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Rejection of Periodic Disturbances Based on Adaptive Repetitive Model Predictive Control

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Abstract: The paper presents an adaptive strategy to reject periodic disturbances with unknown period based on a combination of model predictive control and repetitive control. A novel period estimator is presented. For the integer period case, the estimator is designed based on integer programming. For the non-integer period case, it is designed based on a two-step optimization, namely integer programming followed by a constrained least square method. With the estimated period, feedforward compensation is made to improve the tracking performance asymptotically. Simulation results are given to show the effectiveness of the algorithm.

Keywords: feedforward control, model predictive control, adaptive repetitive control, integer programming, disturbance rejection, adaptive filter

1. INTRODUCTION

Disturbance rejection is an important aspect in control theory. Periodic disturbance is rather common in industrial processes due to periodic operations, such as continuous steel casting processes and fed-batch fermentation processes, shown in Manayathara et al. (1996) and Valentinotti (2001). There are many ways to deal with the periodic disturbance rejection problem as shown in Bodson (2005). Most of them are based on repetitive control. Repetitive control was originally proposed to reject periodic disturbance(Inoue et al. (1981b)) or track periodic reference(Inoue et al. (1981a)) founded on Internal Model Principle(Francis and Wonham (1976)). It is widely accepted by people in the field of robotics, servo mechanical devices and so on, but seldom used in chemical processes which are generally multi-variable constrained processes. In Natarajan and Lee (2000), Lee combined repetitive control with model predictive control based on an augmented state space model and a periodic Kalman filter and applied it on a simulated moving bed process with periodic operations. In Lee et al. (2001), they extended the method to deal with periodic continuous processes with constraints. When combined with model predictive control, repetitive control becomes more suitable to be applied in chemical processes.

The benefits of combining model predictive control with repetitive control or iterative learning control attracts many people's attention. In Gupta and Lee (2006), a period-robust repetitive model predictive control was proposed to deal with mismatch between actual period and period used. In Shi et al. (2007), iterative learning control was combined with generalized predictive control based on a two dimensional model resulted in a more unified design and the 2D-GPILC(Two Dimensional Generalized Predictive Iterative Learning Control) was applied on batch processes with single and multi-cycle.

Most of the above work assumed the period is known. In this paper, we consider the case that the period is unknown. A period estimation problem is transferred into the design problem of an adaptive filter based on the analysis of the prediction error. The non-integer period case is considered by using a two step optimization. Based on the obtained filter, repetitive control is plugged into GPC.

The paper is organized in the following way. In next section, the basic formulation of the problem is given. In the third section, methods of period identification for both integer and non-integer case are introduced. The control algorithm of the adaptive repetitive model predictive control is given. Then simulation results and conclusion are followed.

2. PROBLEM FORMULATION

An industrial process is often modeled as an ARMAX model as follows.

$$A(q^{-1})y(t) = B(q^{-1})u(t-1) + \varepsilon(t) \tag{1}$$

where

$$A(q^{-1}) = 1 + a_1 q^{-1} + a_2 q^{-2} + ... + a_n q^{-n}$$

and

$$B(q^{-1}) = 1 + b_1 q^{-1} + b_2 q^{-2} + ... + b_m q^{-m}$$

n and m are the orders of the model, and $\varepsilon(t)$ is white noise. The parameters of A and B are identified based on system identification test.

⋆

Following a similar idea in Shi et al. (2007), a prediction model can be derived for model predictive control at time t.

$$(A_1 \ A_2) \begin{pmatrix} y(|_t^{t-n+1}) \\ y(|_{t+N_1}^{t+1}) \end{pmatrix} = (B_1 \ B_2) \begin{pmatrix} u(|_{t-1}^{t-m+1}) \\ u(|_{t+N_2-1}^{t}) \end{pmatrix} \quad (2)$$

N_1 and N_2 are denoted as prediction horizon and control horizon respectively, $N_2 \leq N_1$. $y(|_b^a)$ denotes $[y(a) \ y(a+1) \ ... \ y(b-1) \ y(b)]^T$ and

$$(A_1 \mid A_2) = \begin{pmatrix} a_n & a_{n-1} & a_{n-2} & \dots & a_1 & | & 1 & 0 & \dots & 0 & 0 \\ 0 & a_n & a_{n-1} & \dots & a_2 & | & a_1 & 1 & \dots & 0 & 0 \\ 0 & 0 & a_n & \dots & a_3 & | & a_2 & a_1 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & | & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & * & | & * & * & \dots & a_1 & 1 \end{pmatrix} \quad (3)$$

$$(B_1 \mid B_2) = \begin{pmatrix} b_m & b_{m-1} & b_{m-2} & \dots & b_2 & | & b_1 & 0 & \dots & 0 & 0 \\ 0 & b_m & b_{m-1} & \dots & b_3 & | & b_2 & b_1 & \dots & 0 & 0 \\ 0 & 0 & b_m & \dots & b_4 & | & b_3 & b_2 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & | & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & * & | & * & * & \dots & b_2 & b_1 \end{pmatrix} \quad (4)$$

Based on this, the predicted output could be obtained by simple manipulation.

$$\begin{aligned} y_p(|_{t+N_1}^{t+1}) =& A_2^{-1} B_1 u(|_{t-1}^{t-m+1}) + A_2^{-1} B_2 u(|_{t+N_2-1}^{t}) \\ & - A_2^{-1} A_1 y(|_t^{t-n+1}) \end{aligned} \quad (5)$$

Denote y_r as the reference, diagonal and positive definite matrix Q and R as the weight matrix, then the unconstrained GPC can be formulated as

$$\begin{aligned} \min_{u(|_{t+N_2-1}^{t})} & [y_r(|_{t+N_1}^{t+1}) - y_p(|_{t+N_1}^{t+1})]^T Q [y_r(|_{t+N_1}^{t+1}) - y_p(|_{t+N_1}^{t+1})] \\ & + u(|_{t+N_2-1}^{t})^T R u(|_{t+N_2-1}^{t}) \end{aligned}$$

which is a basic GPC structure(Clarke et al. (1987)). When there are measurable disturbances $d_m(t)$, a feedforward GPC can be formulated as follows.

$$\begin{aligned} \min_{u(|_{t+N_2-1}^{t})} & [y_r(|_{t+N_1}^{t+1}) - G_d d_m(|_{t+N_1}^{t+1}) - y_p(|_{t+N_1}^{t+1})]^T Q \\ & \times [y_r(|_{t+N_1}^{t+1}) - G_d d_m(|_{t+N_1}^{t+1}) - y_p(|_{t+N_1}^{t+1})] \\ & + u(|_{t+N_2-1}^{t})^T R u(|_{t+N_2-1}^{t}) \end{aligned}$$

where G_d is the transfer function from the measurable disturbance to the output of the plant. That's the basic structure for feedforward MPC(Rawlings (1999)).

However, generally disturbances are not measurable. For processes with repetitive nature, such as periodic processes and batch processes, the feedforward part can be estimated based on input and output information of last cycle or batch, as shown in Shi et al. (2007) and Lee et al. (2001). This motivation induces a MPC combined with iterative learning control or repetitive control. The formulation of GPC for periodic disturbances rejection can follow this framework. Denote T as the period of disturbance. The new prediction model is given in equation (6)

$$\begin{aligned} y_p(|_{t+N_1}^{t+1}) =& A_2^{-1} B_1 u(|_{t-1}^{t-m+1}) + A_2^{-1} B_2 u(|_{t+N_2-1}^{t}) \\ & - A_2^{-1} A_1 y(|_t^{t-n+1}) + \text{Correction} \end{aligned} \quad (6)$$

$$\begin{aligned} \text{Correction} =& y(|_{t+N_1-T}^{t+1-T}) - A_2^{-1} B_1 u(|_{t-1-T}^{t-m+1-T}) \\ & + A_2^{-1} B_2 u(|_{t+N_2-1-T}^{t-T}) - A_2^{-1} A_1 y(|_{t-T}^{t-n+1-T}) \end{aligned}$$

The correction is actually the prediction error from time $t+1-T$ to $t+N_1-T$. Further, denote

$$\Delta u(|_{t+N_2-1}^{t}) = u(|_{t+N_2-1}^{t}) - u(|_{t+N_2-1-T}^{t-T})$$

the optimization part can be given as

$$\begin{aligned} \min_{u(|_{t+N_2-1}^{t})} & [y_r(|_{t+N_1}^{t+1}) - y_p(|_{t+N_1}^{t+1})]^T Q [y_r(|_{t+N_1}^{t+1}) - y_p(|_{t+N_1}^{t+1})] \\ & + \Delta u(|_{t+N_2-1}^{t})^T R \Delta u(|_{t+N_2-1}^{t}) \end{aligned} \quad (7)$$

Based on equation (6) and equation (7), the control law for this unconstrained repetitive GPC is induced as follows by taking first derivative.

Denote

$$G = A_2^{-1} B_2$$

Then

$$\begin{aligned} & u(|_{t+N_2-1}^{t}) = u(|_{t+N_2-1-T}^{t-T}) + (R + G^T Q G)^{-1} G^T Q \\ & \times \{ y_r(|_{t+N_1}^{t+1}) - y(|_{t+N_1-T}^{t+1-T}) - A_2^{-1} B_1 [u(|_{t-1}^{t-m+1}) \\ & - u(|_{t-1-T}^{t-m+1-T})] + A_2^{-1} A_1 [y(|_t^{t-n+1}) - y(|_{t-T}^{t-n+1-T})] \} \end{aligned} \quad (8)$$

Here considering the case of $T < N_1$, $y(t+N_1-1-T)$ is not available at time t. Compensation can still be done by using $y(t+N_1-\alpha T)$ and $u(t+N_2-\alpha T)$, where

$$\alpha = \lceil \frac{N_1}{T} \rceil \quad (9)$$

$\lceil * \rceil$ means to round up to the nearest integer. In order to have a unified expression, we can also keep all the value of $u(|_{t+N_2-1}^{t})$ and manually assign

$$\begin{aligned} & y(|_{t+N_1+1}^{t+1}) = A_2^{-1} B_1 u(|_{t-1}^{t-m+1}) + A_2^{-1} B_2 u(|_{t+N_2-1}^{t}) - \\ & A_2^{-1} A_1 y(|_t^{t-n+1}) + y(|_{t+N_1-T}^{t+1-T}) - A_2^{-1} B_1 u(|_{t-1-T}^{t-m+1-T}) \\ & + A_2^{-1} B_2 u(|_{t+N_2-1-T}^{t-T}) - A_2^{-1} A_1 y(|_{t-T}^{t-n+1-T}) \end{aligned} \quad (10)$$

Then Equ.(8) is always established and the effect is the same as method shown in Equ.(9). As MPC usually does, only the value of $u(t)$ is sent to the plant and the value of $y(t+1)$ is updated as the measured output of the plant. Next, the key problem is how to identify the period T on-line.

3. PERIOD IDENTIFICATION OF THE PERIODIC DISTURBANCES

3.1 Integer period case

The estimation of the period is mainly based on an analysis of the prediction error. As prediction error is caused by model mismatch and disturbances, when model mismatch is not significant compared to the magnitude of disturbances, the prediction error is also of periodic form. Denote $e_p(t)$ as the prediction error, then

$$e_p(t) = A(q^{-1}) y(t) - B(q^{-1}) u(t) \quad (11)$$

Next, we will show how to transfer a period estimation problem into a coefficient identification problem of an adaptive filter. Furthermore, this filter will be used in MPC directly. Firstly, we only consider the case when the period is an integer.

Denote N as as the order of the filter w, and

$$w = [w_1, \ w_2, ..., \ w_N]^T \in \mathbb{Z}^{N \times 1}$$

Assume $N > T$, and denote H as the estimation horizon,

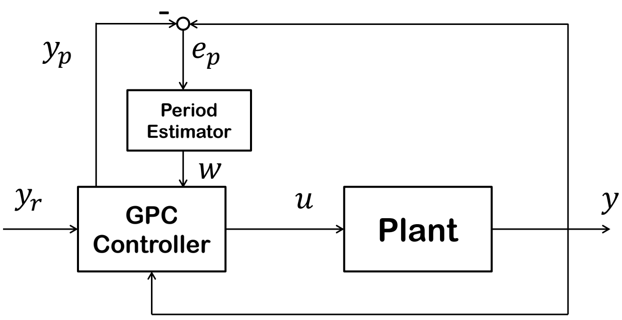

Fig. 1. Block diagram of adaptive repetitive MPC

$$C(t) = \begin{pmatrix} e_p(t-H+1) \\ . \\ . \\ . \\ e_p(t-1) \\ e_p(t) \end{pmatrix}$$

$$D(t) = \begin{pmatrix} e_p(t-N-H+1) & ... & e_p(t-1-H) & e_p(t-H) \\ e_p(t-N-H+2) & ... & .. & e_p(t-H+1) \\ . & . & . & . \\ . & . & . & . \\ . & . & . & . \\ e_p(t-N) & ... & e_p(t-2) & e_p(t-1) \end{pmatrix} \tag{12}$$

Then the optimization part can be formulated as

$$\begin{aligned} &\min_{[w_1,\ w_2, ...,\ w_N]} [C(t) - D(t)w]^T [C(t) - D(t)w] \\ &\text{st.} \qquad \sum w_i = 1 \\ &\qquad\qquad w_i = \{0, 1\} \qquad\qquad i = 1,\ ...,\ N \end{aligned} \tag{13}$$

To $\forall i \in [0, H]$,$(e_p(t-N-H+i),\ \ldots,\ e_p(t-H+i))w$ can be interpreted as the prediction of $e_p(t-H+i+1)$ based on identified w. So with the above constraints, by minimizing $\|e_p(t-H+i) - [e_p(t-H-N+i), e_p(t-H-N+i+1), \ldots, e_p(t-H+i-1)]w\|_2^2$ for H steps,which is a moving horizon estimation, we can get w as

$$w = [0, ..0, \underbrace{1, 0, ...0}_{T}]^T \tag{14}$$

With the filter obtained, the prediction model can be revised as

$$\begin{aligned} y_p(|_{t+N_1}^{t+1}) =& A_2^{-1} B_1 u(|_{t-1}^{t-m+1}) + A_2^{-1} B_2 u(|_{t+N_2-1}^{t}) \\ &- A_2^{-1} A_1 y(|_{t}^{t-n+1}) + A_2^{-1} f(|_{t+N_1}^{t+1}) \end{aligned} \tag{15}$$

$$f(t) = [e_p(t-N), e_p(t-N+1), \ldots, e_p(t-1)]w$$

So the filter here is actually used to pick up a proper compensation from $[e_p(t-N), e_p(t-N+1), \ldots, e_p(t-1)]$, $f(t)$ can be considered as a compensation item which is similar to the correction term in Equ.(6). Further, denote

$$\begin{aligned} U(t) =& [u(t-N), u(t-N+1), \ldots, u(t-1)] \\ Y(t) =& [y(t-N), y(t-N+1), \ldots, y(t-1)] \end{aligned}$$

and

$$\Delta_w u(|_{t+N_2-1}^{t}) = u(|_{t+N_2-1}^{t}) - U(|_{t+N_2-2}^{t-1})w \tag{16}$$

the optimization part could be formulated as

$$\begin{aligned} \min_{u(|_{t+N_2-1}^{t})} & [y_r(|_{t+N_1}^{t+1}) - y_p(|_{t+N_1}^{t+1})]^T Q [y_r(|_{t+N_1}^{t+1}) - y_p(|_{t+N_1}^{t+1})] \\ & + \Delta_w u(|_{t+N_2-1}^{t})^T R \Delta_w u(|_{t+N_2-1}^{t}) \end{aligned} \tag{17}$$

The induced control law is as follows

$$\begin{aligned} u(|_{t+N_2-1}^{t}) =& U(|_{t+N_2-2}^{t-1})w + (R + G^T Q G)^{-1} G^T Q \\ & \times \{y_r(|_{t+N_1}^{t+1}) - Y(|_{t+N_1-1}^{t})w - A_2^{-1} B_1 [u(|_{t-1}^{t-m+1}) \\ & - U(|_{t-2}^{t-m})]w + A_2^{-1} A_1 [y(|_{t}^{t-n+1}) - Y(|_{t-1}^{t-n})w]\} \end{aligned} \tag{18}$$

Considering the period may vary, and the past information may not be as efficient as the newest one, we can further improve the optimization as follows

$$\begin{aligned} &\min_{[w_1, w_2, ..., w_N]} [C(t) - D(t)w]^T \mathbf{P}_w [C(t) - D(t)w] + w^T \mathbf{Q}_w w \\ &\text{st.} \qquad \sum w_i = 1 \\ &\qquad\qquad w_i = \{0, 1\} \qquad\qquad i = 1,\ ...,\ N \end{aligned} \tag{19}$$

where $\mathbf{P}_w$ and $\mathbf{Q}_w$ are defined as

$$\mathbf{P}_w = \begin{pmatrix} \alpha^{H-1} & 0 & ... & 0 \\ 0 & \alpha^{H-2} & ... & 0 \\ . & . & . & . \\ . & . & . & . \\ . & . & . & . \\ 0 & 0 & ... & \alpha^0 \end{pmatrix} \tag{20}$$

$$\mathbf{Q}_w = \theta \begin{pmatrix} \beta^0 & 0 & ... & 0 \\ 0 & \beta^1 & ... & 0 \\ . & . & . & . \\ . & . & . & . \\ . & . & . & . \\ 0 & 0 & ... & \beta^{N-1} \end{pmatrix} \tag{21}$$

$\alpha \in (0, 1]$, $\beta \in [0, 1]$ and $\theta \geq 0$, so $\mathbf{P}_w$ and $\mathbf{Q}_w$ are both positive semi-definite. α is taken as a forgetting factor to give priority to the latest information, which may improve the ability to adapt to any period variation of $e_p(t)$. $\mathbf{Q}_w$ is the weight matrix of the coefficients. Based on this structure, when $H > mT$ and $m \in \mathbb{Z}^+$, by properly assign the value of β, w can be forced to converge to

$$w = [0, ..0, \underbrace{1, 0, ...0}_{T}]^T$$

instead of jumping among different mT. The parameter θ is used to balance the weight of the two parts in the objective function.

Next, we will consider the non-integer case, which is also very common when T can not be divided by the sample rate of the control system.

3.2 Non-integer period case

Considering the non-integer period case, here a two-step optimization will be adopted. Firstly, it will be treated in the same way as the integer period case and a group of w can be obtained as

$$w = [0, ..0, \underbrace{1, 0, ...0}_{T_a}]^T$$

T_a is the approximation of the real period T, since T is not an integer. Then do a relaxation around the '1' as

$$\widehat{w} = [0, \ldots, \widehat{w}_1, \underbrace{\widehat{w}_2, \widehat{w}_3, \ldots, 0}_{T_a}]^T$$

This relaxation actually helps us to turn a non-integer period into a combination of three integer period with optimized weights. Since a non-integer can always interpreted

by two neighbor integer, here we only need to relax it into three free variables.
Then the second step optimization can follow this as

$$\begin{aligned} \min_{[\widehat{w}_1,\ \widehat{w}_2,\ \widehat{w}_1]} \quad & [C(t) - D(t)\widehat{w}]^T [C(t) - D(t)\widehat{w}] \\ \text{st.} \quad & \sum \widehat{w}_i = 1 \\ & \widehat{w}_i \geq 0 \qquad i = 1,\ ...,\ N \end{aligned} \tag{22}$$

Based on this convex optimization, the compensation can be made not solely rely on one point, so the performance can be improved for the non-integer case.

3.3 stablity analysis

The stability analysis of repetitive MPC is always a tough problem. This is partly because of the difficulty in stability analysis of MPC itself. In the proposed adaptive algorithm, by restricting the coefficients of the estimator w_i satisfying the following two constraints, the estimation error of the disturbances can be proved to be bounded.

$$\sum_{i=1}^{i=N} w_i = 1 \tag{23}$$

$$w_i \in [0, 1] \quad i = 1,\ 2, \ldots, N \tag{24}$$

With the bounded estimation error, some robust MPC can be adopted to guarantee the stability. When the disturbance and model error is not too large, in practice, a well-tuned regular MPC can also be considered to be stable.
In this periodic disturbance case, we may further seek a way to explore the period-wise convergence, which is more important, and this will be the follow-up work. Here due to the limited space, only a brief analysis is given.

4. SIMULATION

The simulation is based on a true process model (25) and its model (26) used in GPC. Disturbances are treated as output disturbances.

$$y(t+1) = \frac{2.651 + 5.298q^{-1} + 0.5805q^{-2}}{1 - 1.454q^{-1} + 0.5285q^{-2} - 0.04736q^{-3}} u(t) \tag{25}$$

$$y(t+1) = \frac{13.81q^{-1}}{1 - 0.9524q^{-1}} u(t) \tag{26}$$

- Case 1: integer unknown period

$$dis(t) = sin(0.2\pi t) + N(0, 0.01) \tag{27}$$

As shown in the formula, the period is 10s. Here take the forgetting factor $\alpha = 0.95$ and the weight of coefficients $\beta = 0.95$, $\theta = 1$ the order of the filter $N = 50$, the estimation horizon $H = 100$. Figure. 2 shows the estimated period T, Figure 3 gives the absolute value of the output error. Before $t = 50s$, it is the output of conventional GPC without any feedforward compensation, and after $t = 50s$, it shows the result of the proposed adaptive repetitive GPC. We can see that the output error is significantly reduced by the proposed method and an accurate estimation of the period can be quickly obtained.

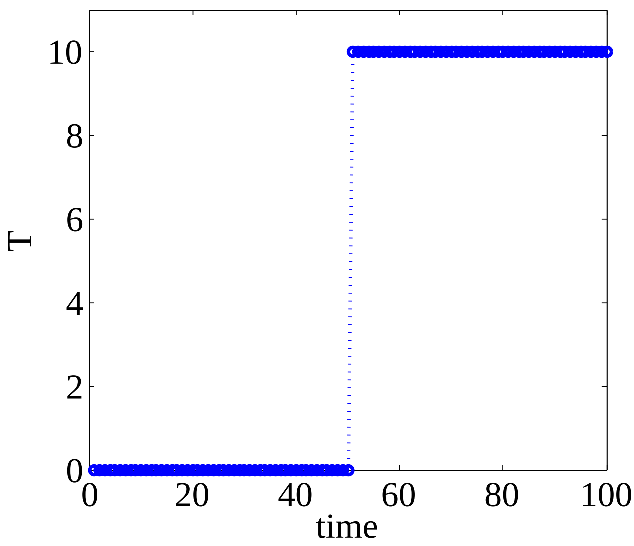

Fig. 2. Estimated period for case 1

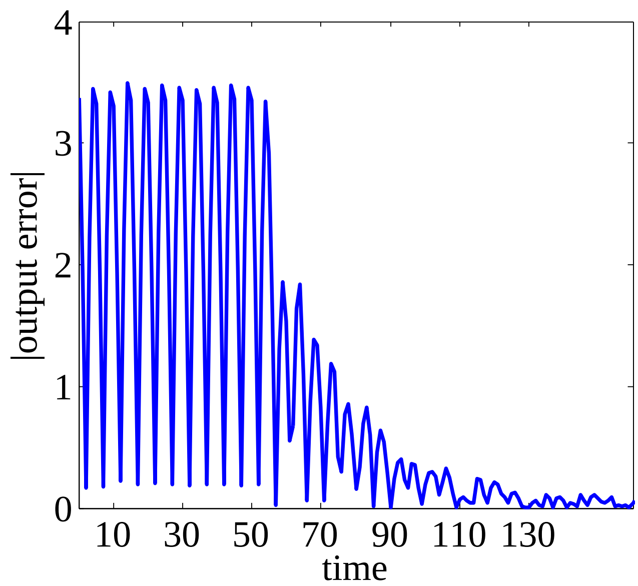

Fig. 3. Output error for case 1

- Case 2: integer unknown and varying period

$$t < 800 \quad dis(t) = sin(0.2\pi t) + N(0, 0.01) \tag{28}$$

$$t \geq 800 \quad dis(t) = sin(0.25\pi t) + N(0, 0.01) \tag{29}$$

Keep the settings for each parameter as in the last case. From Fig. 4 we can see when the period of disturbances changes, the estimated value quickly track the changes. Fig 5 shows the output error. Next consider the effect of forgetting factor. Fig.6 and 7 show the comparison results of the case that $\alpha = 0.95$ and $\alpha = 0.35$. We can conclude that smaller forgetting factor gives us better transient performance with faster response, but the resulted T is not quite stable. The stability is worse. We can further see the effect of the penalty term with Fig. 8. The comparison is between $\theta = 0$, which means there is no penalty term, and $\theta = 1$. We can see that when there is no penalty term, the estimated T may jump among different mT as illustrated in the former section. With a small penalty term, the estimation has better stability.

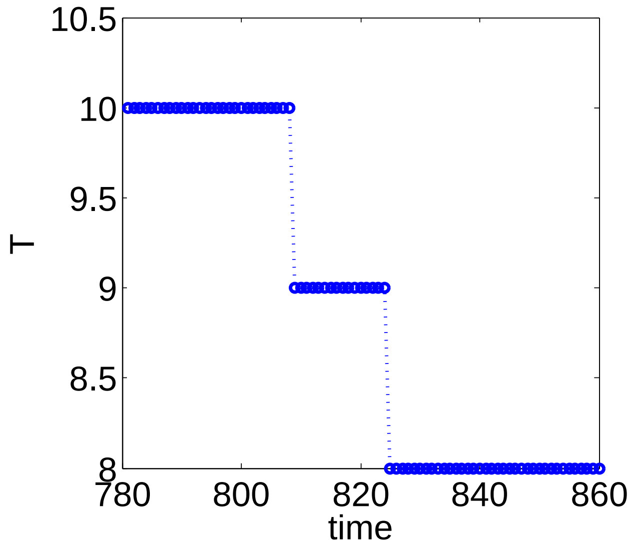

Fig. 4. Estimated period for case 2

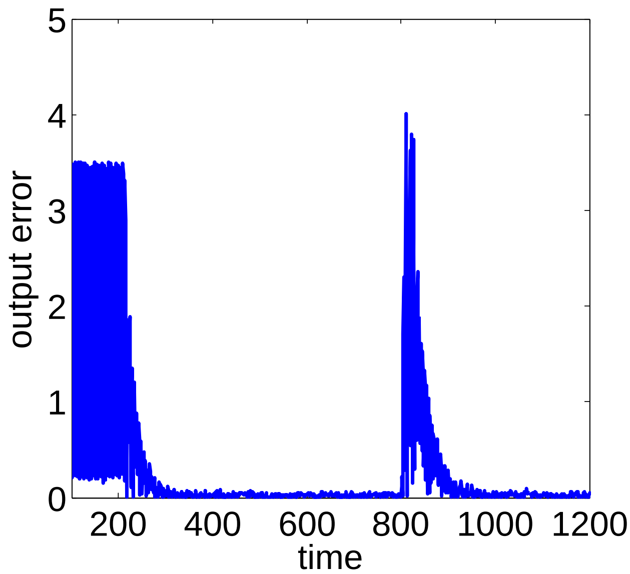

Fig. 5. Output error for case 2

- Case 3: non-integer unknown period

$$dis(t) = sin(2\pi/26.8t) + N(0, 0.01) \quad (30)$$

It is clear with this example the period $T = 26.8$. Fig.10 shows the coefficient for case 3 based on double optimization. This result indicates the feedforward compensation relies on multi-former information, which is similar to high order iterative learning control. Fig. 11 shows that the output error of this two step double optimization can be much smaller than the integer programming when the period is not an integer.

5. CONCLUSION

An adaptive repetitive control based on Generalized Predictive Control with a period estimator is proposed for disturbance with unknown and varying period. The non-integer case is discussed together with a period estimator based on a two step double optimization. Simulation results show the effectiveness of both the period estimator

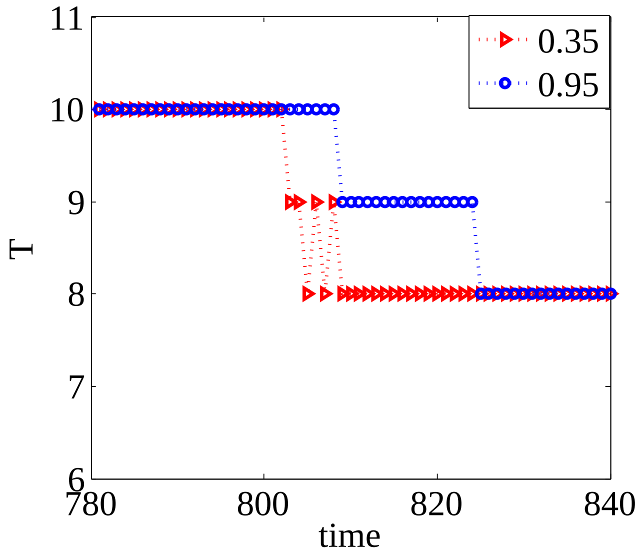

Fig. 6. Comparison between different fogetting factor: estimated T for case 2

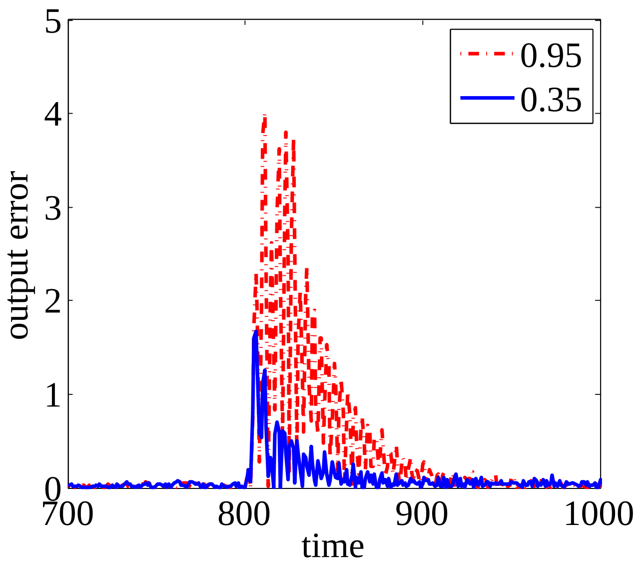

Fig. 7. Comparison between different forgetting factor: Output error for case 2

and the adaptive repetitive control algorithm. In addition, the method can be further extended to constrained case. The tuning method for the parameters of the period estimator and stability analysis of the algorithm is worth further consideration.

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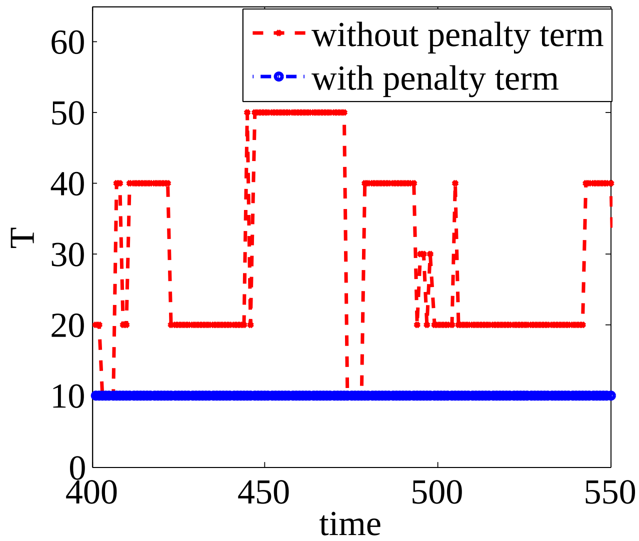

Fig. 8. Effect of the penalty term

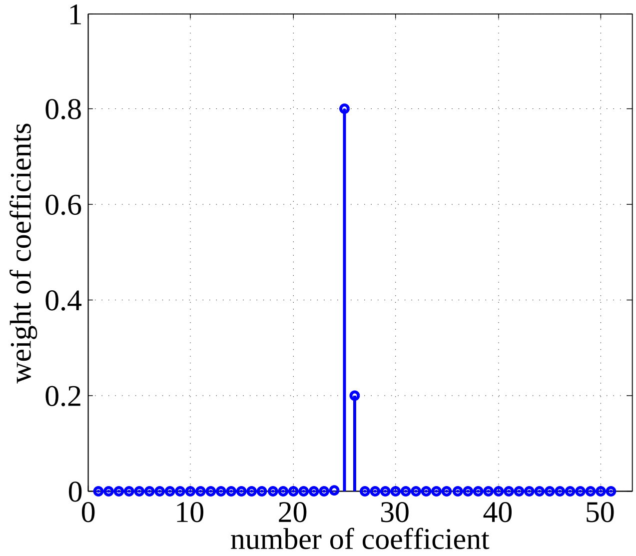

Fig. 10. coefficient of the filter for case 3

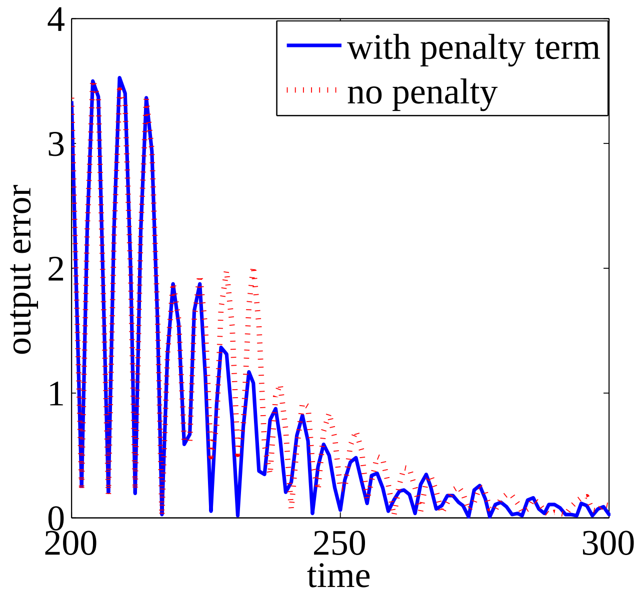

Fig. 9. Effect of the penalty term: output error

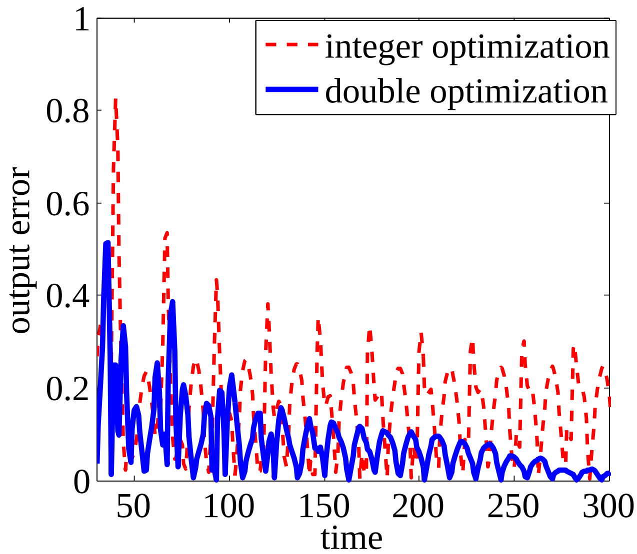

Fig. 11. Comparison between integer programming with double optimization for case 3

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