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Subspace Identification of Unstable Systems by MON4SID algorithm

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Abstract: In this paper, we consider a closed loop subspace identification problem. Here the open loop processes are unstable. By using the subspace identification algorithm, the closed loop system is first identified. The plant dynamics are extracted from the identified closed loop system. Three unstable processes are simulated and identified by the MON4SID algorithm.

Keywords: Subspace identification, closed loop, unstable process

1. INTRODUCTION

Subspace identification is used to identify the linear time invariant state space models from measured (input/output) data. It uses the concepts of systems theory and linear algebra. It will not encounter problems like convergence, slow convergence or numerical instability. Due to its several good properties, it has gained popularity in industrial applications. Subspace identification algorithms such as Numerical algorithms for Subspace State Space System Identification (N4SID), Multivariable Output Error State sPace (MOESP) (Verhaegen, 1994) and Canonical Variate Analysis (CVA) (Larimore, 1990) are not iterative (Van Overschee and De Moor, 1996), so it is faster than the classical identification methods such as Prediction Error Methods (PEM). The main features of subspace identification methods are simple parameterization for Multiple Input Multiple Output (MIMO) systems and non-iterative numerical solution. Closed loop identification plays a very important role when the open loop process is unstable. N4SID and MOESP are biased under closed loop identification. (Verhaegen, 1993) proposed a closed loop subspace identification method to overcome the above mentioned biased problems. Different kinds of closed loop identification methods are available and these are broadly categorized into three main types such as direct, indirect and joint input output identification method (Forssell & Ljung, 1999). An indirect method (Pouliquen, et. al. 2010) is developed to identify the dynamics of plant. In literature, only stable systems are being identified using MON4SID (Miranda & Garcia, 2009) algorithm, however, no paper has reported the identification of unstable systems using this algorithm. We illustrated the identification procedure by considering a case study of a first order bioreactor and some second order unstable systems.

1.1 Open Loop Subspace Identification

A linear time invariant dynamic system is described by the state space model in the innovation form

$$\begin{aligned} x_{k+1} &= Ax_k + Bu_k + Ke_k \\ y_k &= Cx_k + Du_k + e_k \end{aligned} \tag{1}$$

Where $u_k \in \Re^m$, $y_k \in \Re^l$ and $x_k \in \Re^n$ are denoted as input, output and state vectors respectively. The matrices A, B, C, D and K are system, input, output, direct feed through and the noise matrices with appropriate dimensions respectively. $e_k \in \Re^l$ denotes the zero-mean white innovation process.

1.2 Problem Statement for Open Loop Identification

Given s samples of the input sequence {u(0), …, u(s-1)} and output sequence {y(0), … , y(s-1)}
Estimate the system order and the system matrices A, B, C, D and noise covariance matrices Q, R and S.
By successive substitution of state equation in output equation and stacking the equations in matrix form gives the subspace matrix equation. The subspace matrix equation is given below

$$Y_f = \Gamma_i X_f + H_i^d U_f + H_i^s E_f \tag{2}$$

$$Y_p = \Gamma_i X_p + H_i^d U_p + H_i^s E_p \tag{3}$$

$$X_f = A^i X_p + \Delta_i^d U_p + \Delta_i^s E_p \tag{4}$$

Where subscript ‘f’ stands for future and ‘p’ stands for past. i is a number of block rows. The above equations play a very important role in the development of subspace identification. Description of the different terms included in the above equation are given below

$$U_p = U_{0|i-1} = \begin{pmatrix} u_0 & u_1 & \dots & u_{j-1} \\ u_1 & u_2 & \dots & u_j \\ \dots & \dots & \dots & \dots \\ u_{i-1} & u_i & \dots & u_{i+j-1} \end{pmatrix} \in \Re^{mi\times j} \tag{5}$$

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$$U_f = U_{i|2i-1} = \begin{pmatrix} u_i & u_{i+1} & \dots & u_{i+j-1} \\ u_{i+1} & u_{i+2} & \dots & u_{i+j} \\ \dots & \dots & \dots & \dots \\ u_{2i-1} & u_{2i} & \dots & u_{2i+j-2} \end{pmatrix} \in \Re^{mi \times j} \quad (6)$$

The output data can be similarly stacked to give matrices Y_p, and Y_f. E_p and E_f can be constructed in a similar way.
The states are defined as

$$X_p = X_0 = \begin{pmatrix} x_0 & x_1 & \dots & x_{j-1} \end{pmatrix} \quad (7)$$

$$X_f = X_i = \begin{pmatrix} x_i & x_{i+1} & \dots & x_{i+j-1} \end{pmatrix} \quad (8)$$

The extended observability matrix

$$\Gamma_i = \begin{pmatrix} C \\ CA \\ CA^2 \\ \dots \\ CA^{i-1} \end{pmatrix} \in \Re^{li \times n} \quad (9)$$

The lower triangular block-Toeplitz matrices H_i^d and H_i^s are given by

$$H_i^d = \begin{pmatrix} D & 0 & 0 & \dots & 0 \\ CB & D & 0 & \dots & 0 \\ CAB & CB & D & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ CA^{i-2}B & CA^{i-3}B & CA^{i-4}B & \dots & D \end{pmatrix} \quad (10)$$

$$H_i^s = \begin{pmatrix} I & 0 & 0 & \dots & 0 \\ CK & I & 0 & \dots & 0 \\ CAK & CK & I & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ CA^{i-2}K & CA^{i-3}K & CA^{i-4}K & \dots & I \end{pmatrix} \quad (11)$$

The reversed extended controllability matrices Δ_i^d and Δ_i^s are given by

$$\Delta_i^d = \begin{pmatrix} A^{i-1}B & A^{i-2}B & \dots & AB & B \end{pmatrix} \quad (12)$$

$$\Delta_i^s = \begin{pmatrix} A^{i-1}K & A^{i-2}K & \dots & AK & K \end{pmatrix} \quad (13)$$

In subspace identification literature, the following short hand notation is often used

$$W_p = \begin{pmatrix} Y_p \\ U_p \end{pmatrix} \in (li \times mi) \times j \quad (14)$$

The following two projections plays vital role in subspace identification algorithms
Orthogonal projection: The orthogonal projection (Van Overschee and De Moor, 1996) of the row space of A onto the row space of B is denoted by A/B and can be defined as

$$A/B = AB^T(BB^T)^{\dagger}B \quad (15)$$

Property: $A/A^{\perp} = 0$ (16)

where $\bullet^{\dagger}$ denotes the Moore-Penrose pseudo inverse of the matrix $\bullet$.
Oblique projection: The oblique projection (Van Overschee and De Moor, 1996) of the row space of A along the row space of B on the row space of C are defined as

$$A/_B C = A\begin{pmatrix} C^T & B^T \end{pmatrix}\left[\begin{pmatrix} CC^T & CB^T \\ BC^T & BB^T \end{pmatrix}^{\dagger}\right].C \quad (17)$$

Properties: $A/_A B = 0$ and $C/_B C = C$ (18)

1.3 Closed Loop Identification

State space form of the closed loop identification is given by the following difference equation:

$$x_{k+1}^c = A_c x_k^c + B_c(r_k - y_k)$$
$$u_k = C_c x_k^c + D_c(r_k - y_k) \quad (19)$$

Where, 'c' stands for controller. r_k is the exogenous input, u_k the input control, y_k the plant, w_k the process noise and v_k the measurement noise. A_c, B_c, C_c and D_c are matrices with appropriate dimensions.

1.4 Problem Statement for Closed Loop Identification

Given (r_k, u_k and y_k), a set input and output measurements. Identify the deterministic part of plant, that is, determine the order of the unknown system, the system matrices (A, B, C, and D) up to within a similarity transformation.

2. SUBSPACE IDENTIFICATION METHOD

Subspace identification algorithms always consist of two main steps. In the first step, the extended observability matrix and state sequences are retrieved from the weighted projection of the future outputs Y_f into the orthogonal complement of future inputs U_f. Second step determines the state space model using either of the extended observability matrix or state sequence. Algorithms which use the extended observability matrix to obtain state space model are MOESP, IV-SID and basic-4SID. Algorithms which use state sequence to find the system matrices are N4SID and CVA.

Inside MOESP family, there is the Past Output MOESP (PO-MOESP) method, which solves the state space model by means of an approximation of the extended observability matrix Γ_i. MOESP is biased under closed loop condition, which requires special treatment whereas in MON4SID, there are no issues with biasness. To solve this problem, (Verhaegen, 1993) proposed a closed loop subspace identification method. Based on it, the plant and the controller models are estimated. But, here it is necessary to provide the order of the controller. Similarly in the N4SID case (Van Overschee; De Moor, 1997) it is necessary to know a limited number of impulse response samples of the controller and, via direct identification, the plant model is estimated. MON4SID does not require any information regarding the controller.

2.1 MON4SID identification method

In this section, MON4SID method is discussed. To solve equation (2), it is used the POMOESP method to calculate the extended observability matrix Γ_i and N4SID method is employed to calculate the system matrices A, B, C, D through the least squares method. Therefore, it is necessary to eliminate the last two terms in the right side of equation (1). That is done in two steps: first eliminating the term $H_i^d U_f$ in (1), performing an orthogonal projection of equation (2) into the row space of $U_f^{\perp}$, which yields:

$$Y_f / U_f^{\perp} = \Gamma_i X_f / U_f^{\perp} + H_i^d U_f / U_f^{\perp} + H_i^s E_f / U_f^{\perp} \quad (20)$$

And by the orthogonal property (16), equation (20) can be simplified to

$$Y_f / U_f^{\perp} = \Gamma_i X_f / U_f^{\perp} + H_i^s E_f / U_f^{\perp} \quad (21)$$

Second, to eliminate the noises in equation (21), an instrumental variable Wp is defined. Multiplication of (21) by yields:

$$Y_f / U_f^{\perp} W_p = \Gamma_i X_f / U_f^{\perp} W_p + H_i^s E_f / U_f^{\perp} W_p \quad (22)$$

As it is assumed that the noise is uncorrelated with past input and output past data, which means that $E_f / U_f^{\perp} W_p = 0$. Therefore, equation (22) can be simplified to

$$Y_f / U_f^{\perp} W_p = \Gamma_i \hat{X}_f \quad (23)$$

In equation (23), $X_f / U_f^{\perp} W_p = \hat{X}_f$ is the estimate of the Kalman filter state. Equation (23) indicates that the column space of Γ_i can be calculated by the SVD decomposition of $Y_f / U_f^{\perp} W_p$.

Γi, given in (23), can be derived from a simple LQ factorization of a matrix constructed from the block-Hankel matrices U_f, U_p and Y_f, Y_p in the form:

$$\begin{pmatrix} U_f \\ W_p \\ Y_f \end{pmatrix} = \begin{pmatrix} L_{11} & 0 & 0 \\ L_{21} & L_{22} & 0 \\ L_{31} & L_{32} & L_{33} \end{pmatrix} \begin{pmatrix} Q_1 \\ Q_2 \\ Q_3 \end{pmatrix} \quad (24)$$

and the orthogonal projection in the left side of (23) can be computed by matrix L_{32} . The SVD of L_{32} can be given as:

$$L_{32} = \begin{pmatrix} U_1 & U_2 \end{pmatrix} \begin{pmatrix} S & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} V_1^T \\ V_2^T \end{pmatrix} = U_1 S V_1^T \quad (25)$$

The order of the system is equal to the number of non-zero singular values in S. The column space of U_1 approximates that of Γ_i in a consistent way. That is:

$$\Gamma_i = U_1 \quad (26)$$

The system (1) can be written as:

$$\begin{pmatrix} \tilde{X}_{i+1} \\ Y_{i|i} \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} \tilde{X}_i \\ U_{i|i} \end{pmatrix} + \begin{pmatrix} r_1 \\ r_2 \end{pmatrix} \quad (27)$$

In equation (27), suppose (ideally) that X_{i+1} and X_i are given, then the system matrices (A, B, C, D) could be computed through the least squares method. Therefore, the problem now is to find the state sequences.

$\Theta_i = Y_f /_{U_f} W_p$ is the oblique projection, which is achieved by performing an oblique projection of equation (2), along the row space of U_f onto the row space of W_p, that is:

$$Y_f /_{U_f} W_p = \Gamma_i X_f /_{U_f} W_p + H_i^d U_f /_{U_f} W_p + H_i^s E_f /_{U_f} W_p \quad (28)$$

The last two terms of equation (28) are zero, $U_f /_{U_f} W_p = 0$ by the property of oblique projection (18); $E_f /_{U_f} W_p = 0$ by the assumption that the noise is uncorrelated with input and output past data. Thus equation (28) can simplified as

$$Y_f /_{U_f} W_p = \Gamma_i \tilde{X}_i \quad (29)$$

where $\tilde{X}_i = X_f /_{U_f} W_p$. Then equation (29) cab be written as

$$\Theta_i = \Gamma_i \tilde{X}_i \quad (30)$$

The oblique projection Θ_i given in (30) can be computed from (24) by:

$$\Theta_i = Y_f /_{U_f} W_p = L_{32} (L_{22})^{-1} \begin{pmatrix} L_{21} & L_{22} \end{pmatrix} \begin{pmatrix} Q_1 \\ Q_2 \end{pmatrix} \quad (31)$$

An estimate of the state sequence X is given by:

$$X = (\Gamma_i)^* L_{32} (L_{22})^{-1} W_p \quad (32)$$

Define the following matrices with j-1 columns as

$$X_{i+1} = [x(i+1)......x(i+j-1)] \quad (33)$$

$$X_i = [x(i)......x(i+j-2)] \quad (34)$$

$$U_{i\backslash i} = [u(i)......u(i+j-2)] \quad (35)$$

$$Y_{i\backslash i} = [y(i)......y(i+j-2)] \quad (36)$$

Thus, the system matrices can be estimated from equation (27).

2.2 MON4SID algorithm

1. Compute the matrices U_f, U_p, and Y_f, Y_p and the LQ factorization given in (24).
2. Compute the SVD of the matrix L_{32} from equation (24).
3. Determine the system order by inspection of the singular values in S given in (25)
4. Determine Γ_i from equation (26) and the state sequence X from (32), determine X_{i+1} and X_i

5. Compute the matrices A, B, C and D from equation (27).

3. SIMULATION STUDIES

In this section, we provide simulation examples of some of unstable processes. Performance of MON4SID algorithm is compared with the identification algorithm PEM. The exogenous input is a Gaussian white noise sequence with mean zero and variance 1. The number of columns in the block Hankel matrices is 640. 1000 samples are collected and the number of block rows i = 20.

Case study 1: Here, we consider a first order bio-reactor problem. An open-loop unstable plant is studied. For this simulation, the transfer functions of plant and controller are respectively given by

$$G_P = \frac{-5.859e^{-s}}{5.888s-1} \tag{37}$$

$$G_C = -0.7356(1+\frac{1}{5.452s}) \tag{38}$$

Estimates of the poles are shown in the (Fig. 1). Frequency response plots are shown in (Fig. 2). We can see that the MON4SID algorithm identifies the unstable bioreactor effectively. The validation data is performed by testing the identified system to impulse response which is presented in (Fig. 3).

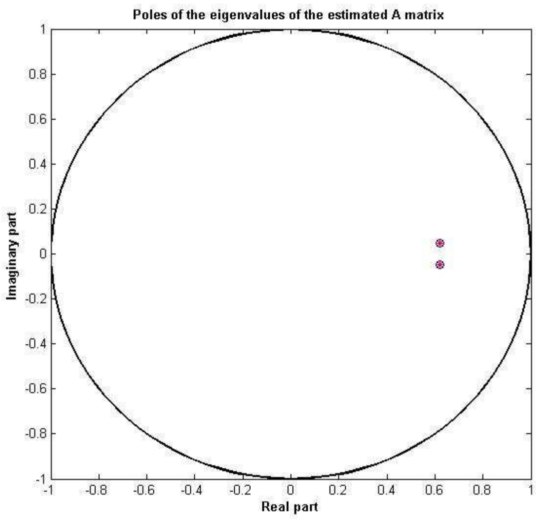

Fig. 1. Plot of the Eigen values of the estimates of ‘A’ (‘o’ denotes real and ‘*’ denotes estimated)

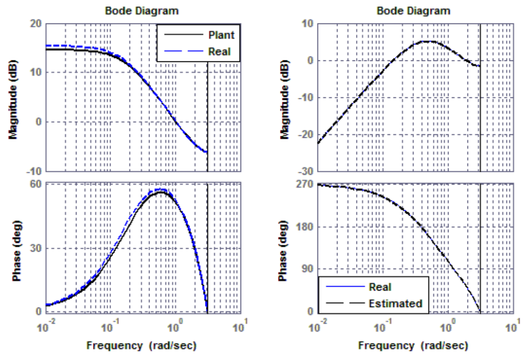

Fig. 2. Bode plots of the plant (left) and closed loop system (right)

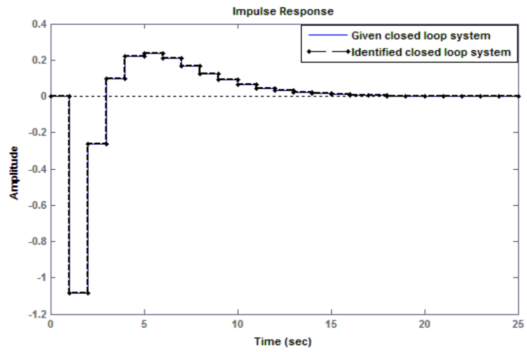

Fig. 3. Impulse response of identified model

Case study 2: We consider a second order plant with two real poles 1 and 0.333. The open-loop plant transfer function and controller transfer function are given by

$$G_P = \frac{2(s+0.2)e^{-0.3s}}{(s-1)(s-0.333)} \tag{39}$$

$$G_C = 1.33(1+\frac{1}{1.5s}+0.14s) \tag{40}$$

The above system is simulated and generated output data. (Pseudo Random Binary Signal) PRBS is used as the exogenous input to excite the process. The results are shown in the following (Fig. 4, 5 and 6). Figure 4 shows the poles of true pant and the estimated one, where ‘o’ and ‘*’represent the poles of the real plant and estimated one respectively. Here too, the proposed algorithm works well for identifying the second order unstable systems. The phase plot of both true and estimated plant shows the same trend (Fig. 5). The deviation may be accounted for the presence of two unstable poles. Impulse response of the closed loop system is presented in (Fig. 6).

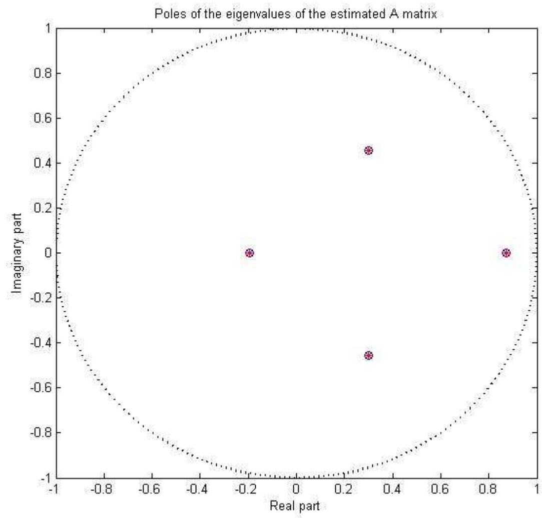

Fig. 4. Plot of the Eigen values of the estimates of 'A' ('o' denotes real and '*' denotes estimated)

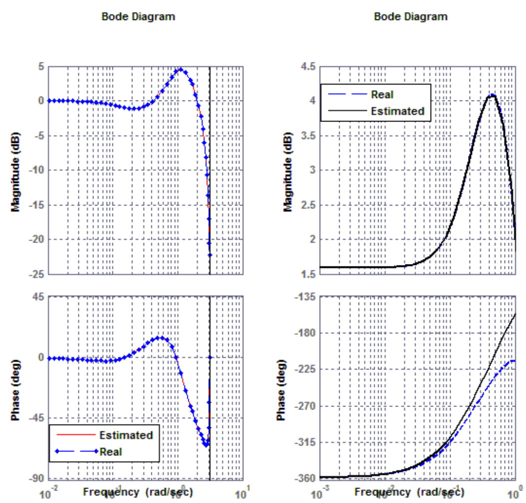

Fig. 5. Bode plots of the closed loop system (left) and plant (right)

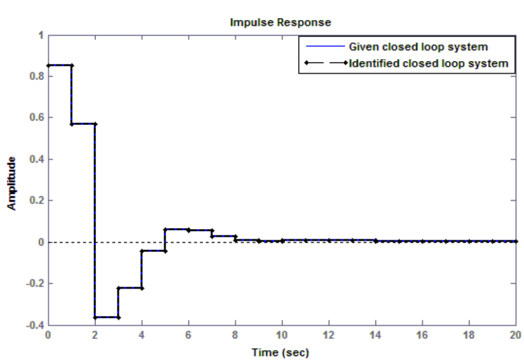

Fig. 6. Impulse response of identified model

Case study 3: We consider a second order plant with complex conjugate poles 0.3649 ± 1.4013i. Suppose plant and controller transfer functions are given by the following

$$G_P = \frac{3.87(s+1.897)e^{-0.1s}}{s^2 - 0.7298 + 2.097} \tag{41}$$

$$G_C = 2.3\left(1 + \frac{1}{0.7s} + 0.045s\right) \tag{42}$$

The above system is simulated and generated the measurement data. This data is used for the identification. We see from the (Fig. 7 and 8) that the identified results obtained from the MON4SID algorithm are quite good. In (Fig. 8), bode plots are shown. Magnitude plots of both real and identified plant are matching accurately but phase plots of estimated plant is deviated from the real one due to the presence of two unstable poles. And the resulted data is validated through the impulse response (Fig. 9).

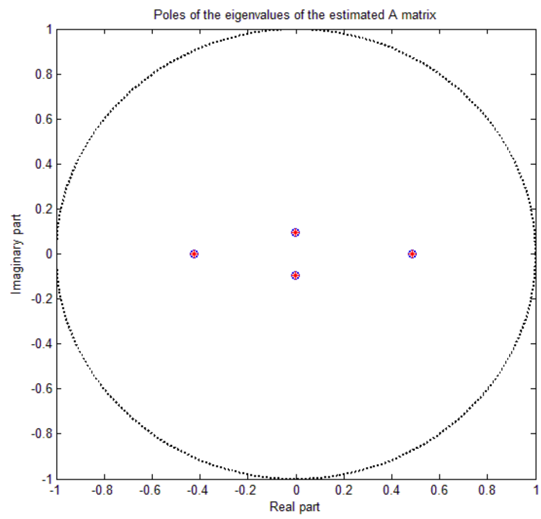

Fig. 7. Plot of the eigen values of the estimates of 'A' ('o' denotes real and '*' denotes estimated)

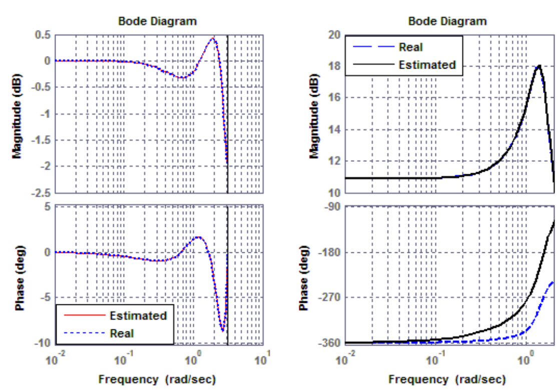

Fig. 8. Bode plots of the closed loop system (left) and plant (right)

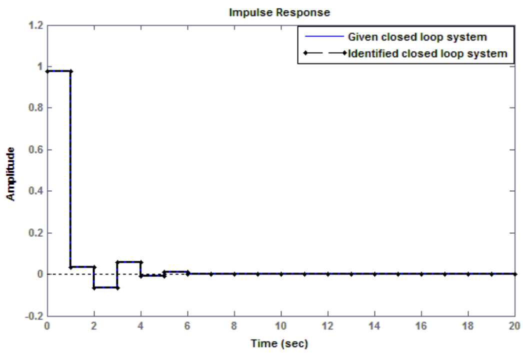

Fig. 9. Impulse response of identified model

4. CONCLUSION

Unstable systems are simulated and identified by MON4SID algorithm. Three case studies are considered. The results are compared by means of Bode plot and estimated poles with the real pole of the true plant. The magnitude diagrams of both real and estimated plant are matching but the phase plots are not matching for the last two case studies. The phase plots of both the real and the estimated plants show same trend up to certain frequency. After that phase plot of estimated plant is deviating from the real one. This may be due to the presence of two unstable poles or the loss of numerical accuracy in the estimation.

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