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Analytical Design of Centralized PI Controller for High Dimensional Multivariable Systems ⋆

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Abstract: This paper presents a simple analytical method for the design of full matrix PI controller based on the direct synthesis approach. By proposing the practically desired closed-loop diagonal transfer function to reduce interactions between individual loops, analytical expressions for PI controller are derived for several common types of process models, including first order plus time delay models and second order plus time delay models. Compared with the existing direct synthesis approaches, the proposed controller design method requires no approximation of the inverse of process model or Maclaurin's series expansion. Furthermore, it is applicable to high dimensional multivariable systems with satisfactory performance and robustness. Several examples are introduced to demonstrate the effectiveness and simplicity of the design method.

Keywords: Multivariable system; High dimension; PI controller; Centralized control

1. INTRODUCTION

Multivariable proportional-integral-derivative (PID) control has attracted considerable attention in the literature of process control due to its remarkable effectiveness and simplicity of implementation. Differing from traditional single-input and single-output (SISO) PID tuning techniques, the tuning of multivariable PID controller has difficulty in coping with interactions between control loops. Therefore, much research has been focused on how to design multivariable PID controller efficiently by taking loop interactions into account (Mayne (1973), Luyben (1986), Wang et al. (1997), Wang et al. (2008), Escobar and Trierweiler (2013)). In practice, multi-loop (decentralized) control and centralized control are the most common control strategies. In multi-loop control, the multivariable processes are decomposed into multiple SISO loops and accordingly the controllers are designed in diagonal form (Shiu and Hwang (1998), Chen and Seborg (2003), He et al. (2005), Huang et al. (2003), Xiong and Cai (2006)). Although multi-loop control has less tuning parameters, simple structure, reasonable performance and robustness, it is only applicable to the systems with modest interactions.

When the interactions in different channels of the process are strong, it is necessary to design off-diagonal controllers to eliminate the interactions, and in such cases the full dimensional matrix controllers are usually preferred (Dong and Brosilow (1997), Wang et al. (2000), J. Garrido (2012)). Among the performance indices used for tuning centralized PID controller parameters, the criterion to keep the controlled variable response close to the desired closed-loop response has gained widespread acceptance in modern industries because of its simplicity and successful practical applications. The IMC (internal model control)-PID tuning method and the direct synthesis method are typical of the tuning methods based on achieving a desired response. Lieslehto (1996) presented a centralized PID controller based on internal model control (IMC) SISO design and Wang et al. (1996) proposed the multivariable PID controller tuning as an optimization problem based on the desired closed-loop transfer function matrix, but both methods present limitations when the process incorporates significant time delays. In the direct synthesis method, to get the full controller matrix directly, the inverse of the process transfer function has to be known. By calculating directly the inverse of simple process model (2-by-2 or 3-by-3), Morilla et al. (2008) proposed a multivariable controller for two-input and two-output systems, and Garrido et al. (2009) developed a centralized controller for a nonlinear boiler-turbine unit. However, when the system dimension is high, it is difficult to find a suitable solution for the inverse of the process model under fixed control structures. To approximate the inverse of the process transfer function, Xiong et al. (2007) introduced effective relative gain array (ERGA) approach to design the full dimensional controller and Kumar et al. (2012) used relative normalized gain array (RNGA) and relative average residence time array (RARTA) to obtain the centralized PID controller using Maclaurin series expansion. However, the approximation way to obtain the inverse of the process transfer function inevitably introduces modeling

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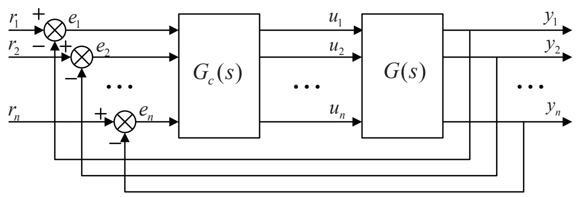

Fig. 1. Closed-loop multivariable control system

errors, which leads to more conservativeness for higher dimensional processes.

This paper proposes a novel decoupling design method for multivariable processes. By proposing the practically desired closed-loop diagonal transfer function to reduce interactions between individual loops, analytical expressions for PI controllers are derived for several common types of process models, including first order plus time delay (FOPTD) models and second order plus time delay (SOPTD) models. Compared with the existing multivariable controller design methods, the proposed method requires no approximation of the inverse of process model and Maclaurin series expansion. The effectiveness of the proposed design approach is verified by several typical multivariable industrial processes.

2. PROBLEM STATEMENT

Consider an n-inputs and n-outputs open-loop stable and physically proper multivariable system with time delay, as shown in Figure 1, where r_i, $i = 1, 2, ..., n$ are the reference inputs, e_i, $i = 1, 2, ..., n$ are the errors between feedback and reference, u_i, $i = 1, 2, ..., n$ are the manipulated variables, y_i, $i = 1, 2, ..., n$ are the system outputs, $G(s)$ is process transfer function matrix and $G_c(s)$ is full dimensional controller matrix, both of which have compatible dimensions.

$$G(s) = \begin{bmatrix} g_{11}(s) & g_{12}(s) & \cdots & g_{1n}(s) \\ g_{21}(s) & g_{22}(s) & \cdots & g_{2n}(s) \\ \vdots & \vdots & \ddots & \vdots \\ g_{n1}(s) & g_{n2}(s) & \cdots & g_{nn}(s) \end{bmatrix}$$

$$G_c(s) = \begin{bmatrix} g_{c,11}(s) & g_{c,12}(s) & \cdots & g_{c,1n}(s) \\ g_{c,21}(s) & g_{c,22}(s) & \cdots & g_{c,2n}(s) \\ \vdots & \vdots & \ddots & \vdots \\ g_{c,n1}(s) & g_{c,n2}(s) & \cdots & g_{c,nn}(s) \end{bmatrix}$$

From Fig. 1, it is easy to see that the closed-loop transfer function matrix between outputs and set-points can be determined as:

$$H(s) = (I + G(s)G_c(s))^{-1}G(s)G_c(s) \tag{1}$$

A multivariable controller is derived from equation (1) as follows:

$$G_c(s) = G^{-1}(s)(H^{-1}(s) - I)^{-1} \tag{2}$$

In multivariable systems, the required ideal structure of matrix $H(s)$ is in diagonal form, which reveals the system is perfectly decoupled and that each output can track its reference independently. As described in Vu and Lee. (2010), $H(s)$ cannot achieve diagonal structure by any decentralized controller, because the system transfer function $G(s)$ is a non-diagonal matrix. Therefore, a centralized controller is a better choice to give better performance. However, it is rather difficult to find the inverse of open-loop transfer function matrix with direct synthesis method, particularly, when the system dimension is high.

This paper aims to establish the relations between PI controller tuning parameters and steady and dynamic characteristics of open-loop system without calculating $G^{-1}(s)$ directly. Next section will present the design algorithm to obtain the tuning relations for the PI controller.

3. CONTROL SYSTEM DESIGN

In this section, formulae for the calculation of PI controller tuning parameters are derived for the desired closed-loop response models of the types FOPTD and SOPTD.

From equation (2), the desired ideal closed-loop transfer function should be in the form of

$$H(s) = \begin{bmatrix} h_{11}(s) & & 0 \\ & \ddots & \\ 0 & & h_{nn}(s) \end{bmatrix} \tag{3}$$

where h_{ii} is a diagonal element of $H(s)$ and corresponds to the desired closed-loop transfer function of each loop.

Then $(H^{-1}(s) - I)^{-1}$ can be expressed as

$$(H^{-1}(s) - I)^{-1} = \begin{bmatrix} \frac{h_{11}}{1 - h_{11}} & & \\ & \ddots & \\ & & \frac{h_{nn}}{1 - h_{nn}} \end{bmatrix} \tag{4}$$

According to IMC theory, the desired closed-loop diagonal transfer function (Wang et al. (2003), Liu et al. (2007), Morari and Zafiriou (1989)) is expressed in the form of

$$h_{ii} = \frac{e^{-d_i s}}{(\lambda_i s + 1)^{m_i}} \prod_{k=1}^{q_i} \frac{z_k - s}{z_k^* + s}, i = 1, 2, ..., n \tag{5}$$

where d_i is the largest time delay of i-th row elements of $G(s)$, λ_i is adjustable parameter that provides the tradeoff between performance and robustness, m_i is the relative order of the numerator and denominator in $g_{ij}(s)$, z_k and z_k^* denote the Right Half Plane (RHP) zeros and the corresponding complex conjugate of RHP zeros of the i-th diagonal element of the process transfer function matrix, respectively, and q_i is the number of RHP zeros.

Substituting equation (4) into equation (2) yields

$$g_{c,ji} = \frac{adjG_{ji}}{|G|} \frac{h_{ii}}{1 - h_{ii}}, i, j = 1, 2 \ldots, n \tag{6}$$

where $adjG_{ij}$ is i-th row and j-th column element of the adjugate matrix of $G(s)$, $|G|$ is the determinant of $G(s)$.

The standard PI controller is given by

$$g_{c,ji}(s) = k_{C,ji} + \frac{k_{I,ji}}{s} \tag{7}$$

According to equation (6) and equation (7), it can be obtained that

$$k_{C,ji} + \frac{k_{I,ji}}{s} = \frac{adjG_{ji}}{|G|}\frac{h_{ii}}{1-h_{ii}} \quad (8)$$

Multiplying both sides by s, we have

$$sk_{C,ji} + k_{I,ji} = \frac{adjG_{ji}}{|G|}\frac{sh_{ii}}{1-h_{ii}} \quad (9)$$

Taking the derivative of both sides of equation (9), it yields

$$\begin{aligned} k_{C,ji} &= \left(\frac{adjG_{ji}}{|G|}\frac{sh_{ii}}{1-h_{ii}}\right)' = \frac{adjG_{ji}}{|G|}\left(\frac{sh_{ii}}{1-h_{ii}}\right)' \\ &-\frac{1}{|G|^2}\frac{sh_{ii}}{1-h_{ii}}\sum_{p=1}^{n}\left(\sum_{q=1}^{n}(adjG_{jq})g'_{qp}\right)adjG_{pi} \end{aligned} \quad (10)$$

where g'_{qp} is the first derivative of g_{qp}.

Letting $s=0$ and solving equation (9) and equation (10), the controller parameters can be calculated as

$$k_{I,ji} = \frac{adjK_{ji}}{|K|}\frac{sh_{ii}}{1-h_{ii}}\bigg|_{s=0} \quad (11)$$

$$\begin{aligned} k_{C,ji} &= \frac{adjK_{ji}}{|K|}\left(\frac{sh_{ii}}{1-h_{ii}}\right)'\bigg|_{s=0} - \left(\frac{sh_{ii}}{1-h_{ii}}\bigg|_{s=0}\right) \\ &\frac{1}{|K|^2}\sum_{p=1}^{n}\left(\sum_{q=1}^{n}(adjK_{jq})\left(g'_{qp}\big|_{s=0}\right)\right)adjK_{pi} \end{aligned} \quad (12)$$

where

$$|K| = \begin{vmatrix} k_{11} & \cdots & k_{1n} \\ \vdots & \ddots & \vdots \\ k_{n1} & \cdots & k_{nn} \end{vmatrix}$$

$adjK_{ji}$, $adjK_{jq}$ and $adjK_{pi}$ are simplified as a single formula using subscripts v and w, defining:

$$adjK_{vw} = (-1)^{v+w}\begin{vmatrix} k_{1,1} & \cdots & k_{1,v-1} & k_{1,v+1} & \cdots & k_{1,n} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ k_{w-1,1} & \cdots & k_{w-1,v-1} & k_{w-1,v+1} & \cdots & k_{w-1,n} \\ k_{w+1,1} & \cdots & k_{w+1,v-1} & k_{i+1,v+1} & \cdots & k_{w+1,n} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ k_{n,1} & \cdots & k_{n,v-1} & k_{n,v+1} & \cdots & k_{n,n} \end{vmatrix}$$

k_{vw} is the steady gain of process transfer function.

3.1 FOPDT case

The FOPTD process model is given by the transfer function:

$$\begin{bmatrix} \frac{k_{11}}{\tau_{11}s+1}e^{-\theta_{11}s} & \cdots & \frac{k_{1n}}{\tau_{1n}s+1}e^{-\theta_{1n}s} \\ \vdots & \ddots & \vdots \\ \frac{k_{n1}}{\tau_{n1}s+1}e^{-\theta_{n1}s} & \cdots & \frac{k_{nn}}{\tau_{nn}s+1}e^{-\theta_{nn}s} \end{bmatrix}$$

Accordingly, equation (5) becomes

$$h_{ii} = \frac{e^{-d_i s}}{\lambda_i s+1} \quad (13)$$

where $d_i = \max(\theta_{ij}, j=1,...,n)$

With equation (13), following conditions hold:

$$\frac{sh_{ii}}{1-h_{ii}}\bigg|_{s=0} = \frac{se^{-d_i s}}{\lambda_i s+1-e^{-d_i s}}\bigg|_{s=0} = \frac{1}{\lambda_i+d_i} \quad (14)$$

$$\left(\frac{sh_{ii}}{1-h_{ii}}\right)'\bigg|_{s=0} = \frac{-2\lambda_i d_i - d_i^2}{2(\lambda_i+d_i)^2} \quad (15)$$

$$g'_{ij}\big|_{s=0} = \left(\frac{k_{ij}}{\tau_{ij}s+1}e^{-\theta_{ij}s}\right)'\bigg|_{s=0} = -k_{ij}\tau_{ij} - k_{ij}\theta_{ij} \quad (16)$$

Substituting equations (14-16) into equations (11-12), we have the following tuning relations for the PI controller:

$$k_{I,ji} = \frac{adjK_{ji}}{|K|}\frac{1}{\lambda_i+d_i} \quad (17)$$

$$\begin{aligned} k_{C,ji} &= -\frac{adjK_{ji}}{|K|}\frac{2\lambda_i d_i + d_i^2}{2(\lambda_i+d_i)^2} - \frac{1}{|K|^2}\frac{1}{\lambda_i+d_i} \\ &\sum_{p=1}^{n}\left(\sum_{q=1}^{n}(adjK_{jq})(-k_{qp}\tau_{qp} - k_{qp}\theta_{qp})\right)adjK_{pi} \end{aligned} \quad (18)$$

3.2 SOPDT case

For multivariable process described by the following SOPTD models

$$\begin{bmatrix} \frac{k_{11}e^{-\theta_{11}s}}{s^2+2\zeta\omega_{n,11}s+\omega_{n,11}^2} & \cdots & \frac{k_{1n}e^{-\theta_{1n}s}}{s^2+2\zeta\omega_{n,1n}s+\omega_{n,1n}^2} \\ \vdots & \ddots & \vdots \\ \frac{k_{n1}e^{-\theta_{n1}s}}{s^2+2\zeta\omega_{n,n1}s+\omega_{n,n1}^2} & \cdots & \frac{k_{nn}e^{-\theta_{nn}s}}{s^2+2\zeta\omega_{n,nn}s+\omega_{n,nn}^2} \end{bmatrix}$$

According to IMC theory, the desired closed-loop diagonal transfer function h_{ii} is the same as equation (13).

g'_{ij} is computed as

$$\begin{aligned} g'_{ij}\big|_{s=0} &= \left(\frac{k_{ij}}{s^2+2\zeta\omega_{n,ij}s+\omega_{n,ij}^2}e^{-\theta_{11}s}\right)'\bigg|_{s=0} \\ &= -\frac{k_{ij}\theta_{ij}}{\omega_{n,ij}^2} - \frac{2k_{ij}\zeta_{ij}}{\omega_{n,ij}^3} \end{aligned} \quad (19)$$

Substituting equation (19) and equations (14-15) into equations (11-12), and solving for PI controller parameters yields the following design formulae:

$$k_{I,ji} = \frac{adjK_{ji}}{|K|}\frac{1}{\lambda_i+d_i} \quad (20)$$

$$\begin{aligned} k_{C,ji} &= -\frac{adjK_{ji}}{|K|}\frac{2\lambda_i d_i + d_i^2}{2(\lambda_i+d_i)^2} - \frac{1}{|K|^2}\frac{1}{\lambda_i+d_i} \\ &\sum_{p=1}^{n}\left(\sum_{q=1}^{n}(adjK_{jq})(-\frac{k_{pq}\theta_{pq}}{\omega_{n,pq}^2} - \frac{2k_{pq}\zeta_{pq}}{\omega_{n,pq}^3})\right)adjK_{pi} \end{aligned} \quad (21)$$

Remark: From equations (17-18) and equations (20-21), it can be seen that tuning relations for PI controller parameters are directly derived for FOPTD and SOPTD models according to the desired closed-loop response. Different from the existing direct synthesis method, the proposed design method need not use equivalent transfer function to approximate $G^{-1}(s)$ so that better performance can be achieved through more accurate controller tuning relations.

What's more, there is no need to compute the inverse of the process model or to tune controller parameters by Maclaurin series expansion. The straightforward design procedure makes it easier to understand by engineers and applicable to practical applications.

3.3 Performance and robustness of control system

To analyze the performance of the control system, the following performance indices are used. Firstly, the integral absolute error (IAE) criterion is considered to evaluate the closed-loop performance, which is defined as:

$$IAE = \int_0^\infty |e(t)|\, dt \tag{22}$$

where

$$e(t) = r(t) - y(t)$$

Secondly, the following integral of the time-weighted absolute error (ITAE) criterion is used to evaluate the closed-loop performance over long periods of time:

$$ITAE = \int_0^\infty t\,|e(t)| dt \tag{23}$$

Finally, to investigate the robust stability of the resulting control system, a renowned method for robust stability is used for a fair comparison with other existing controller design methods (Lee et al. (2005), Vu and Lee (2010), Skogestad and Poslethwaite (1996)). The robust stability can be examined under output multiplication uncertainty, since it is often less restrictive than input uncertainty in terms of control performance (Skogestad and Poslethwaite (1996)). For a system with an output uncertainty as $G(s)[I + \Delta_o(s)]$, the closed-loop system is stable if

$$\gamma < 1/\bar{\sigma}[(I + G(j\omega)G_c(j\omega))^{-1}G(j\omega)G_c(j\omega)] \tag{24}$$

where $\|\Delta_o(j\omega)\|$ represents the multiplicative output uncertainties. For a fair comparison with other methods, $\|\Delta_o(j\omega)\|$ should be kept the same or larger than those of compared methods. Note that a system with a larger implies having more stability margin.

4. SIMULATION STUDIES

Example 1. To verify the effectiveness and superior performance of this proposed method, an industrial-scale polymerization reactor model (Chien et al. (1999)) is used, which is given by

$$G(s) = \begin{bmatrix} \dfrac{22.89e^{-0.2s}}{4.572s+1} & \dfrac{-11.64e^{-0.4s}}{1.807s+1} \\ \dfrac{4.689e^{-0.2s}}{2.174s+1} & \dfrac{5.8e^{-0.4s}}{1.801s+1} \end{bmatrix}$$

The equivalent transfer function based centralized PI control by direct synthesis method, such as Xiong et al. (2007) and Kumar et al. (2012), are employed here for comparison. In tuning the controller parameters, λ_i of the proposed method is adjusted to obtain the same value of γ or larger than that of others. The control parameters, together with the performance indices, are listed in Table 1. The closed-loop system responses are shown in Fig. 2, where the unit set-points changes in r_1 occur at t=0 and r_2 at t=10. It is apparent from Fig. 2 that the proposed PI controller provides a good performance with fast and well-balanced response in comparison with the other three methods. The effectiveness of the proposed PI controller is also confirmed by its smaller IAE and ITAE values given in Table 1. Model predictive control (MPC) (Bemporad et al. (2004)) is used to give the evidence of the validity of the proposed approach in practice, designed with the following tuning parameters: N=100, W=diag{0.1,0.1}, u_{min}=[0 -0.2], u_{max}=[0.3 0.2], P=10, M=4 and sample time is 0.2.

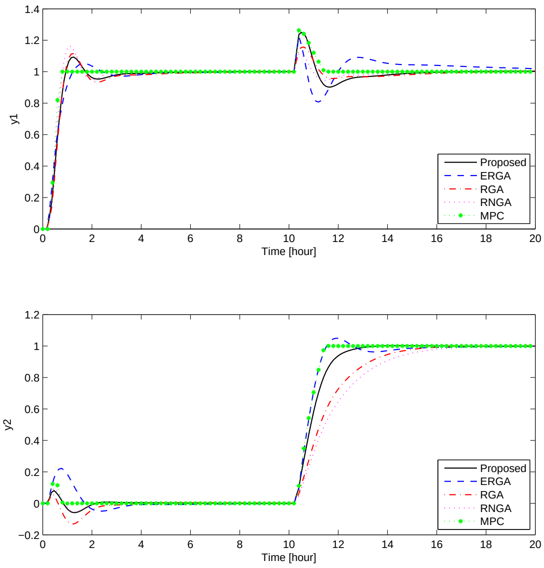

Fig. 2. Closed-loop responses to the sequential step changes in the set-point for the ISP reactor

To further investigate the robustness in comparison to different methods, a perturbation uncertainty of 40% is inserted in the process gains and time delays. As seen from Table 1, the smallest IAE and ITAE values verify that the proposed controller affords a good robust performance consistently.

Example 2. This example is cited to illustrate that the proposed method can be easily applied to a 4×4 system with satisfactory performance and robustness. This interactive 4×4 process is the experimental centralized HVAC system model of four rooms that appears in Shen et al. (2010). The transfer function of HVAC system is

$$\begin{bmatrix} \dfrac{-0.098e^{-17s}}{122s+1} & \dfrac{-0.036e^{-27s}}{149s+1} & \dfrac{-0.014e^{-32s}}{158s+1} & \dfrac{-0.017e^{-30s}}{155s+1} \\ \dfrac{-0.043e^{-25s}}{147s+1} & \dfrac{-0.092e^{-16s}}{130s+1} & \dfrac{-0.011e^{-33s}}{156s+1} & \dfrac{-0.012e^{-34s}}{157s+1} \\ \dfrac{-0.012e^{-31s}}{153s+1} & \dfrac{-0.016e^{-34s}}{151s+1} & \dfrac{-0.102e^{-16s}}{118s+1} & \dfrac{-0.033e^{-26s}}{146s+1} \\ \dfrac{-0.013e^{-32s}}{156s+1} & \dfrac{-0.015e^{-31s}}{159s+1} & \dfrac{-0.029e^{-25s}}{144s+1} & \dfrac{-0.108e^{-18s}}{128s+1} \end{bmatrix}$$

Table 1. Controller parameters and performance indices for ISP reactor

Tuning method	Loop	$k_{C,ij}$ ($j=1$)	$k_{I,ij}$ ($j=1$)	$k_{C,ij}$ ($j=2$)	$k_{I,ij}$ ($j=2$)	λ_i	γ	IAE_s Nominal	IAE_s +40%	$ITAE_s$ Nominal	$ITAE_s$ +40%
Proposed	$i=1$	0.2072	0.0543	0.2329	0.0621	0.17	0.86	2.1872	3.7300	2.0787	6.1359
	$i=2$	-0.1599	-0.0439	0.1447	0.1222	0.60					
ERGA	$i=1$	0.3137	0.0686	0.2203	0.1013	-	0.80	2.5390	4.3170	4.1459	9.6012
	$i=2$	-0.0369	-0.0204	0.2439	0.1354	-					
RGA	$i=1$	0.1644	3.8774	0.143	0.0383	0.53	0.86	2.8806	3.9438	3.8771	7.1827
	$i=2$	-0.1922	-0.0538	0.0843	0.0764	1.20					
RNGA	$i=1$	0.2402	0.0688	0.1073	0.0327	0.25	0.83	2.8953	3.8790	4.0858	6.7424
	$i=2$	-0.1792	-0.0556	0.0838	0.0643	1.50					
MPC	-	-	-	-	-	-	-	1.6975	3.1452	0.9707	6.4746

IAE_s and $ITAE_s$ denote the total sum of each loop's IAE and $ITAE$ respectively, +40% represents the plant model under +40% uncertainty.

Using equations(11-12), the controller parameters are obtained as

$$G_c(s) = \begin{bmatrix} -23.03 - 0.2244/s & 6.3731 + 0.0846/s & 0.9021 + 0.0154/s & 1.6856 + 0.0201/s \\ 7.9110 + 0.1027/s & -27.09 - 0.2478/s & 0.8901 + 0.0092/s & 0.8369 + 0.0070/s \\ 0.7810 + 0.0068/s & 1.7224 + 0.0231/s & -19.55 - 0.1892/s & 4.2471 + 0.0530/s \\ 0.9979 + 0.0109/s & 1.5886 + 0.0180/s & 3.9825 + 0.0477/s & -20.24 - 0.1746/s \end{bmatrix}$$

The decoupling tuning results presented by Shen et al. (2010) and MPC (Bemporad et al. (2004)) are applied for comparison with ours. Closed-loop time responses of the proposed method in comparison with Shen et al. (2010) and MPC are shown in Fig. 3, where unit step changes in r_1 occur at t_1=0, r_2 at t_2=1000, r_3 at t_3 = 2000 and r_4 at t_4=3000, respectively. It can be seen from Fig. 3 that the set point response is improved in the proposed method compared to that of Shen et al. (2010) and MPC (Settings: N=4000, W=diag{0.1,0.1 0.1 0.1}, P=50, M=2 and T_s=10). The IAE indices of each loop are listed in Table 2, and it is shown that the IAE values are lower for the proposed method. This indicates that response is fast and there are less oscillations in both response and interactions. For quantitative performance measurement, the sum of IAE values is listed in Table 2. The overall response in terms of IAE values is better in the case of proposed method. The performance analysis with respect to robustness shows that the proposed method gives a better robust performance compared to that of the other mentioned control techniques.

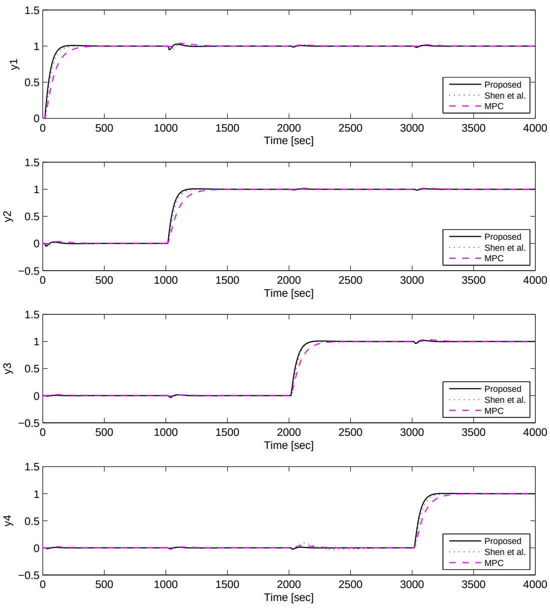

Fig. 3. Closed-loop responses to the sequential step changes in the set-point for Example 2

5. CONCLUSION

In this paper, a novel full dimensional controller for multivariable systems has been developed. Based on traditional internal model control theory, the centralized PI controller can be established directly without any equivalent transfer function matrix. Simulation results for typical industrial processes show that the proposed method provides better or compatible performance compared to other existing methods. The advantage of the proposed method is even more significant when applied to higher dimensional processes with complicated interaction modes. The method can also be easily understood and implemented by practicing engineers.

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Table 2. IAE performance for Example 2

Tuning method	Set-piont change	*IAE* y_1	y_2	y_3	y_4	λ_i	γ	IAE_s Nominal	+40%
Proposed	r_1	58.647	3.8388	1.1652	1.3763	23.50	1.04	259.8509	321.3711
	r_2	3.6672	56.815	0.9426	1.1576	19.50			
	r_3	0.7322	2.3207	60.872	2.7382	23.50			
	r_4	1.0431	1.5178	1.7023	61.314	27.00			
Shen et al.	r_1	69.869	3.9805	3.6532	1.5939	-	0.8	332.6827	388.6992
	r_2	3.9947	68.405	2.4727	1.0941	-			
	r_3	1.1152	3.3313	70.923	2.3726	-			
	r_4	1.4263	3.617	19.713	74.229	-			
MPC	-	-	-	-	-	-	-	491.5454	375.6415

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