Preprints of the 10th IFAC International Symposium on Dynamics and Control of Process Systems
The International Federation of Automatic Control
December 18-20, 2013. Mumbai, India

PID versus MPC Performance for SISO Dead-time Dominant Processes ⋆

Y. A. Sha'aban * **B. Lennox** ** **D. Laurí** ***

* *Control Systems Group, University of Manchester, Manchester, UK, (e-mail: yusuf.shaaban@ postgrad.manchester.ac.uk).*
** *Control Systems Group, University of Manchester, Manchester, UK, (e-mail: barry.lennox@manchester.ac.uk)*
*** *Control Systems Group, University of Manchester, Manchester, UK, (e-mail: david.lauripla@manchester.ac.uk)*

Abstract: Proportional-Integral-Derivative (PID) controllers are used extensively in the process industries for regulating single-input, single output (SISO) processes, with Model Predictive Controllers (MPC) typically being reserved for use on large scale systems. However, in recent years there has been suggestions that MPC may offer benefits when applied to SISO systems at the regulatory level. This paper compares the performance of PID and MPC when they are both applied to first and second order, SISO systems that contain a time delay. From the comparison it can be concluded that improved performance can be achieved by using MPC for, in some cases, very small time delays. Both PID and MPC are shown to be robust to plant-model mismatch.

Keywords: PID, PI, MPC, dead-time, FOPDT, SOPDT, SISO

1. INTRODUCTION

The simplicity and effectiveness of PID controllers has made its use widespread in industrial applications. Over the past thirty years or so, PID controllers have continued to gain popularity; it is typically considered to be the first choice of controller for most applications. According to Åström and Hägglund (2001) over 90% of all control loops are of the PI/PID type. Because of its popularity, ease of implementation, and availability in off the shelf hardware and software, practitioners are more comfortable with this control strategy. Initially neglected by the research community, PID controllers have received renewed attention during the last two to three decades. This interest in PID has seen the emergence of many new tuning methods: Åström et al. (1988), Åström and Hägglund (2001), O'Dwyer (2009) and Seborg (2011).

Despite the vast literature on PID tuning, a significant percentage of controllers in automatic mode are poorly tuned, Ender (1993). Hence, optimal performance is not always attained. The need for high quality products, reduced energy consumption (fuel and electricity), increasing market competition, lower cost and legislation to cut down emissions, makes the need for improved process control performance imperative.

There are certain applications in which PID is known to perform poorly. For example, an area where PID may not be the best option is for systems which have a time delay that is large compared to the time constant of the process, O'Dwyer (2000). Generally, PID controllers are recommended for non-delay dominant processes, O'Dwyer (2001). O'Dwyer (2000, 2002) presented a survey of PID compensation for time delayed processes, which highlighted a significant quantity of research that had been published on developing PID controllers that were suitable for time delayed processes.

Model predictive control has received significant attention from both industry and academia and is regarded as the only advanced control scheme that has had a notable impact on industry, see Maciejowski (2002). The traditional way of implementing MPC is that it is applied to large scale processes and provides set points to PID controllers at the regulatory level. However, developments in computing and optimization has seen the implementation of MPC controllers at the regulatory layer, even for systems with small time constants, e.g. Wills and Heath (2005), Valencia-Palomo and Rossiter (2011a,b, 2012).

The predictive capability inherent to MPC enables it to cater for process time delay systematically and therefore it could be a sensible alternative to PID for systems containing relatively large time delays or other complex dynamics. In fact, as MPC is now available in many off the shelf products, it offers a possible alternative to PID as a general control tool for application to systems at the regulatory level. The focus of this paper is in understanding the effect of time delays on the performance of PID and MPC systems and identifying the types of processes where MPC may offer general improvements to PID.

Specifically, in this paper a study of the effect that increasing the time delay, relative to the time constant for two SISO first order plus dead-time (FOPDT) systems and a second order plus dead-time (SOPDT) process is

⋆ Financial support from Petroleum Technology Development Fund (PTDF) is gratefully acknowledged.

carried out. Both PID controllers and MPC controllers were implemented on these processes and the performance of the control systems was compared. The paper is organised as follows. Section 2 gives a discussion on PID controllers for systems with time delay. Section 3 briefly discuses the MPC strategy used in this work. Section 4 presents simulation results and discussion. Conclusions are provided in section 5.

2. PID FOR TIME DELAY PROCESSES

Various techniques for specifying PI/PID parameters exist; O'Dwyer (2009), for example, identified 245 tuning rules. PID parameters can be specified using iterative methods, tuning rules or analytical techniques. While the iterative methods are time consuming, tuning rules and analytical methods are suitable for non-delay-dominant processes: O'Dwyer (2000), Isermann (1989).

Whenever a reasonable model of a process is available, model based control approaches may provide improved control performance. Internal Model Control (IMC), Garcia and Morari (1982), for example, is known for its ability to handle un-modelled dynamics and process uncertainties: Rivera et al. (1986), O'Dwyer (2000). Although IMC is not routinely applied to regulate process systems, it is now used extensively as a tuning tool for PID controllers, Yu et al. (2011). The advantage it has over more traditional tuning methods, such as Ziegler-Nichols, is that the PID parameters are specified to produce desired closed-loop dynamics. However, for processes with significant time delays, the performance of PID regulators, tuned using the IMC method decreases because of modelling errors introduced by approximations made to the dead-time. A summary of IMC controller tuning is presented by Chien (1990).

In this work, the PID controllers were tuned using the IMC method, as this is consistent with what is now routinely implemented in industry, and also using an iterative technique that identified the optimal PID parameters which gave the minimum mean square error (MSE) for a setpoint change. The latter approach is typically not suitable for real applications as it tends to produce controllers with aggressive behaviour. However, it was used in this study as it provides an upper measure of the performance achievable with PID. The iterative tuning approach used

Table 1. GA parameter values

GA parameters	Value
Population Size	20
Crossover fraction	0.8
Generations	100
Fitness function	Mean squared error (MSE)

a Genetic Algorithm (GA) to identify the optimal PID parameters. A GA is a search algorithm inspired by the theory of evolution, see Ünal et al. (2013), which as a result of its parallel search approach, has good speed of convergence. The performance of a GA depends on the values of various parameters, such as crossover frequency. The values used for these parameters in this work is shown in Table 1. This is to limit the search space and allow for adequate variation in offspring population, Ünal et al. (2013). For the initial conditions, the IMC obtained PID parameters were used.

The desired closed-loop process time constant, τ_c, is key to IMC design, Seborg (2011). Rivera et al. (1986), Skogestad (2003) and Fruehauf et al. (1994) suggested that it should be specified according to the expressions in (1), (2) and (3) respectively:

$$\tau_c/\theta > 0.8 \text{ and } \tau_c > 0.1\tau \quad (1)$$
$$\tau_c = \theta \quad (2)$$
$$\tau > \tau_c > \theta \quad (3)$$

where τ and θ are the process time constant and time delay respectively.

For time delay dominant processes, the expression in (3) is not applicable for values of $\theta/\tau > 1$, as it violates the relationship when $\theta > \tau$. Seborg (2011) suggested a value of closed loop time constant, $\tau_c = \tau/3$, but for dead-time dominant processes this will lead to aggressive control action. Hence, for this work the following guidelines, based on (1) and (2) are used. If the lower and upper constraints on τ_c are defined as $\tau_{c_{min}}$ and $\tau_{c_{max}}$ respectively. Then,

$$\tau_c = \max(\tau/3,\ \theta) \quad (4)$$
$$\tau_{c_{min}} = \max(0.1\tau,\ 0.5\theta) \quad (5)$$
$$\tau_{c_{max}} = 2\tau_c \quad (6)$$

Given the process defined in (7), the parameters of the ideal parallel-PID controller structure, (8), can be computed using various expressions.

$$G(s) = \frac{Ke^{-\theta s}}{\tau s + 1} \quad (7)$$
$$G_c(s) = K_p + \frac{K_I}{s} + K_D s \quad (8)$$

Using the expressions for IMC-based PID obtained from Seborg (2011), the following IMC-based PI controller parameters were obtained:

$$K_p = \frac{\tau}{K\,(\tau_c + \theta)} \quad (9)$$
$$K_I = \frac{K_p}{\tau} \quad (10)$$

The limits for the parameters based on (5) and (6) will then be given as:

$$K_{p_{min}} = \frac{\tau}{K\left(\tau_{c_{max}} + \frac{\theta}{2}\right)} \qquad K_{I_{min}} = \frac{K_I}{3} \quad (11)$$
$$K_{p_{max}} = \frac{\tau}{K\,(\tau_{c_{min}} + \theta)} \qquad K_{I_{max}} = 3K_I \quad (12)$$
$$K_{D_{min}} = \frac{K_D}{3} \qquad K_{D_{max}} = 3K_D \quad (13)$$

The constraints in (11) and (12) define the search space for the GA algorithm. For the second order plant defined in (14) the IMC tuning parameters are selected using the

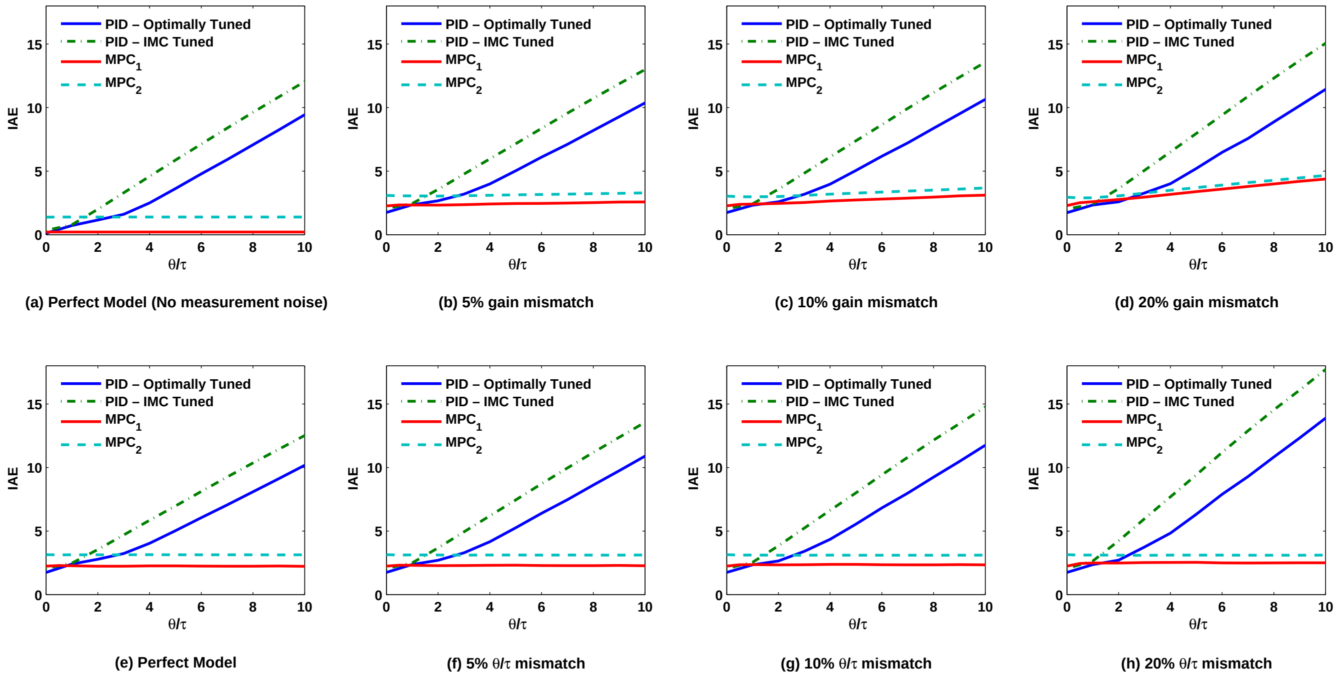


Fig. 1. Plots of IAE against $\frac{\theta}{\tau}$ for $G(s) = \frac{2e^{-\theta s}}{s+1}$ with mismatch in gain and $\frac{\theta}{\tau}$

expressions in (15) - (17), Seborg (2011). The limits for the GA parameters can be computed using (11)– (13) and substituting $\tau^2 = \tau_1\tau_2$.

$$G(s) = \frac{K\left(\tau_3 s + 1\right)e^{-\theta s}}{\left(\tau_1 s + 1\right)\left(\tau_2 s + 1\right)} \tag{14}$$

$$K_p = K\frac{\tau_1 = \tau_2 - \tau_3}{\tau_c + \theta} \tag{15}$$

$$K_I = \frac{K_p}{\tau_1 + \tau_2 - \tau_3} \tag{16}$$

$$K_D = K_p\frac{\tau_1\tau_2 - \left(\tau_1 + \tau_2 - \tau_3\right)\tau_3}{\tau_1 + \tau_2 - \tau_3} \tag{17}$$

3. MODEL PREDICTIVE CONTROL

At each time step in MPC, a finite horizon optimal control problem is solved; the first element in the open-loop optimal sequence obtained is then selected as the current control input. Most recent developments and research on MPC is related to state space formulation of MPC, for which there are several different formulations that exist; see for example Qin and Badgwell (2003), Wang (2004). In this paper, the formulation in Wang (2004) was used. The models given in (7) and (14) can be converted to discrete state space format and the augmented velocity format (18) and (19) respectively, Wang (2004):

$$\begin{aligned} x_p(k+1) &= A_p x_p(k) + B_p u(k) \\ y(k) &= C_p x_p(k) \end{aligned} \tag{18}$$

$$\begin{aligned} x(k+1) &= Ax(k) + B\Delta u(k) \\ y(k) &= Cx(k) \end{aligned} \tag{19}$$

Where

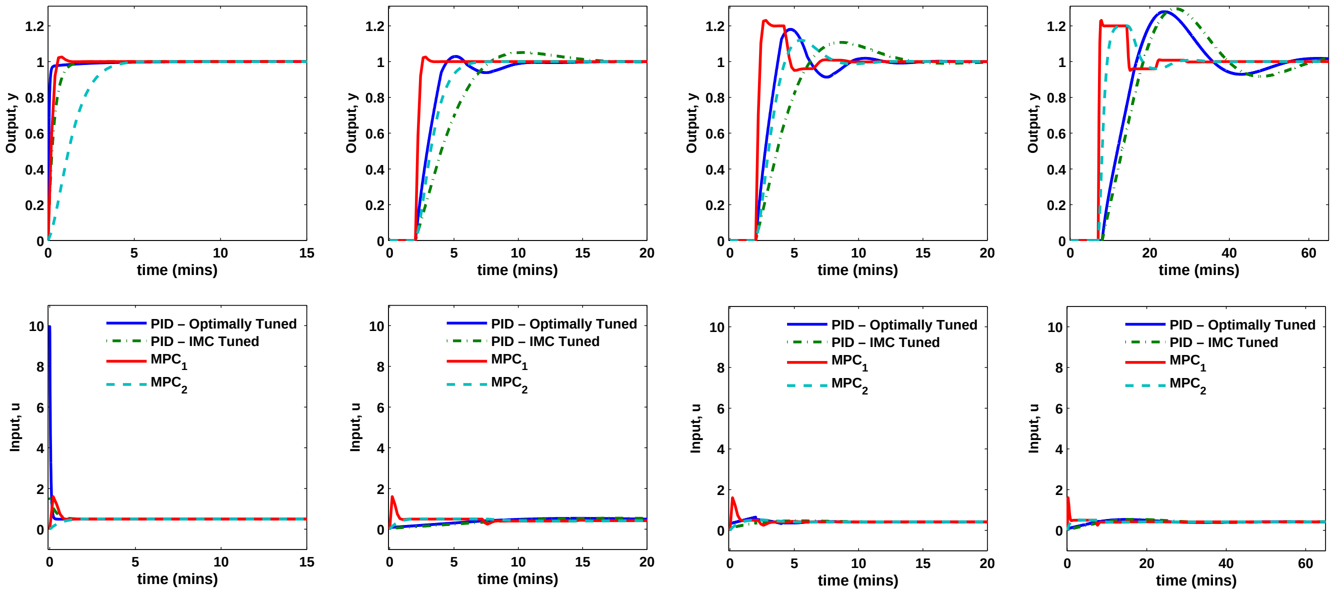
$$\begin{aligned} A &= \begin{bmatrix} & A_p 0^T_{n_{out}} \\ C_p A_p & I_{n_{out}} \end{bmatrix}; \qquad B = \begin{bmatrix} B_p \\ C_p B_p \end{bmatrix} \\ C &= \begin{bmatrix} 0_{n_p} & I_{n_{out}} \end{bmatrix}; \\ x(k)^T &= \begin{bmatrix} \Delta x_p(k)^T & y(k)^T \end{bmatrix} \\ \Delta x_p(k) &= x_p(k) - x_p(k-1) \end{aligned}$$

The cost function used with MPC, which penalizes the tracking error as well as the change in manipulated variable, is defined in (20):

$$J = \sum_{i=1}^{P} \| r(k+1) - y(k+i) \|^2 + \sum_{i=1}^{M} \| \Delta u \|^2_{r_w} \tag{20}$$

Where r_w, $r(.)$, P and M are the input rate weighting, set-point, prediction and control horizon respectively. The cost function defined in (20) with the augmented velocity model in (19) was used to design the model predictive control strategy used in this work.

The prediction horizon used in the MPC cost function was selected to be approximately equal to the settling time of the process as shown in (21). A control horizon of 3 was used in all the simulations. A zero output weighting was used. Whilst it was possible to improve the performance of the MPC by adjusting these parameters, these values were selected to give an indication of the performance that was achievable using MPC. For many processes, no significant improvement is obtained beyond $M = 3$, Rossiter (2003).



(a) Perfect Model, θ/τ =0 (b) Perfect Model, θ/τ =2 (c) 20% gain mismatch, θ/τ =2 (d) 20% gain mismatch, θ/τ =8

Fig. 2. Sample plots of manipulated and control variables for the first order system

$$P = \frac{(\theta + 5\tau)}{T_s} \tag{21}$$

Where T_s is the sampling period. With these choices of P and M, MPC tuning is achieved using the weighting, r_w.

$$IAE = \int_{\theta}^{t_s} |e(t)|\, dt \tag{22}$$

In this work the performance of the control systems was quantified using the Integral Absolute Error (IAE), defined by (22), where e is the difference between the set-point and output between the time delay, θ, and settling time, t_s, of the process, following a step change in set-point.

4. SIMULATION AND RESULTS

To compare the performance of the PID and MPC controllers, the models defined in (7) and (14) were used, with three different dynamic process models used. To begin, a FOPDT system with $K = 2$ and $\tau = 1$ was used. The ratio of the process time delay to time constant, $\frac{\theta}{\tau}$ was varied over a range of values by varying θ over the range shown in (23):

$$\frac{\theta}{\tau} = [0 : 10] \tag{23}$$

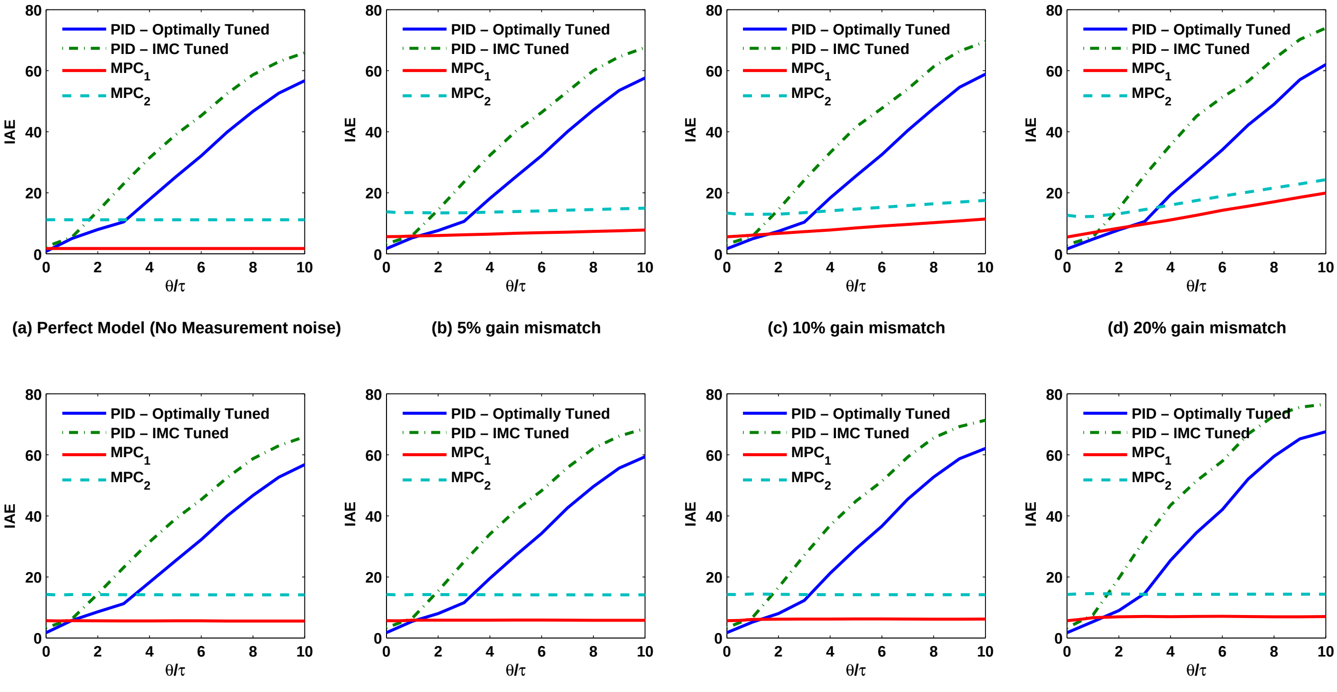
The corresponding models were then used to tune PID and MPC controllers using the methods described in sections 2 and 3 respectively. Following the design of the controllers, set-point step changes were made and the performance of the system evaluated using the Integral Absolute Error (IAE) as the performance measure. Two different MPC controllers were tuned. An aggressive MPC controller and a more conservative controller labelled as MPC_1 (with $r_w = 0.1$) and MPC_2 (with $r_w = 100$) respectively.

The controllers where also implemented when the plant had a 5%, 10% and 20% mismatch in process model gain, K, and $\frac{\theta}{\tau}$, and white noise with a signal to noise ratio 20 was added to the output measurements. A sampling time of $T_s = 0.2s$ was used. The results of this are shown in Fig. 1. This figure shows that in the case where there is no plant-model mismatch, the performance of PID and MPC for relatively small time delays is comparable. However, as the time delay increases, the performance of PID degrades sharply. In all cases, the performance of MPC and PID is only affected slightly by the increase in plant-model mismatch. Furthermore, the increase in the time delay to time constant ratio is seen to degrade the performance of the PID controllers significantly, whereas for MPC the effect of MPC, is as expected, minimal. Sample responses are shown in Fig. 2.

In a second study, PID and MPC controllers were applied to a second first order system with $K = 1$ and $\tau = 7$. The performance of the controllers were analysed using both time constant and gain mismatch as in the first study; white noise with signal-to-noise ratio of 20 was applied. A sampling time of $T_s = 1s$ was used. The plots of the IAE are shown in Fig. 3.

In this study, the performance trend is consistent to that of the previous study, with significant improvements in control observed with MPC when the ratio of time delay to time constant exceeds a value of approximately 2. This is consistent with dead-time compensation results, which suggests an improvement in performance with dead-time compensation when $\theta/\tau > 1$, Ingimundarson and Hägglund (2002).

In the final study, the controllers were tuned for second order system with parameters; $K = 2$, $\tau_1 = 3$, $\tau_2 = 10$ and $\tau_3 = 0$. PID and MPC controllers were applied, as before, with 5%, 10% and 20% mismatch in process gain and θ/τ; and measurement noise also white with signal-to-



(a) Perfect Model (No Measurement noise) (b) 5% gain mismatch (c) 10% gain mismatch (d) 20% gain mismatch

(e) Perfect Model (f) 5% θ/τ mismatch (h) 10% θ/τ mismatch (h) 20% θ/τ mismatch

Fig. 3. Plots of IAE against $\frac{\theta}{\tau}$ for first order system, $G(s) = \frac{e^{-\theta s}}{7s+1}$

noise ratio of 20. A sampling time of $T_s = 2s$ was used. The plots of the IAE against θ/τ are shown in Fig.4. As with the first two cases, the performance of MPC is maintained as θ/τ increases. Furthermore, the performance of the IMC tuned PID controller degrades very quickly even for very small time delays. This is an important result as it suggests that for industrial processes, which will almost certainly be of high order, even for very small delays, there may be significant benefit in using MPC to regulate SISO systems. However, there is a need for a much more thorough and systematic study before definitive benefits can be established and this is the subject of on-going research.

5. CONCLUSIONS

In this paper, a study in to the effect that process time delay has on the performance of PI/PID and MPC controllers was conducted. The study has shown that for the two first order systems investigated in this paper, the performance of the PI controller tuned using IMC degraded almost linearly with the time delay and when the delay exceeded approximately twice the time constant, MPC was found to provide much improved performance. However, for the second order system, the IMC tuned PID controller was found to be much more sensitive to the time delay and with the time delay exceeding approximately 10% of the time constant, the performance of MPC was found to be significantly better than PID. The optimally tuned PID controller produced slightly improved results compared with the IMC tuned PID controller. However, it should be noted that the optimally tuned PID controller is unlikely to be acceptable in an industrial application as it is too aggressive.

REFERENCES

Chien, I.L. (1990). Consider IMC tuning to improve controller performance. *Chemical Enginering Progress*, 86, 33–41.

Ender, D. (1993). Process control performance: not as good as you think. *Control Engineering*, 40(10), 180–190.

Fruehauf, P.S., Chien, I.L., and Lauritsen, M.D. (1994). Simplified IMC-PID tuning rules. *ISA Transactions*, 33(1), 43–59.

Garcia, C.E. and Morari, M. (1982). Internal model control. a unifying review and some new results. *Industrial & Engineering Chemistry Process Design and Development*, 21(2), 308–323.

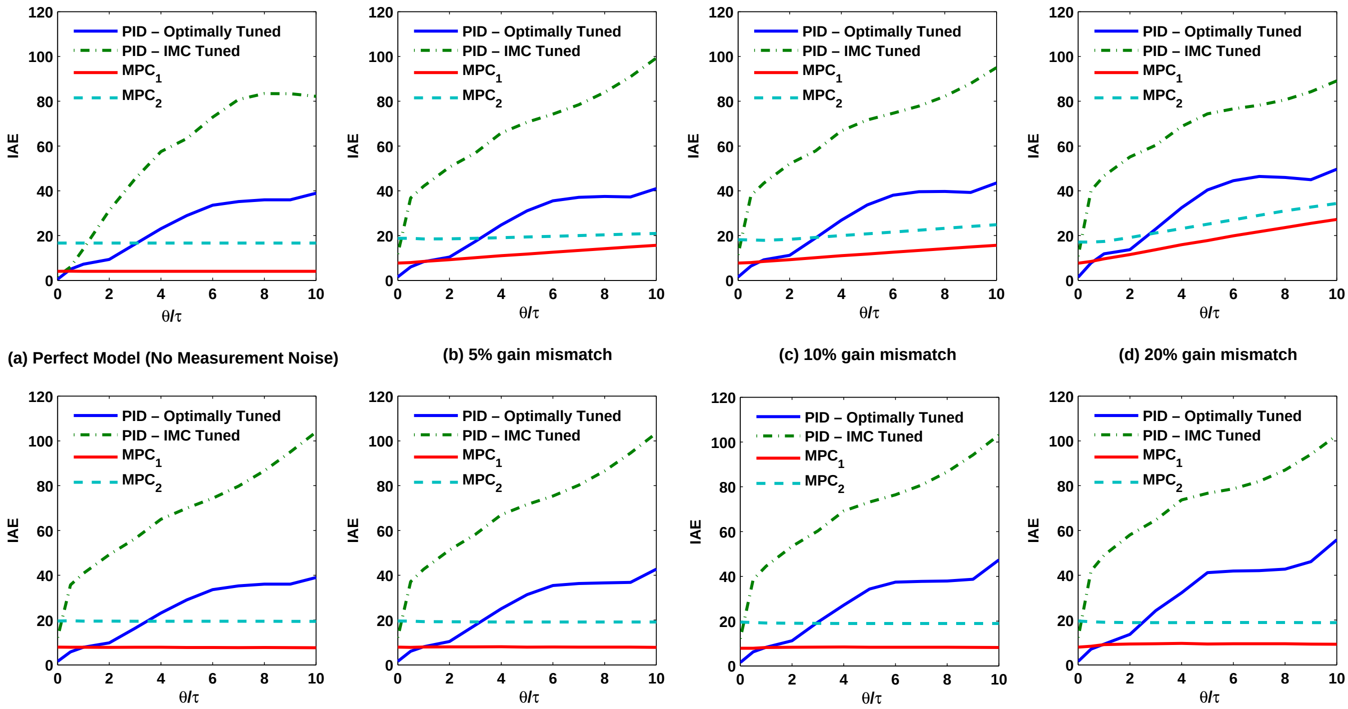
Ingimundarson, A. and Hägglund, T. (2002). Performance comparison between PID and dead-time compensating controllers. *Journal of Process Control*, 12(8), 887–895.

Isermann, R. (1989). *Digital control systems, Volume 1: fundamentails, deterministic control.* Springer-Verlag Berlin Heidelberg.

Maciejowski, J.M. (2002). *Predictive control : with constraints.* Pearson Education, Harlow.

O'Dwyer, A. (2000). PID compensation of time delayed processes: a survey. In *Proceedings of the Irish Signals and Systems Conference*, 5–12. Dublin.

O'Dwyer, A. (2001). PI and PID controller tuning rule design for processes with delay, to achieve constant gain and phase margins for all values of delay. In *Proceedings*



(a) Perfect Model (No Measurement Noise) (b) 5% gain mismatch (c) 10% gain mismatch (d) 20% gain mismatch

(e) Perfect Model (f) 5% θ/τ mismatch (g) 10% θ/τ mismatch (h) 20% θ/τ mismatch

Fig. 4. Plots of IAE against $\frac{\theta}{\tau}$ for second order system, $G(s) = \frac{2e^{-\theta s}}{(3s+1)(10s+1)}$

of the Irish Signals and Systems Conference, 96–101. National University of Ireland, Maynooth.

O'Dwyer, A. (2002). PID compensation of time delayed processes 1998-2002: a survey. In *Proceedings of the American Control Conference*, 1494–1499. Denver, Colorado, USA.

O'Dwyer, A. (2009). *Handbook of PI and PID: Controller Tuning Rules*. Imperial College Press, London.

Qin, S. and Badgwell, T.A. (2003). A survey of industrial model predictive control technology. *Control Engineering Practice*, 11(7), 733–764.

Åström, K.J., Hägglund, T., et al. (1988). *Automatic tuning of PID controllers*. Instrument Society of America.

Åström, K. and Hägglund, T. (2001). The future of PID control. *Control Engineering Practice*, 9(11), 1163–1175.

Rivera, D.E., Morari, M., and Skogestad, S. (1986). Internal model control: PID controller design. *Industrial & Engineering Chemistry Process Design and Development*, 25(1), 252–265.

Rossiter, J.A. (2003). *Model-based predictive control : a practical approach*. CRC Press, Boca Raton.

Seborg, D.E. (2011). *Process dynamics and control*. Wiley, Hoboken, N.J.

Skogestad, S. (2003). Simple analytic rules for model reduction and PID controller tuning. *Journal of Process Control*, 13(4), 291–309.

Ünal, M., Ak, A., Topuz, V., and Erdal, H. (2013). *Optimization of PID Controllers Using Ant Colony and Genetic Algorithms*. Springer.

Valencia-Palomo, G. and Rossiter, J. (2011a). Efficient suboptimal parametric solutions to predictive control for PLC applications. *Control Engineering Practice*, 19(7), 732–743.

Valencia-Palomo, G. and Rossiter, J. (2011b). Programmable logic controller implementation of an auto-tuned predictive control based on minimal plant information. *ISA Transactions*, 50(1), 92–100.

Valencia-Palomo, G. and Rossiter, J. (2012). Novel programmable logic controller implementation of a predictive controller based on laguerre functions and multiparametric solutions. *Control Theory Applications, IET*, 6(8), 1003–1014.

Wang, L. (2004). A tutorial on model predictive control: Using a linear velocity-form model. *Developments in Chemical Engineering and Mineral Processing*, 12(5-6), 573–614.

Wills, A.G. and Heath, W.P. (2005). Application of barrier function based model predictive control to an edible oil refining process. *J of Process Control*, 15(2), 183–200.

Yu, Z., Wang, J., Huang, B., and Bi, Z. (2011). Performance assessment of PID control loops subject to setpoint changes. *J of Process Control*, 21(8), 1164–1171.