt{n} \tag{5}$$

$$a(n) = a_0 + \beta\sqrt{n} \tag{6}$$

where U_{b0}, a_0, α, β, and η are constants.

3.2 First-Principle Model for Casting Period

Molten Steel in Ladle It is assumed that volumetric flow Q from the ladle to the tundish is constant and the depth of the molten steel in the ladle decreases by ΔH_m during Δt. In addition, the outflow temperature is assumed to be the average temperature $T_{in}(t)$ within $0 \leq z \leq \Delta H_m$. It is also assumed that the increase of the temperature difference $k(t)$ stops at the end of transportation period. On the other hand, the heat radiation continues and $T_m(t)$ decreases to $T_m(t + \Delta t)$. This decrease in $T_m(t)$ corresponds to the parallel shift from $T_m(z, t)$ to $T_m(z, t + \Delta t)$: $-\Delta H_m$ in z axial direction and $\Delta \overline{T}_m$ in T_m axial direction as shown in Fig. 4. The $T_m(z, t)$ distributes over the following region:

$$-\frac{Q}{\pi R_i^2}(t - t_1) \leq z \leq H_m - \frac{Q}{\pi R_i^2}(t - t_1) \tag{7}$$

where t_1 is the time at the end of transportation.

Molten Steel in Tundish It is assumed that the inflow to the tundish is equal to the outflow from the tundish and also the depth of the molten steel in the tundish is constant. The CFD simulations have indicated that TD temp is distributed in the flow direction (Odenthal (2010)). Thus, the tundish is modeled as a compartment model consisting of N_t isothermal baths connected in series as shown in Fig. 5. The heat balance of the k-th bath is

$$\begin{aligned}\rho_m c_m W H \frac{L}{N_t}\frac{dT_t^{(k)}(t)}{dt} &= \rho_m c_m Q T_t^{(k-1)}(t) \\ &-\rho_m c_m Q T_t^{(k)}(t) - S_t U_t (T_t^{(k)}(t) - T_{am}) \\ &-W\frac{L}{N_t}\varepsilon_{mol-lin}\sigma((T_t^{(k)}(t))^4 - T_{a2}^4) \\ &-W\frac{L}{N_t}h_3(T_t^{(k)}(t) - T_{a2})\end{aligned} \tag{8}$$

$$S_t = \begin{cases} W\dfrac{L}{N_t} + 2H\dfrac{L}{N_t} + WH & (k = 1, N_t) \\ W\dfrac{L}{N_t} + 2H\dfrac{L}{N_t} & (k = 2, 3, \cdots, N_t - 1) \end{cases} \tag{9}$$

where W, H, and L denote the width, the height, and the length of molten steel in the tundish, respectively. $T_t^{(k)}$ is TD temp in the k-th bath. S_t denotes the contact area between the molten steel and the tundish, U_t the overall heat transfer coefficient of the tundish, $\varepsilon_{mol-lin}$ the emissivity of the molten steel, and h_3 the heat transfer coefficient between the molten steel and the air. $T_t^{(0)}(t)$ is equal to $T_{in}(t)$ because the molten steel poured from the ladle flows into the first bath. The left side of "(8)" represents the time change of the molten steel enthalpy. The first to fifth terms of the right side represent the inflow enthalpy, the outflow enthalpy, the heat conduction from the molten steel to the tundish wall, the radiation from the molten steel to the tundish wall and the heat conduction from the molten steel to the air in the tundish, respectively. The tundish wall temperature is assumed to be equal to the air temperature T_{a2}, which is assumed to be constant. In addition, the influence of former batch on the measurement of TD temp is assumed to be negligible.

3.3 Parameter Fitting

The first-principle model contains eleven parameters to be identified, i.e., a_0, h_1, h_2, h_3, T_{a1}, T_{a2}, U_{b0}, U_t, α, β, and η. The first ten of these parameters were estimated through the least squares algorithm using real process data. The η was given a fixed value because the other term accompanying η, i.e., $U_b(n)$, was indirectly estimated through U_{b0} and α. A total of 1270 samples were used for parameter estimation. The input variables of the first-principle model were the number of ladle usage, the weight of the molten steel, RH temp, transportation time, and casting time.

4. RANDOM FORESTS (RF)

RF is an ensemble classifier that consists of many decision trees (Breiman (2001)). RF combines Breiman's bagging idea and the random selection of split features, proposed in (Ho (1995)) and (Ho (1998)). Bagging is a mechanism to improve stability and accuracy of classification and regression models. Given a training set D of size N, bagging generates M new training sets $D_m^*(m = 1, 2, \cdots, M)$, whose size is N, by random sampling from D with replacement.

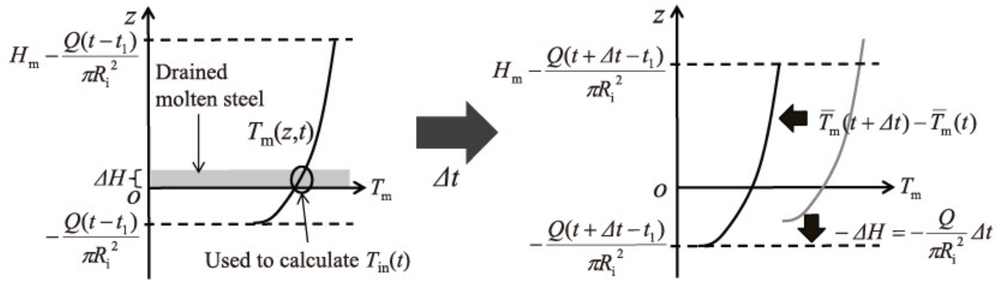

Fig. 4. Model of molten steel temperature in ladle during casting period

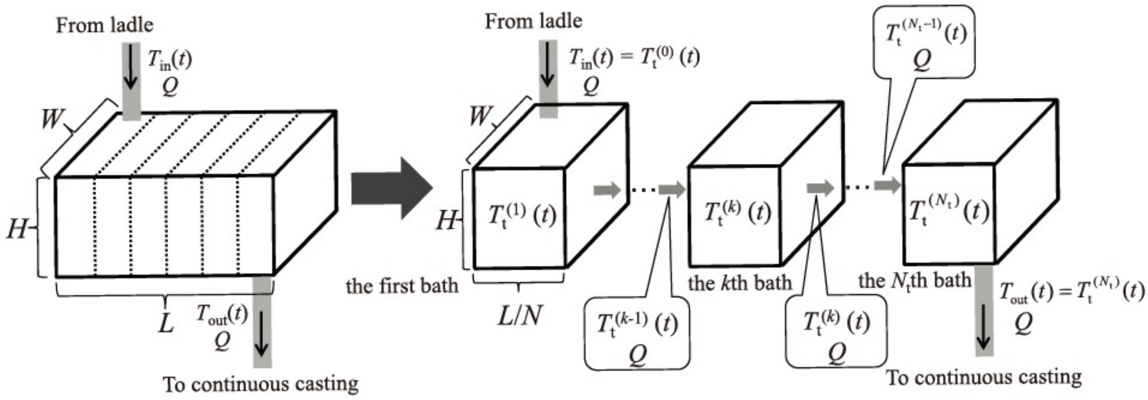

Fig. 5. Compartment model of molten steel in tundish

The set D_m^* is expected to have about two-third of the unique datasets in D and the rest is duplicated. The newly created training datasets are called bootstrapped samples while the fraction of original data that is not bootstrapped is termed out-of-bag (OOB) data. In addition, at each node, feature variables, i.e., split features, are randomly selected and splitting is performed using these features one by one to find the best split. RF creates multiple trees; each tree is trained by using the bootstrapped samples. RF for regression is formed by growing trees on $(\boldsymbol{x}, y) \in D_m^*$ such that the predictions $\hat{f}(\boldsymbol{x})$ are numerical values as opposed to class labels in classification. OOB data is used for error calculation of the respective trees.

Suppose OOB data D_{OOB} of size N_{OOB} such that $(\boldsymbol{x}_j, y_j) \in D_{\mathrm{OOB}}(j = 1, 2, \cdots, N_{\mathrm{OOB}})$ and there are K_j trees that did not use sample $\boldsymbol{x}_j$ during their construction. Averaging predictions at $\boldsymbol{x}_j$ over K_j trees, the RF OOB prediction is derived:

$$\hat{f}_{\mathrm{OOB}}(\boldsymbol{x}_j) = \frac{1}{K_j} \sum_{k=1}^{K} \hat{f}_k(\boldsymbol{x}_j) I[(\boldsymbol{x}_j, y_j) \in D_{\mathrm{OOB}}] \quad (10)$$

where I is the indicator function. The integrated mean-squared prediction error for $\hat{f}_{OOB}$ is

$$mspe[\hat{f}_{\mathrm{OOB}}] = \frac{1}{N_{\mathrm{OOB}}} \sum_{j=1}^{N_{OOB}} (y_j - \hat{f}_{\mathrm{OOB}}(\boldsymbol{x_j}))^2 \quad (11)$$

For validation data V of size N_v such that $(\boldsymbol{x}_v, y_v) \in V(v = 1, 2, \cdots, N_v)$, the RF prediction at $\boldsymbol{x}_v$ is the average prediction of K trees.

$$\hat{f}(\boldsymbol{x}_v) = \frac{1}{K} \sum_{k=1}^{K} \hat{f}_k(\boldsymbol{x}_v) \quad (12)$$

5. GRAY-BOX MODELS

In the outer gray-box modeling method (Okura et al. (2012)), TD temp T_{TD} is predicted through the first-principle model f_{fp} and then the error e of f_{fp} is predicted through a statistical model f_{st}. In other words, TD temp is predicted by adding the output of the statistical model to that of the first-principle model.

$$\hat{T}_{\mathrm{fp}} = f_{\mathrm{fp}}(T_{\mathrm{RH}}, \boldsymbol{x}_{\mathrm{fp}}) \quad (13)$$

$$e = T_{\mathrm{TD}} - \hat{T}_{\mathrm{fp}} \quad (14)$$

$$\hat{e} = f_{\mathrm{st}}(\boldsymbol{x}_{\mathrm{st}}) \quad (15)$$

$$\hat{T}_{\mathrm{TD}} = \hat{T}_{\mathrm{fp}} + \hat{e} \quad (16)$$

where $\hat{T}_{\mathrm{fp}}$ denotes TD temp predicted through the first-principle model. RH temp T_{RH} and $\boldsymbol{x}_{\mathrm{fp}}$ are input variables for the first-principle model. $\boldsymbol{x}_{\mathrm{st}}$ is input variables for the statistical model, whose output is the predicted error $\hat{e}$; it includes measured variables of processes from the converter to the tundish.

In the inner gray-box model, statistical models are used inside the first-principle model to estimate two parameters, i.e., overall heat transfer coefficients of ladle and tundish, for each batch separately. The two out of ten parameters were selected on the basis of effectiveness in reducing the error. A positive error e shows that the heat loss calculated by the first-principle model is larger than the actual heat loss in the plant. Thus U_{b} and U_{t} are decreased for positive error e and increased for negative error e. $\boldsymbol{x}_{\mathrm{RF}}$, the measured variables of processes from the converter to the tundish excluding RH temp, was used to construct random forests (RF) models f_{RF1} and f_{RF2} that estimate U_{b} and U_{t}, respectively.

$$\hat{U}_{\mathrm{b}} = f_{\mathrm{RF1}}(\boldsymbol{x}_{\mathrm{RF}}) \tag{17}$$

$$\hat{U}_{\mathrm{t}} = f_{\mathrm{RF2}}(\boldsymbol{x}_{\mathrm{RF}}) \tag{18}$$

For a certain dataset, $\hat{U}_{\mathrm{b}}$ and $\hat{U}_{\mathrm{t}}$ estimated by the RF models are better than the fixed values; the prediction error was decreased significantly. In order to compensate the remaining error e_{r}, a combined gray-box model is developed with another RF model f_{RF3}. Finally, TD temp is predicted by adding the output of f_{RF3} to the output $\hat{T}_{\mathrm{fp}}^{\mathrm{op}}$ of the inner gray-box model.

$$\hat{T}_{\mathrm{fp}}^{\mathrm{op}} = f_{\mathrm{fp}}(T_{\mathrm{RH}}, \boldsymbol{x}_{\mathrm{fp}}, \hat{U}_{\mathrm{b}}, \hat{U}_{\mathrm{t}}) \tag{19}$$

$$e_{\mathrm{r}} = T_{\mathrm{TD}} - \hat{T}_{\mathrm{fp}}^{\mathrm{op}} \tag{20}$$

$$\hat{e}_{\mathrm{r}} = f_{\mathrm{RF3}}(\boldsymbol{x}_{\mathrm{RF}}) \tag{21}$$

$$\hat{T}_{\mathrm{TD}} = \hat{T}_{\mathrm{fp}}^{\mathrm{op}} + \hat{e}_{\mathrm{r}} \tag{22}$$

The symbols $\boldsymbol{\theta}$, y and $\boldsymbol{x}$ used in Fig. 2 can be defined as follows:

$$\boldsymbol{\theta} = \{U_{\mathrm{b}}, U_{\mathrm{t}}\} \tag{23}$$

$$y = T_{\mathrm{TD}} \tag{24}$$

$$\boldsymbol{x} = \{T_{\mathrm{RH}}, \boldsymbol{x}_{\mathrm{RF}}\} \tag{25}$$

5.1 Model Validation

TD temp prediction performance of the first-principle model, statistical model developed with RF, the outer gray-box model, the proposed inner gray-box model, and the proposed combined gray-box model was compared by applying them to real process data. The number of samples was 1588; 1270 samples (80 %) were used for modeling and the other 318 samples (20 %) were used for validation. For each validation sample, the parameters U_{b} and U_{t}, and the error of the first-principle model and the inner gray-box model were estimated through the statistical models f_{RF1}, f_{RF2} and f_{RF3}, respectively. Fig. 6 shows the prediction results. In addition, the results of five models are summarized in Table 1. The prediction performance was evaluated on the basis of the root-mean-square error (RMSE) and the correlation coefficient (r) between reference TD temp and predicted TD temp.

The performance of the proposed combined gray-box model is superior to the other models. The combined gray-box model achieved the highest prediction accuracy and its RMSE is 37 %, 17 %, 6 % and 4 % smaller than that of the first-principle model, the statistical model, the outer gray-box model and the inner gray-box model, respectively. In particular, it is important that the prediction performance of the RF model alone was lower than the gray-box model, in which the RF model was used to estimate the values of U_{b}, U_{t} and e_{r}. Although RF can build a nonlinear process model, its direct application is not always the best approach because of its weaker interpretability than the gray-box models.

Table 1. Prediction results of TD temp

Modeling method	RMSE	r
First-principle model	2.73	0.74
Statistical model (RF)	2.08	0.86
Outer gray-box model	1.84	0.88
Inner gray-box model	1.81	0.88
Combined gray-box model	1.73	0.89

6. CONCLUSIONS

In the present work, a new gray-box model to predict the molten steel temperature in a tundish (TD temp) in a steel making plant was proposed and was applied to the real process data. In the proposed approach, statistical technique, i.e., random forests (RF), was used to estimate two parameters in the first-principle model, i.e., overall heat transfer coefficients of ladle and tundish. The use of the statistical models for the parameters update makes the first-principle model able to model the heat losses during transportation and casting periods, more precisely. Another RF model was used to compensate the error of the inner gray-box model. The results in TD temp prediction show the advantage of the proposed gray-box model over the first-principle model, the statistical model and the conventional gray-box model.

ACKNOWLEDGMENTS

This study was partially supported by the grant from ISIJ as an activity of research group, High Precision Process Control via Large Scale Database and Simulation Models, and also by Japan Society for the Promotion of Science (JSPS), Grant-in-Aid for Scientific Research (C) 24560940.

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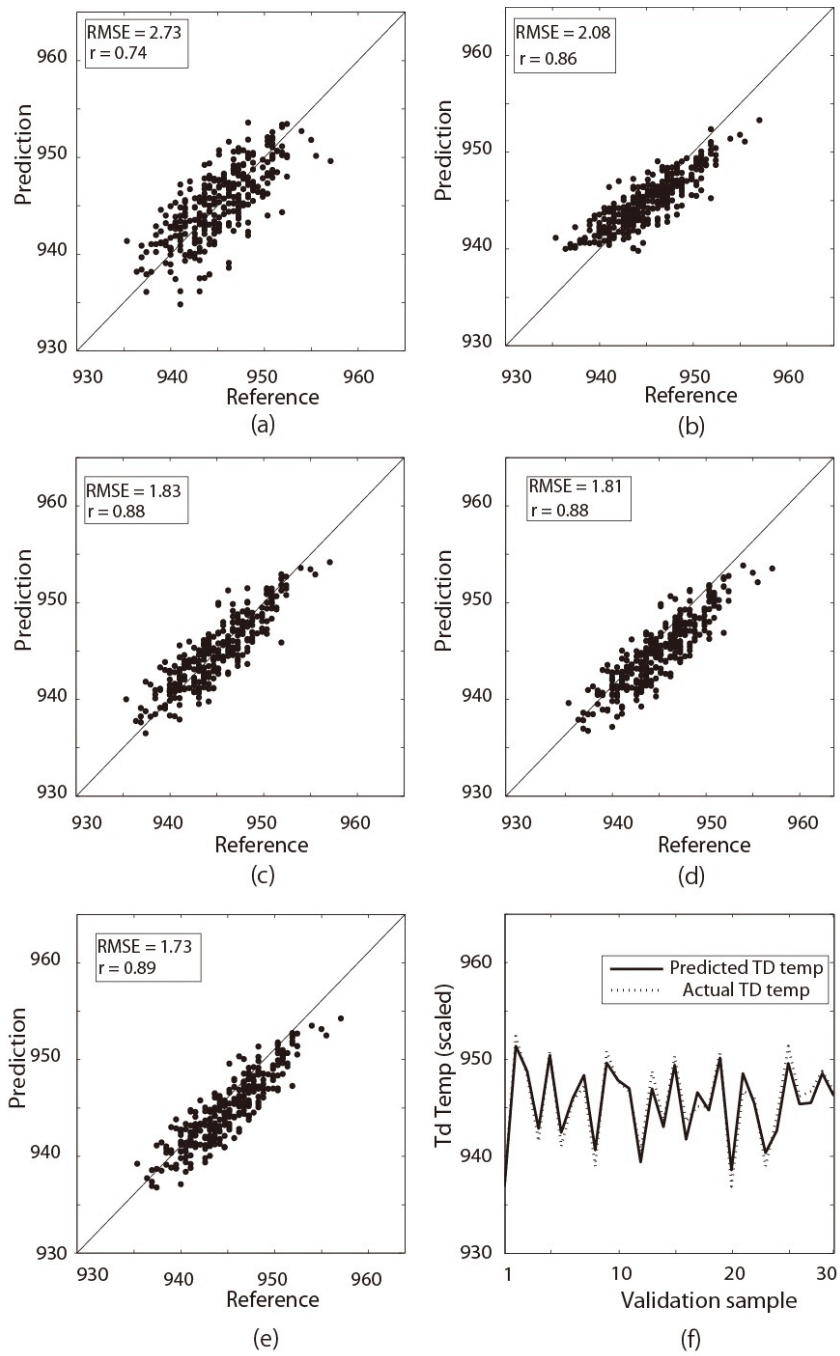

Fig. 6. Predicted TD temp through (a) first-principle model, (b) RF model, (c) outer gray-box model, (d) inner gray-box model and (e) combined gray-box model, and (f) predicted and real TD temp

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