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Scheduling Incorporating Waste Management using Decomposition Approaches

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Abstract: With the shift to high-value, low volume production, the problem of short term production scheduling for multipurpose/multiproduct batch processes has been realized as an important problem in industrial plant operations. Also, any industrial batch process invariably involves the production of harmful wastes along with the useful products. Due to stricter environmental regulations, these wastes cannot be disposed off without prior treatment. However, most scheduling problems considered in literature deal with objectives such as maximization of profit due to product sales, minimization of time span, minimize product tardiness, etc. with no consideration to minimizing downstream waste production cost. So in this study, the problem of short term batch scheduling is approached with the dual objective of maximizing profit and at the same time minimizing the downstream waste treatment cost. Since the problem of short term batch scheduling with waste management is inherently complex, a number of different decomposition approaches to solve complex multi-level optimization problems are presented. The model co-ordination approach is applied to a case example and the results elucidate the fact that the optimal solution is achieved at significantly lesser computational complexity, and agrees with the solution obtained when the optimization problem is solved without decomposition. The case example also illustrates the effectiveness and efficiency of model coordination approach in terms of computational effort.

1. INTRODUCTION

Large systems are complex processes built up by combination of many interacting smaller subsystems. Due to the complexities involved in such systems, it has always been a daunting task to design a single integrated optimizing system for the entire process. This is because of the computational effort and time required for this design is excessive. Consequently, it is necessary to decompose the problem into a number of smaller interconnected problems and solve it using a multi level optimization scheme. Once the integrated optimization problem is decomposed to multi level form with distinct tasks assigned to units on each level, a coordination strategy is decided so that interacting sub systems on any level can be treated independently. Coordination amounts to devising an iterating scheme among the sub problem optimizations such that the final solution is that of the original integrated problem. Mesarovic *et. al.* (1970) listed five co-ordination modes for higher level (master level) subsystems although only two of these have received significant attention: model co-ordination and goal co-ordination. Both these methods will be explained in greater detail in later section.

1.1 Background on batch scheduling

Batch processes play an important role in chemical processing industry. A significant proportion of the world's chemical production is still made in batch plants both in terms of volume and value and it does not seem likely that this proportion will decline in value. In batch processes, large numbers of chemical products are produced to satisfy human demand in daily life. The need for scheduling operations arises from the competing alternatives available to utilize limited resources. Productivity of a batch process depends a lot on scheduling of various tasks involved in production. Scheduling is a decision making process to determine the locations, times and sequences for processing activities with finite units and resources to achieve a certain objective subject to a diverse set of utilization constraints. Techniques proposed to solve batch production scheduling problems are almost as complex and diverse as the problem itself. Although an extensive review of all the existing short term scheduling formulations can be found in Mendez *et. al.* (2006), a summary of some of the major studies on batch scheduling relevant to this paper is presented. Early attempts to deal with batch scheduling relied on the discretization of the time horizon into a number of intervals of equal duration. Kondili *et. al. (*1993) used discrete time representation to formulate the short term scheduling as a mixed integer linear program. Mendez and Cerda (2003) proposed a MILP approach based on a continuous time domain representation for optimal short term scheduling of non-sequential multipurpose batch processess. Pinto and Grossmann (1994) proposed a slot-based continuous time formulation to solve the problem of minimizing the earliness of specific orders. Giannelos and Georgiadis (2002) proposed an STN represented, unit-specific event based formulation. Ierapetritou and Floudas (1998) used some time sequencing constraints in a mixed integer linear programming (MILP) model and decoupled the task events from the unit events.

1.2 Downstream waste management Due to strict environmental regulations in today's process industry, manufacturers are forced to find the best ways to reduce the environmental impacts of their plants and at the same time maximize their profits. To account for environmental considerations in process design, planning and scheduling, different methodologies are proposed in the literature. Most of these methodologies are focused towards either waste minimization as an explicit objective or using life cycle approach to reduce production of harmful wastes and at the same time maximize the productivity. In this paper, instead of waste minimization or LCA analysis, we explicitly focus on downstream waste treatment cost.

When the production / manufacturing is mainly based on batch operations, waste generation can also be expected to follow a similar pattern as the chemical production. Batch chemical industry therefore produces waste according to the process recipes in small amounts and in a discontinuous way. However, the waste treatment operations run continuously and imply large units which cannot be easily shut down or restarted as a function of available waste. The challenge is then to provide enough feed for continuous processing using available wastes. As the different waste treatment units have different requirements and cannot be shut down for small duration, a mixing of waste with different quantities can allow a continuous feeding respecting input requirements. A way to solve the problem therefore is to store the wastes and perform the mixing of wastes in order to provide the required flows. Waste mixing makes it possible to provide the required flow to a treatment unit and at the same time helps fulfil the input requirement for that unit. The goal therefore is to formulate and model the aforementioned waste management process and obtain a set of waste streams which can be continuously fed to waste treatment plants and at the same time, these streams should be optimal with respect to cost of the treatment operations. Also integration of this management model with the scheduling model is to be done so as to obtain the optimal cost for the whole batch processing plant (i.e. including waste management).

In this paper, we consider the integration of the traditional batch scheduling problem to include waste management, so as to evolve a production policy that is optimal with respect to both objectives of manufacturing and waste management. We show that the integrated problem is computationally complex and necessitates the use of decomposition approaches to realize overall optimality. We propose the use of the model co-ordination method for the decomposition task and demonstrate the efficacy of the method for complexity reduction. The overall optimization approach is validated on an extended formulation of traditional batch scheduling problem to include waste management, and key computational advantages of the proposed method in terms of reduced computational complexity are highlighted.

The rest of the paper is structured as follows: In the next section we briefly provide an overview of the decomposition approaches relevant to the problem at hand. Section 3 introduces the extended formulation of the batch scheduling problem to include waste management. Key results of the joint optimization strategy are presented in Section 4, followed by summarizing remarks.

2. Decomposition Approaches

A general two level hierarchical system is shown in Figure 1. Infimal level solves the nonlinear programming problem defined by Equations (2.1) and (2.2) decomposed into two smaller interacting nonlinear programming sub-problems.

$$\min f(l,x) \tag{2.1}$$

$$subject\ to\ g(l,x) = 0 \tag{2.2}$$

The supremal sub-system is responsible for coordinating the efforts of the interacting infimal sub-problems to achieve the overall goal. In model coordination approach, the supremal level selects values of the interactions, $\delta_i = \mu_i$, and the infimal level solves for the set points l_i, given μ_i.

Supremal problem:

$$\min_{\mu} f(\mu, y) \tag{2.3}$$

Infimal sub-problems:

$$\min_{l_i} f_i(l_i, x_i, \mu_i) \tag{2.4}$$

$$subject\ to\ g_i(l_i, x_i, \mu_i) = 0 \tag{2.5}$$

In goal coordination approach, a Lagrange term is added to the overall objective function as shown in equation (2.6). The Lagrange term is then expanded and separated to obtain the infimal objective functions.

$$f' = f + \delta^T (\mu - z) \tag{2.6}$$

The supremal problem solves for the values of δ to maximize f' and the infimal sub-problems solve for l_i and μ_i.

Supremal problem:

$$\max_{\delta} f'(\delta, y) \tag{2.7}$$

Infimal sub-problems:

$$\min_{l_i \mu_i} f_i\ (l_i, x_i, \mu_i, \delta) \tag{2.8}$$

$$subject\ to\ g_i(l_i, x_i, \mu_i) = 0 \tag{2.9}$$

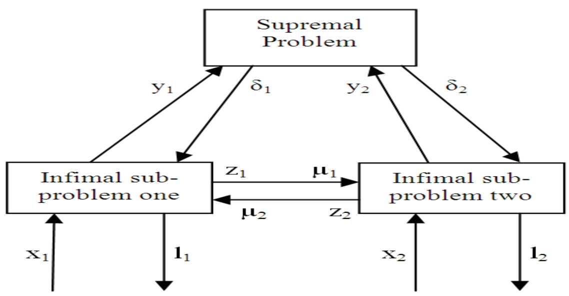

Figure 2.1. Two-level Hierarchical system.

In this study, the method of model coordination is applied to the integrated optimization problem of batch scheduling and waste management and it is verified through the results that optimal solution obtained for the decomposed problem is same as that obtained for the integrated problem. Further two example cases are studied which indicate advantages of using model coordination approach over the integrated approach in terms of CPU time.

3. Scheduling incorporating waste management objectives using decomposition approaches

The problem of short term production scheduling is an important problem in the batch process industry. Considerable amount of work has been done in the area of short term scheduling for batch plants in the last few decades. Most scheduling problems considered in literature deal with objectives such as maximization of production in terms of profit due to product sales, minimization of time span for production, satisfying product demands or minimization of product tardiness. However, every industrial batch process invariably involves lots of harmful waste which is produced along with the useful products and cannot be disposed into environment without prior treatment and seldom is the aspect of downstream waste treatment considered while scheduling an industrial batch process. Therefore, in this study we integrate the scheduling with the downstream waste treatment and highlight the importance of incorporating the waste treatment cost to the scheduling objectives in order to obtain schedules which satisfy environmental constraints.
Since the complex engineering system involving scheduling and downstream waste treatment is inherently multi level, model coordination technique is also applied to obtain the optimal solution which is then compared with the solution for the integrated problem.

3.1 Problem statement

The short term scheduling and the waste management problem for systems of batch processes that is considered in this study can be stated as follows. Given (i) the production recipe, (ii) the available units and their capacity limits, (iii) waste treatment units and their capacity limits, (iv) the available storage capacity for each of the materials and (v) the time horizon under consideration, then the objectives are (i) Firstly, to study the impact of the downstream waste treatment on the upstream scheduling problem and how it can affect the overall cost of the whole process of scheduling and waste treatment, (ii) Secondly, to discuss the applicability of model coordination technique to complex optimization problems and its advantages in terms of computational effort required to reach the optimal solution.

3.2 Mathematical Formulation

In the following sub-sections the mathematical model for the scheduling and waste management problem is presented in detail.

3.2.1 Mathematical model for scheduling problem

For the deterministic short term scheduling, the model proposed by Ierapetritou and Floudas (1998) based on continuous time representation, is used in this study. It results in smaller models in terms of binary and continuous variables and constraints.

3.2.1.1 Constraints

Allocation constraints

$$\sum_{i \epsilon I_j} wv(i,n) \le 1, \forall j \epsilon J,\ n \epsilon N \quad (3.1)$$

Capacity constraints

$$V_{ij}^{min}\, wv(i,n) \le B(i,j,n) \le V_{ij}^{max}\, wv(i,n), \quad \forall i \epsilon I,\ j \epsilon J_i,\ n \epsilon N \quad (3.2)$$

Storage constraints

$$ST(s,n) \le ST(s)^{max}, \quad \forall s \epsilon S,\ n \epsilon N \quad (3.3)$$

Material balances

$$ST(s,n) = ST(s,n-1) + \sum_{i \epsilon I_s} \rho_{si}^{p} \sum_{j \epsilon J_i} B(i,j,n-1) + \sum_{i \epsilon I_s} \rho_{si}^{c} \sum_{j \epsilon J_i} B(i,j), \forall s \epsilon S,\ n \epsilon N \quad (3.4)$$

Duration constraints

$$T^f(i,j,n) = T^s(i,j,n) + \alpha_{ij}\, w(i,n) + \beta_{ij}\, B(i,j,n), \quad \forall i \epsilon I,\ j \epsilon J_i,\ n \epsilon N \quad (3.5)$$

Sequence constraints: Same tasks in the same unit

$$T^s(i,j,n+1) \ge T^f(i,j,n) - H(2 - wv(i,n) - \textstyle\sum_{i \epsilon I_j} wv(i,n)), \forall i \epsilon I,\ j \epsilon J_i, n \epsilon N, n \ne N \quad (3.6)$$

$$T^s(i,j,n+1) \ge T^s(i,j,n), \forall i \epsilon I,\ j \epsilon J_i\ \ n \epsilon N,\ n \ne N \quad (3.7)$$

$$T^f(i,j,n+1) \ge T^f(i,j,n), \forall i \epsilon I,\ j \epsilon J_i,\ n \epsilon N,\ n \ne N \quad (3.8)$$

Sequence constraints: Different tasks in the same unit

$$T^s(i,j,n+1) \ge T^f(i',j,n) - H(2 - wv(i',n) - \sum_{i \epsilon I_j} wv(i,n)), \quad \forall j \epsilon J,\ i \epsilon I_j,\ i' \epsilon I_j,\ i \ne i',\ n \epsilon N,\ n \ne N \quad (3.9)$$

Sequence constraints: Different tasks in different units

$$T^s(i,j,n+1) \ge T^f(i',j',n) - H(2 - wv(i',n) - \sum_{i \epsilon I_{j'}} wv(i',n)), \quad \forall j,j' \epsilon J,\ i \epsilon I_j,\ i' \epsilon I_{j'},\ i \ne i',\ n \epsilon N,\ n \ne N \quad (3.10)$$

Sequence constraints: Completion of previous tasks

$$T^s(i,j,n+1) \ge \sum_{n' \epsilon N, n' \le n} \sum_{i' \epsilon I_j} (T^f(i',j,n') - T^s(i',j,n')), \forall i \epsilon I,\ j \epsilon J_i,\ n \epsilon N,\ n \ne N \quad (3.11)$$

Time horizon constraints

$$T^f(i,j,n) \le H, \quad \forall i \epsilon I,\ j \epsilon J_i,\ n \epsilon N \quad (3.12)$$

$$T^s(i,j,n) \le H, \quad \forall i \epsilon I,\ j \epsilon J_i,\ n \epsilon N \quad (3.13)$$

3.2.1.2 Objective: Maximization of profit

$$max \sum_{s \epsilon S_p} price_s\ ST(s,N) \quad (3.14)$$

3.2.2 Mathematical model for waste management problem

3.2.2.1 Constraints

Allocation constraints

$$\sum_{k \epsilon K} xv(k,l) \le 1, \quad \forall l \epsilon L_{k'} \quad (3.15)$$

Capacity constraints for each treatment unit

$$X_l^{min} \le \sum_{k \epsilon K} x(k,l) \le X_l^{max}, \quad \forall l \epsilon L_k \quad (3.16)$$

$$X_l^{min} \le \sum_{k \epsilon K} x(k,l)\, xv(k,l) \le X_l^{max}, \quad \forall l \epsilon L_{k'} \quad (3.17)$$

Capacity constraint for each waste stream

$$\sum_{l\epsilon L_{k'}} x(k,l)xv(k,l) + \sum_{l\epsilon L_k} x(k,l) = W(k) \quad (3.18)$$

3.2.2.2 Objective function

$$\min \sum_{l\epsilon L_{k'}} Cost_l \, (\sum_{k\epsilon K} xv(k,l)x(k,l))^{\delta} + \sum_{l\epsilon L_k} Cost_l \, (\sum_{k\epsilon K} x(k,l))^{\delta} \quad (3.19)$$

This is the general MINLP formulation for the waste management problem (non-linearities in the objective function and capacity constraints). Allocation constraints (Eq. (3.15)) ensure that only one of the waste streams is fed to the treatment units belonging to set $L_{k'}$. Capacity constraints (Eq. (3.16 & 3.17)) are the limitations imposed on the total stream flow into a treatment unit by the treatment unit capacities. Constraint (Eq. (3.18)) states that amount of each waste stream to be treated are given by the upstream scheduling problem. Objective (eq. (3.19)) is to minimize the total waste treatment cost. Cost of treatment a waste in a treatment unit is directly proportional to amount of waste fed to the unit raised to power of some factor.

For a NLP waste management problem, $L_{k'}$, is an empty set which implies no allocation constraints. For a LP waste management problem, $\delta = 1$ and $L_{k'}$ is an empty set.

Although upstream scheduling is a batch process and the downstream waste treatment is a continuous process, scheduling and waste treatment are interdependent as shown in Figure 3.1. Waste treatment process requires continuous supply of wastes for each of the treatment units but it is constrained by the amount of available waste from upstream scheduling. At the same time the maximum amount of waste that can be produced by scheduling process in a time horizon is limited by the treatment capacity of the downstream units. As a result, one cannot solve the scheduling and waste management problems as separate optimization problems and integration of these two have to be done in order to obtain an optimal cost for the overall process. Therefore, in the next section, mathematical formulation for the integrated optimization problem is presented.

3.2.3 Integrated scheduling and waste management optimization problem

In order to integrate the batch scheduling and the waste management optimization problems, capacity constraint for each waste stream (Eq. (3.18)) is modified as

$$\sum_{l\epsilon L_{k'}} x(k,l)xv(k,l) + \sum_{l\epsilon L_k} x(k,l) = ST(s,N),$$
$$\forall\, s \,\epsilon\, S_w,\ k \,\epsilon\, K,\ K = S_w \quad (3.20)$$

Remaining constraints same as in section 3.2.1 and 3.2.2.
Objective function for the integrated problem

$$\min \sum_{l\epsilon L_{k'}} Cost_l \left(\sum_{k\epsilon K} xv(k,l)x(k,l)\right)^{\delta} + \sum_{l\epsilon L_k} Cost_l \left(\sum_{k\epsilon K} x(k,l)\right)^{\delta} - \textstyle\sum_{s\epsilon S_p} price_s \ ST(s,N) \quad (3.21)$$

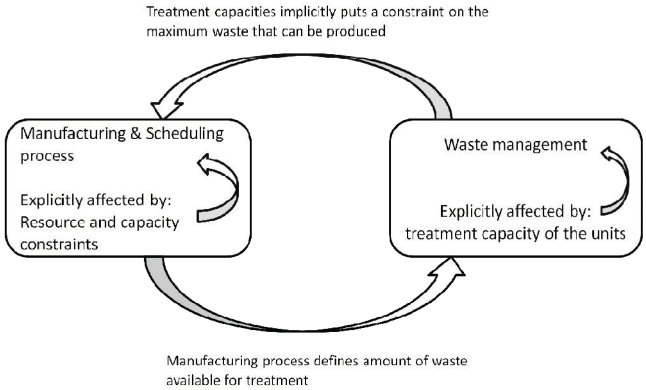

Figure 3.1. Interdependent scheduling and waste management process

4. Case Study

In this case example three different products are produced through five processing stages: heating, reactions 1, 2 and 3, and separation of product 3. The STN representation of the plant flow sheet is shown in Figure 4.1. The data for this example is presented in Table 4.1. The processing times are allowed to vary within ±33 % around the mean values shown in Table 4.1. The time horizon of interest is 8 hours. As indicated in formulation, different production tasks can take place in different units; each reaction is represented by two tasks i.e. each of the three required reactions can be performed in any of reactor 1 or 2. Therefore, in this example there are a total of 8 tasks, heating is task 1, reaction 1 corresponds to task 2 or 3 if it takes place at reactor 1 or 2, respectively, reaction 2 corresponds to task 4 or 5 if it takes place at reactor 1 or 2, respectively, and reaction 3 corresponds to task 6 or 7 if it takes place at reaction 1 or 2, respectively. Finally separation is task 8.

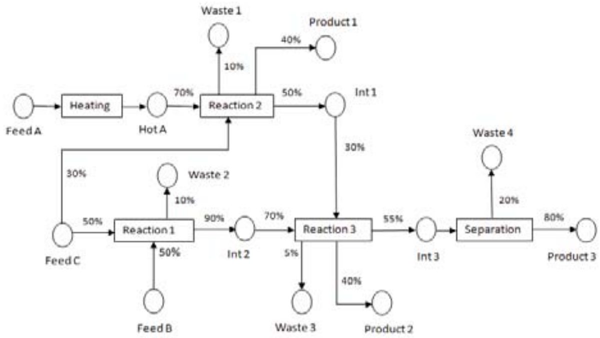

Figure 4.1. State task network representation

Waste streams produced by the production plant are treated in four different treatment units, incinerator, wet air oxidation, waste water treatment and distillation. The data for the capacities and cost for treatment units is presented in Table 4.2.

For this example, the downstream waste management optimization problem is a MINLP problem in this case. The cost factor, δ, for treatment units is equal to 0.8 and stream mixing is not possible for treatment unit 4 (distillation column) i.e. only one of the waste stream can be fed to the distillation unit which implies $L_{k'}$ = [4].

Table 4.1. Data for example

Unit	Capacity	Suitability	Mean Proc. Time, τ_{ij})
Heater	120	Heating	1.0
Reactor 1	60	Rxn. 1,2,3	2.0,2.0,1.0
Reactor 2	100	Rxn. 1,2,3	2.0,2.0,1.0
Still	180	Separation	2.0

State	Storage Capacity	Initial Amount	Price
Feed A	Unlimited	Unlimited	0.0
Feed B	Unlimited	Unlimited	0.0
Feed C	Unlimited	Unlimited	0.0
Hot A	200	0.0	0.0
Int 1	120	0.0	0.0
Int 2	150	0.0	0.0
Int 3	200	0.0	0.0
Prod.1	Unlimited	0.0	10.0
Prod. 2	Unlimited	0.0	10.0
Prod. 3	Unlimited	0.0	10.0
waste 1	Unlimited	0.0	0.0
waste 2	Unlimited	0.0	0.0
waste 3	Unlimited	0.0	0.0
waste 4	Unlimited	0.0	0.0

Table 4.2. Cost and capacities for treatment units

Unit	Min Capacity	Max Capacity	Cost
Incinerator	2	10	200
WAO	5	15	220
WWT	2	20	20
Distillation	1	5	10

For the integrated optimization problem, the capacity constraint (Eq. (3)) is as follows

$$x(k,4)xv(k,4) + \sum_{l \epsilon L_k} x(k,l) = ST(s,N), \qquad \forall\, s \,\epsilon\, S_w,\ k \,\epsilon\, K,\ K = S_w$$

Remaining constraints same as in section 3.1.1 and section 3.2.1.

Objective function for the integrated problem

$$\min Cost_4 (\sum_{k \epsilon K} xv(k,4)x(k,4))^{0.8} + \sum_{l \epsilon L_k} Cost_l (\sum_{k \epsilon K} x(k,l))^{0.8} - \sum_{s \epsilon S_p} price_s\ ST(s,N)$$

4.1 Decomposition using model coordination

As previously mentioned, the integrated problem of waste management and batch scheduling is MINLP for this example. The coordinating or interaction variables to convert this optimization problem to a two level problem are the waste streams flows produced by the scheduling problem which are fed to the treatment units. Therefore the problem is decomposed as follows

First level problem

Sub problem 1 (Scheduling problem)

Following additional constraint is added to the scheduling formulation of section 3.1.

$$ST(s,N) \le \omega(k), \qquad \forall\, s \,\epsilon\, S_w,\ k \,\epsilon\, K$$

Remaining constraints (Eq. (3.1)-(3.13)) and objective function is as follows: $max\ F_1(\omega) = \sum_{s \epsilon S_p} price_s\ ST(s,N)$

Sub problem 2 (Waste management problem)

Constraint (Eq. 3.18) in section 3.2 is modified.

$$x(k,4)xv(k,4) + \sum_{l \epsilon L_k} x(k,l) = \omega(k), \qquad \forall\, k \,\epsilon\, K$$

Remaining constraints (Eq. (3.15)-(3.17)) and objective function is as follows:

$$\min F_2(\omega) = Cost_4 (\sum_{k \epsilon K} xv(k,4)x(k,4))^{0.8} + \sum_{l \epsilon L_k} Cost_l (\sum_{k \epsilon K} x(k,l))^{0.8}$$

Second level problem

Since for the decomposed system, amount of waste streams (**ω**) is a coordinating variable, the objective functions for the scheduling and waste management problems are functions of **ω**. Let the objective function for scheduling problem be F_1 and for the waste management problem be F_2. Then the second level master problem is an unconstrained optimization problem with **ω** as variable to be solved.

$$\min F_2(\boldsymbol{\omega}) - F_1(\boldsymbol{\omega})$$
$$Subject\ to\ LB \le \boldsymbol{\omega} \le UB$$

4.2 Computational results

The integrated optimization problem is a MINLP which involves 483 constraints including the bounds, 206 continuous variables and 44 binary variables. The TOMLAB/MINLPBB code is used for the solution of MINLP formulation. It requires 49338.94 CPU seconds on a Linux system with Intel Core 2 Quad 2.5 GHz processor to reach the optimal solution. The resulting gantt chart is shown in Figure 4.2. It corresponds to an objective function of 151units within the time horizon of 8 hours. Five event points are used to solve the scheduling part of the problem. For the decomposed problem, the first level scheduling sub problem is a MILP which involves 478 constraints including the bounds, 190 continuous variables and 40 binary variables. The TOMLAB/MIPSOLVE is used for the solution of MILP formulation. The first level waste management sub problem is a MINLP and it involves 9 constraints, 16 continuous variables and 4 binary variables. The TOMLAB/MINLPBB code is used for the MINLP formulation. The second level master problem is an unconstrained optimization problem which is solved using TOMLAB/GLBSOLVE code and it involves 4 constraints which are the bounds and 4 variables. The resulting gantt chart is same as obtained for integrated problem (Figure 4.2). The optimal solution corresponds to an objective function of 151units within the time horizon of 8 hours and it takes 10582.28 CPU seconds to reach this solution on a Linux system with Intel Core 2 Quad 2.5 GHz processor.

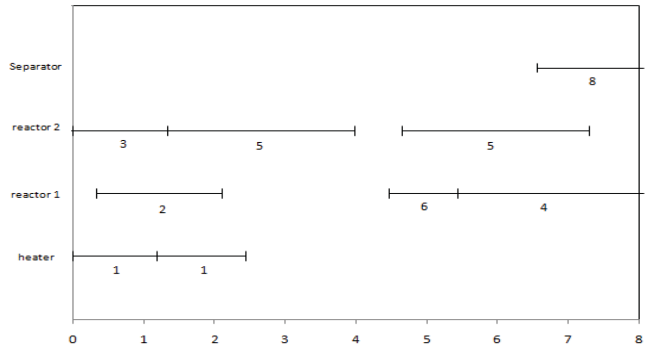

Figure 4.2. Gantt chart for case example

Table 4.3. Result summary for case example

	Integrated optimization	Decomposed-model coordination
NEP	5	5
NC	483	491
NV	250(44)	254(44)
CPU(sec)	49338.94	10582.28
Obj.	151	151

5. Conclusions

In this paper, short term batch scheduling problem incorporating downstream waste treatment cost is solved using decomposition approaches. It is shown from the results of the case example that the downstream waste treatment have a great impact on the overall process of scheduling and in order to minimize the cost for the overall process, one has to solve the scheduling and waste management optimization problem together instead of treating them as independent and separate optimization problems. Results also elucidate the fact that the optimal solution obtained using model coordination approach is same as that obtained by solving the scheduling-waste management optimization problem without decomposition. The case example further illustrates the advantage of using model coordination approach in terms of computational effort required to reach the optimal solution since around 70 % reduction in CPU time was achieved by transforming the integrated optimization problem into a multi-level optimization problem using model coordination.

References

Giannelos, N.F. and Georgiadis, M.C., 2002, "A simple new continuous-time formulation for short-term scheduling of multipurpose batch processes", Ind. Eng. Chem. Res., 41, 2178-2184.

Ierapetritou, M. G. and Floudas, C. A., 1998, "Effective continuous-time formulation for short-term scheduling ·1· multipurpose batch process", Ind. Eng. Chem. Res., 37 (11), 4341-4359.

Kondili, E., Pantelides, C.C. and Sargent, R. W. H., 1993, "A general algorithm for short-term scheduling of batch operations. Part 1. MILP formulation", Comput. Chem. Eng., 17, 211-227.

Mendez, C. A. and Cerda, J., 2003, "An MILP continuous-time framework for short-term scheduling of multipurpose batch processes under different operation strategies", Optimization and Engineering, 4, 7-22.

Mendez, C. A., Cerda, J., Grossmann, I. E., Harjunkoski, I. and Fahl, M., 2006, "State-of-the-art review of optimization methods for short-term scheduling of batch processes", Comput. Chem. Eng., 30, 913-946.

Mesarovic, M. D., Macko, D. and Takahara, Y., 1970, "Theory of Hierarchical, Multilevel, Systems", Academic Press, New York.

Pinto, J and Grossmann, I, 1994, "Optimal Cyclic Scheduling of Multistage Continuous Multiproduct Plants", Comput. Chem. Eng., 18,797

Nomenclature

Indices

i	task
j	unit
k	waste stream
l	waste treatment unit
n	event point representing the beginning of a task
s	state

Sets

I	tasks
I_j	tasks which can be performed in unit j
I_s	tasks which can process state s and either produce or consume
J	units
J_i	units which are suitable for performing task i
K	waste streams
L	waste treatment units
L_k	set of waste treatment units which can be fed multiple number of wastes
$L_{k,}$	set of waste treatment units which can treat only one of the wastes
N	event points within the time horizon
S	set of all involved states s
S_p	set of states s which are the product states
S_w	set of states s which are the waste states

Parameters

V_{ij}^{min}	minimum amount of material processed by task i required to start operating unit j
V_{ij}^{max}	maximum capacity of the specific unit j when processing task i
$ST(s)^{max}$	available maximum storage capacity for state s
X_l^{min}	minimum amount of waste required for the operation of the treatment unit l
X_l^{max}	maximum amount of waste that can be treated in the treatment unit l
$\rho_{si}^{p}, \rho_{si}^{c}$	proportion of state s produced, consumed from task i, respectively
α_{ij}	constant term of processing time of task i at unit j
β_{ij}	variable term of processing time of task i at unit j
δ	cost factor for the waste treatment cost
H	time horizon
$price_s$	price of state s
$W(k)$	amount of waste k available for treatment from the upstream process
$Cost_l$	cost of waste treatment in treatment unit l

Variables

$wv(i,n)$	binary var. that assign the beginning of task i at event point n
$\omega(k)$	amount of waste k available for treatment
$xv(k,l)$	binary variable that assigns the waste k to treatment unit l
$x(k,l)$	amount of waste k treated in treatment unit l
$B(i,j,n)$	amount of material undertaking task i in unit j at event point n
$ST(s,n)$	amount of state s at event point n
$T^s(i,j,n)$	time that task i starts in unit j at event point n
$T^f(i,j,n)$	time that task i finishes in unit j while it starts at event point n