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Simultaneous reduced order multi-parametric moving horizon estimation and model based control

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Abstract: In this paper we apply model order reductions techniques to efficiently implement simultaneous model predictive control and moving horizon estimation for high dimensional chemical processes. Two model approximation schemes that both combine order reduction and linearization are employed and compared. The approach is demonstrated on a benchmark distillation column example model.

Keywords: Model Order Reduction, moving horizon estimation, model predictive control, multi-parametric/explicit control.

1. INTRODUCTION

Multi-parametric/explicit model based control (mp-MPC) is a type of model based control which relies on off-line pre-computation of optimal control values. This approach was developed to reduce online computational costs of model predictive control and consists of generating a parametric map of the optimal control inputs as explicit functions of the state variables of the system (Bemporad et al, 2002). In order to implement this methodology for systems of arbitrary complexity, Pistikopoulos and co-workers developed a step-wise framework (Narciso et al, 2008). This framework aims to bridge the gap between complex models and recent development on multi-parametric/explicit model predictive control. In order to achieve this, model approximation methods (model reduction, system identification) are used to enable or facilitate the application of new theory and tools for the various classes of problems within multi-parametric programming (Pistikopoulos, 2009). The open literature contains a number of studies where multi-parametric/explicit predictive control has successfully been combined with various model order reduction techniques with in-silico test on relatively complex nonlinear mathematical models (Hovland et al, 2006), (Xie et al., 2011), (Rivotti et al, 2012), and (Lambert et al, 2013). Another important issue is state and parameter estimation. While model reduction is efficient at deriving lower dimensional models which preserve input-output behaviours, mp-MPC usually requires full state information. Moving horizon estimation techniques are usually preferred over Kalman filters as they are able to incorporate systems constraints and handle non-Gaussian noise. As part of the suggested framework, moving horizon estimation has been formulated into multi-parametric/explicit form (mp-MHE) (Hedengren and Edgar, 2006) (Darby and Nikolaou, 2007), with recent enhancement addressing robustness against estimation error (Voelker et al, 2013a), (Voelker et al, 2013b). The main aim of this study is to derive multi-parametric/explicit moving horizon estimators for high dimensional systems. In this study, we expand our on-going developments for the simultaneous design of multi-parametric/explicit model predictive controllers and moving horizon estimation by using two model order reduction methodologies. The first methodology employed consists of linearizing the original system and subsequently reducing it whereas the second methodology employs a nonlinear model order reduction technique followed by linearization of the reduced order model. The paper is organized as follows: Firstly we give a description and comparison of the model approximation techniques implemented. Secondly, we present the fundamentals of the mp-MHE and mp-MPC methodologies employed. Lastly, we compare the performance of the two approximation methodologies by implementing simultaneous mp-MHE/mp-MPC on a distillation benchmark model example.

2. MODEL APPROXIMATION METHODOLOGIES

Model approximation is often an important first step for the design of model-based controllers, as it is not always practical or feasible to implement model based control strategies to high fidelity models. In this study we apply a combination of linearization and two efficient model reduction techniques to derive low order models suitable for mp-MPC/MHE methodologies.

2.1 Linear Model Reduction

The first approach consists of linearizing the original system and subsequently using a linear model approximation technique. In this study we will use balanced truncation. Balanced truncation is a model order reduction technique, based on singular value decomposition, which is particularly suitable in the context of state-space dynamic models, linear Model Predictive Control and Multi-parametric controller design. Consider a linear time invariant (LTI) system of the form:

$$x_{t+1} = Ax_t + Bu_t$$
$$y_t = Cx_t \tag{1}$$

Balanced truncation consists of deriving a transformation matrix T so that the resulting system is balanced i.e. the most observable states correspond to the most controllable ones. Following the procedure described in (Antoulas, 2005), we formulate a dynamical system in an equivalent balanced form:

$$\bar{x}_{t+1} = TAT^{-1}\bar{x}_t + TBu_t$$
$$y_t = CT^{-1}x_t \tag{2}$$

It is then possible to truncate the system by retaining the states accounting for most of its dynamical behaviour by partitioning the balanced system into retained and discarded states $\bar{x} = [\bar{x}_1, \bar{x}_2]$ and the resulting reduced order system has the following form:.

$$\bar{x}_{1t+1} = A_{11}\bar{x}_{1t} + B_1 u_t$$
$$y_t = C_1\bar{x}_{1t} \tag{3}$$

Where the matrices in (2) are partitioned as follows:

$$TAT^{-1} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, TB = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}$$
$$CT^{-1} = \begin{bmatrix} C_1 & C_2 \end{bmatrix}, \tag{4}$$

2.2 Nonlinear Model Reduction

The second approach employed is nonlinear balanced truncation, which is a snapshots based technique and an empirical extension of the linear balanced truncation technique (Hahn and al, 2004). Consider a nonlinear system of ODEs of the following form:

$$\dot{x}(t) = f(x(t), u(t))$$
$$y(t) = h(x(t), u(t)) \tag{5}$$

As in linear balanced truncation, the method consists of finding a transformation matrix T in order to project the state vector on a lower order subspace $\bar{x} = Tx$. In order to compute these matrices, empirical gramians or covariance matrices are derived from simulation data from the system.
A balanced system is then obtained from the previously defined empirical gramians as:

$$\dot{\bar{x}}(t) = Tf(T^{-1}\bar{x}(t), u(t))$$
$$y(t) = g(T^{-1}\bar{x}, u) \tag{6}$$

Using a Garlekin projection $P = [I, 0]$ matrix with the same rank as the reduced system, the unimportant states may be set a nominal steady state value and the nonlinear reduced order model:

$$\dot{\bar{x}}_1(t) = PTf(T^{-1}P^T\bar{x}(t), u(t))$$
$$\dot{\bar{x}}_2(t) = \bar{x}_{2ss}(0)$$
$$y = g(T^{-1}\bar{x}, u) \tag{7}$$

Note that in the case of the presence of parametric uncertainty, the system may be reduced by treating the parameters as exogenous inputs in a similar way as the method described above:

$$\dot{x}(t) = f(x(t), u(t), \theta(t))$$
$$y(t) = h(x(t), u(t), \theta(t)) \tag{8}$$

simply by augmenting the vector of input with θ as an exogenous input: $\tilde{u} = \begin{pmatrix} u \\ \theta \end{pmatrix}$ (Sun & Hahn, 2006). Model Order reduction can then be performed on the reduced order model.

2.3 Linearization

mp-MHE and mp-MPC routines require a linear model for their application. In the first case we linearize the system around steady state and reduce the resulting linear model. In the second case the system is firstly reduced via empirical model order reduction and subsequently linearized around the steady state of the reduced order model $\bar{x}_{ss}$. In both cases we use the analytical jacobians of the original and reduced order systems. Because the linearization in the second case is performed with smaller jacobian matrices, its error tends to be smaller than in the first case. Although both systems are linear, the preservation of the transient behaviour in the second case tends to vastly outperform the first case.

3. MULTI-PARAMETRIC MOVING HORIZON ESTIMATION

In this section we briefly introduce the theoretical background necessary for the simultaneous design of a state estimator and an explicit model predictive controller.

3.1 Multi-Parametric Moving Horizon Estimation

Moving horizon estimation (MHE) is an estimation methodology based on optimization. Contrary to Kalman filters, MHE only consider a limited amount of past data. One of the main advantages of moving horizon estimation is the possibility to incorporate system knowledge as constraints in the estimation. In MHE the system states are derived by solving following optimization problem (Rao, 2000):

$$\min_{\hat{x}_{T-N/T},\hat{W}_T} \left\|\hat{x}_{T-N/T} - \underline{x}_{T-N/T}\right\|_{P^{-1}_{T-N/T-1}} + \left\|Y^{T-1}_{T-N} - O\hat{x}_{T-N/T} - \bar{c}bU^{T-2}_{T-N}\right\|^2_{P-1} + \sum_{k=T-N}^{T-1}\|\hat{w}_k\|_{Q_k^{-1}} + \sum_{k=T-N}^{T-1}\|\hat{v}_k\|_{R_k^{-1}}$$

st:

$$\hat{x}_{k+1} = A\hat{x}_k + Bu_k + G\hat{w}_k$$
$$y_k = C\hat{x}_k + \hat{v}_k$$
$$\hat{x}_k \in X, \hat{w}_k \in \Theta, \hat{v}_k \in V \tag{9}$$

where T is the current time, $Q_k \succ 0, R_k \succ 0$, $P_{T-N/T-1} \succ 0$ are the covariances of w_k v_k x_{T-N} assumed to be symetric, N is the horizon length of the MHE, $Y^{T-1}_{T-N} = \left[y^T_{T-N},\ldots,y^T_T\right]^T$ is a vector containing the past $N+1$ measurements and $U^{T-1}_{T-N} = \left[u^T_{T-N},\ldots,u^T_{T-1}\right]^T$ is a vector containing the past N inputs. x, v, w denotes the variables of the system and $\hat{x}, \hat{v}, \hat{w}$ denote the estimated variables of the system and $\hat{x}_{T/T-N}$ and $\hat{W}_T = W^{T-1}_{T-N} = \{\hat{w}\}^{T-1}_{T/T-N}\ \hat{x}_{T-N/T},\hat{W}_T$ denote the decision variable of the optimization problem, respectively the estimated state variable and the noise sequence. $\hat{V}_k$ is the measurement noise.

$$\left\|\hat{x}_{T-N/T} - \underline{x_{T-N/T}}\right\|^2_{P_{T-N/T-1}} + \left\|Y^{T-1}_{T-N} - O\hat{x}_{T-N/T} - \bar{c}bU^{T-2}_{T-N}\right\|^2_{P-1}$$

is described as the smoothed arrival cost (Rao, 2000).

In mp-MHE the problem in (9) is reformulated as a multi-parametric programming problem (Darby and Nikolaou, 2007):

$$\min_{\hat{x}_{T-N/T},\hat{W}^{T-1}_{T-N/T}} \frac{1}{2}\left[\hat{x}^T_{T-N/T}, \hat{W}^{T-1\ T}_{T-N/T}\right] H \begin{bmatrix}\hat{x}_{T-N/T}\\ \hat{W}^{T-1}_{T-N/T}\end{bmatrix} + \theta.f.\begin{bmatrix}\hat{x}_{T-N/T}\\ \hat{W}^{T-1}_{T-N/T}\end{bmatrix}$$
$$\text{st } K.\begin{bmatrix}\hat{x}_{T-N/T}\\ \hat{W}^{T-1}_{T-N/T}\end{bmatrix} \le k \tag{10}$$

The parameters of the multi-parametric programming problem in (10) are the past and current measurements and inputs and the initial guess for the estimated states.

3.2 Simultaneous mp-MPC and mp-MHE

Based on the model approximation techniques described in 3.1, simultaneous reduced order mp-MPC/MHE can be applied in a reduced order fashion. This approach has several advantages. The first advantage is the reduction in computational complexity as both the controller and estimator do longer operate based on full state information. The second advantage has been discussed in (Singh et al, 2005) in the case of extended Kalman filters. It was showed that the use of reduced order observers avoids a estimation error due to poor observability of part of the states. The methodology is illustrated in figure 1 and it is applied to a distillation column example presented in the next section.

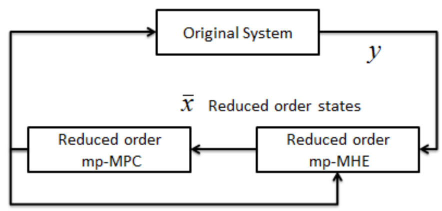

Figure 1: Schematic of simultaneous reduced order mp-MHE/MPC

4. APPLICATION EXAMPLE

4.1 Distillation Column Model

We consider the design of a controller for a simplified model of a distillation column (Benallou et al, 1986). The control problem involves the regulation of the product purity to a fixed set-point, using the reflux ratio as the manipulated variable. A constraint is imposed on the manipulated variable. The inlet concentration is assumed to be the main source of uncertainty of the system and will be included as noise for the moving horizon estimation. A Gaussian distribution centred in 0.5 and with a 3% standard deviation is assumed.

4.2 Moving Horizon Estimators

Multi-parametric moving horizon estimators were built for the following approximated models:

Case 1: The original distillation column model was first linearized, thus yielding a 32 states linear time invariant system. This latter was subsequently reduced to two states via balanced truncation.

Case 2: The original distillation column model was reduced to two states by using empirical nonlinear balanced truncation. The resulting two states system of ODEs was then linearized.

Relative performance of both reduced order model is shown in figure 2. It is evident that the transient response in case 2 gives a better fidelity to the original model.

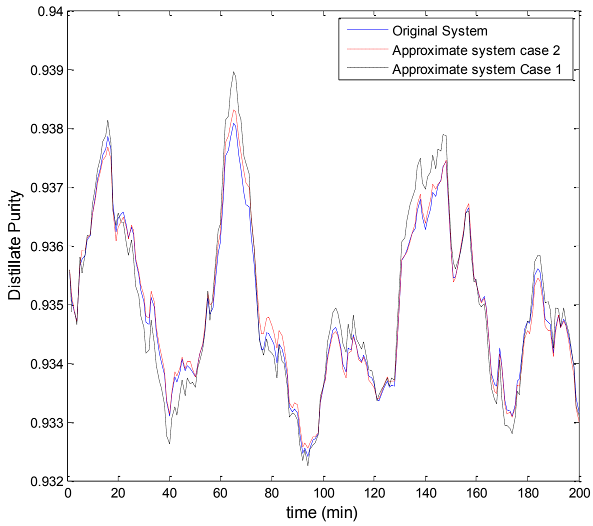

Figure 2: Dynamic simulations of the original system compared to reduced order models

In this work we do not intend to have the original state information for the physical system. Note that the 32-states model cannot be directly used to derive multi-parametric controllers/estimators. Also note that the remaining states are also those required to compute optimal control laws when deriving an mp-MPC controller. Although the first state of the original system of ODEs is measured, it is also possible to reconstruct it from the estimated reduced states. This is shown in figure 3. One can notice that case 2 offers a significantly better estimation than case 1 although both systems are linear. The comparison is also performed on the reduced states. The two lower dimensional subspaces onto which the original states are projected are not the same since the linearization in case 1 is performed around the steady states values of the original states while the linearization in case 2 is carried out on the steady state values for the nonlinear reduced order model. In figure 6 we show the critical regions for the moving horizon estimator based on case 2, which will be used for simultaneous mp-MPC/MHE.

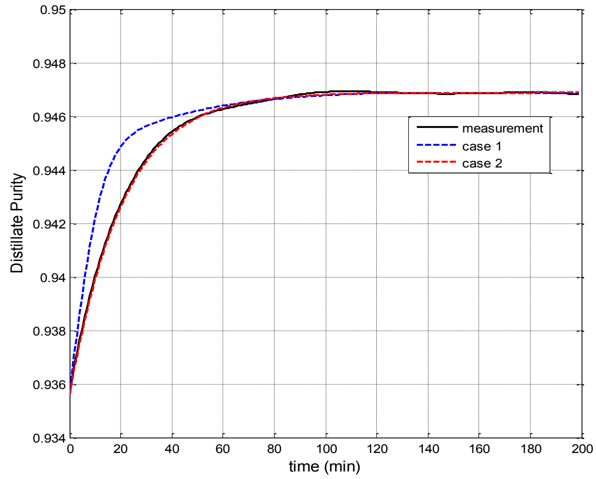

Figure 3: Comparison of reconstructed states for both reduced order models.

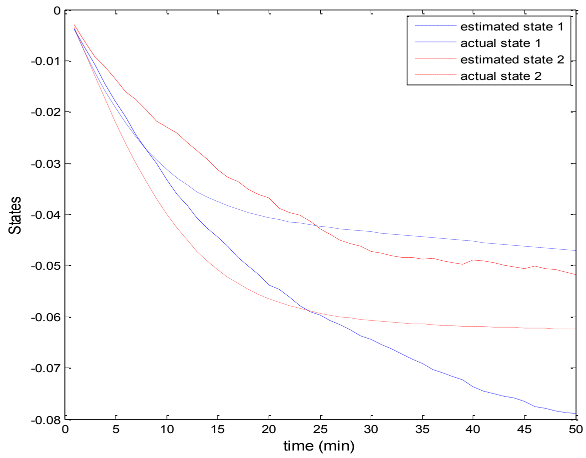

Figure 4: Comparison of actual and estimated reduced order state information for case 1

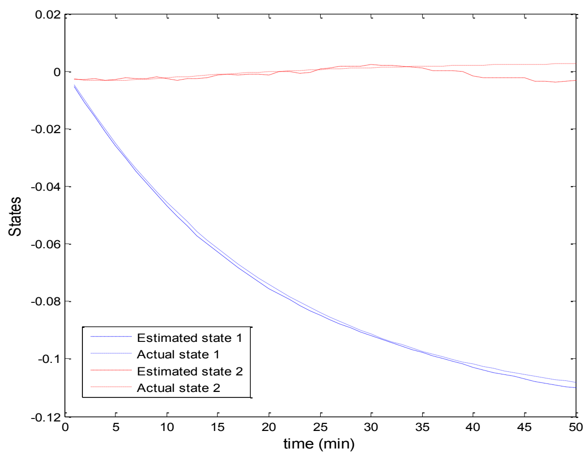

Figure 5: Comparison of actual and estimated reduced order state information for case 2

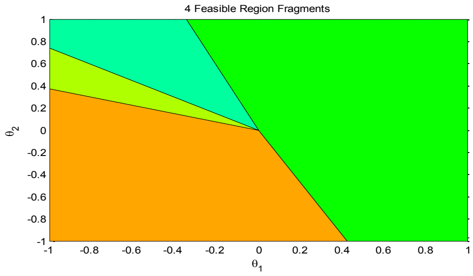

Figure 6: Multi-Parametric critical region maps for the estimator built based on case 2: Projection on the first two parameters, which are the two past measurements in the estimation problem.

Figure 4 and 5 show the performance of both reduced order estimators in their respective subspace. One observation that can be made, having used similar reduction techniques both based on singular value decomposition and balancing of the system, is that the order in which the linearization and reduction steps are performed does matter and nonlinear model order reduction seems to perform better if employed prior to linearization

4.3 Simultaneous mp-MHE/mp-MPC

mp-MHE and mp-MPC were combined and a close-loop simulation, shown in figure 7, was performed to evaluate the performance of the methodology. It can be seen that the estimator provides sufficiently accurate information to the parametric controller to drive the system to the desired set point based only on measurement information. The combination of two reduced order parametric maps is then sufficient to operate a control policy for high order chemical process. A slight offset is observed around the set-point and is mainly due to the noise or uncertainty of the inlet concentration of the column. In the case of high measurement noise (figure 8 and 9), the control profiles are more erratic but the simultaneous implementation of mp-MPC and mp-MHE still achieve the desirable set-point change. Figure 10 displays the critical region map for the mp-MPC controller implemented simultaneously to mp-MHE.

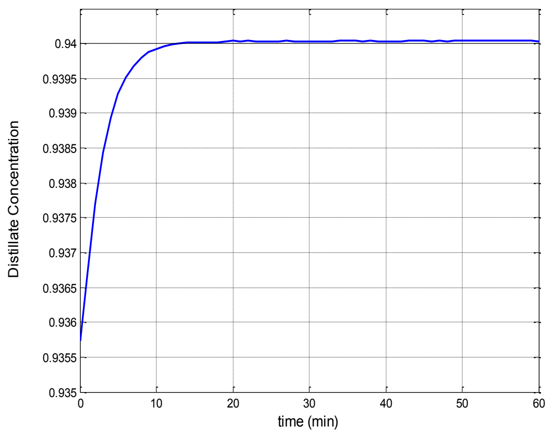

Figure 7: Close loop simulation of a set-point change operated through simultaneous mp-MHE and mp-MPC

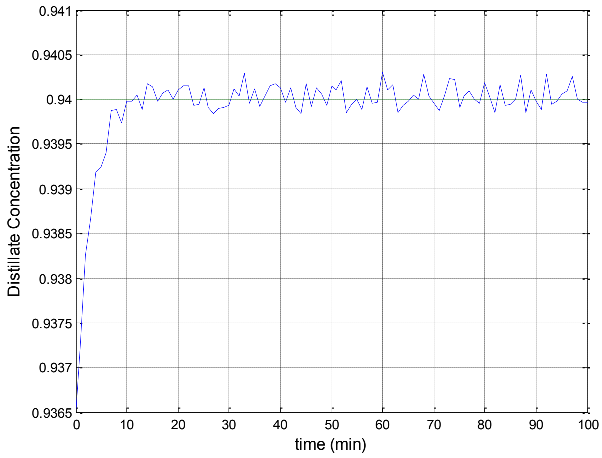

Figure 8: Close loop simulation of a set-point change operated through simultaneous mp-MHE and mp-MPC in the case of high measurement noise.

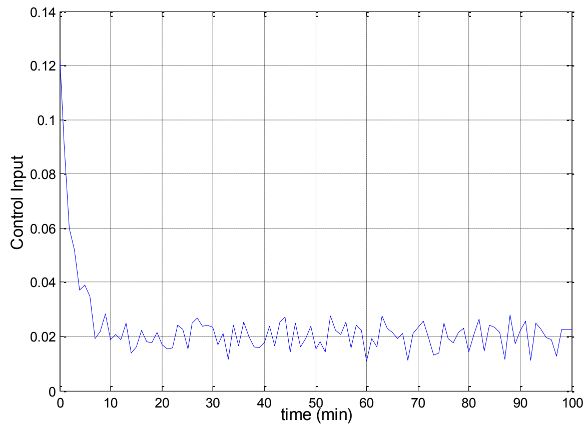

Figure 9: Evolution of the control input variable in the case of high measurement noise.

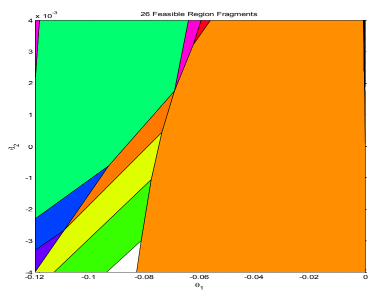

Figure 10: Critical regions for the mp-MPC controller implemented simultaneously to mp-MHE

5. CONCLUSIONS

This work underlines the importance of using reduced order models for simultaneous multi-parametric moving horizon state estimation and mode predictive control. Based on the studied distillation example, the superiority of using nonlinear model order reduction is highlighted when employed prior to linearization. Future work will deal with the use of robust controllers to hedge against the estimation error by incorporating model reduction error metrics in the calculation of mp-MHE error dynamics (Volker et al, 2013a) and combining the reduction methodology to robust tube mp-MPC/MHE (Volker et al, 2013b).

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Appendix A. Linear Reduced Order Models

Case 1:

$$A = \begin{pmatrix} 0.97902 & -0.051464 \\ 0.03223 & 0.92291 \end{pmatrix}$$

$$B = \begin{pmatrix} -0.06874 \\ -0.039587 \end{pmatrix}$$

$$C = \begin{pmatrix} -0.096179 & -0.10013 \end{pmatrix}$$

$$G = \begin{pmatrix} 0.066148324 \\ -0.09309532 \end{pmatrix}$$

Case 2:

$$A = \begin{pmatrix} 0.9546 & 0.05113 \\ -0.04809 & 0.3834 \end{pmatrix}$$

$$B = \begin{pmatrix} -0.09323 \\ -0.0596 \end{pmatrix}$$

$$C = \begin{pmatrix} -0.1009 & 0.06461 \end{pmatrix}$$

$$G = \begin{pmatrix} 0.0097686 \\ 0.045933 \end{pmatrix}$$