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Synthesis of the PID controller using desired closed-loop response

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Abstract: In this paper a design method for proportional-integral-derivative (PID) controller based on internal model control (IMC) principle is proposed. A feedback controller equivalent to internal model control is obtained and then PID controller is derived by an approximate frequency response matching at two low frequency points. A simple and meaningful criterion is provided to choose such low frequency points. The method is illustrated through examples taken from literature.

Keywords: Approximated frequency response model matching; IMC controller; PID controller; process control.

1. INTRODUCTION

The most of the industrial controllers are of proportional-Integral-Derivative (PID) type till today. The performance of the control system is greatly affected by parameters of the PID controller. Many researchers have attempted to develop the design methods for the PID controller (Astrom & Hagglund, 1995). A few of them are the Ziegler-Nichols method (Ziegler & Nichols, 1942), the Cohen-Coon method (Cohen & Coon, 1953), methods based on gain margin and phase margin specifications (Ho et al., 1995), methods based on optimization of integral error criteria (Panagopoulus et al., 2002), (Visioli, 2001), an analytical tuning method which is based on finding the parameters of overall transfer function in some transformed domain to have desired set-point response (Chidambaram & Sree, 2003), method based on IMC and percentage overshoot specification (Ali & Majhi, 2009). The internal model control methods (Rivera et al., 1986), (Shamsuzzoha & Lee, 2007), (Wang et al., 2001) and direct synthesis method (Chen & Seborg, 2002) are the PID controller design methods which are based on achieving the desired closed-loop response. These design methods have only one tuning parameter and the controller required to achieve the desired response is computed analytically which are originally non-PID controllers simplification of which give PID controllers (Shamsuzzoha & Lee, 2007), (Skogestad, 2003), (Shamsuzzoha & Lee, 2008). Generally, such methods reduces the plant order before controller design.

In this paper an IMC-PID design method is proposed in which the desired PID controller is achieved by approximate frequency response matching at low frequency points. Using desired closed-loop response in internal model control architecture the desired controller in conventional unity negative feedback configuration is computed and further simplified to the PID controller by frequency response matching at two low frequency points. The method involves linear algebraic equations and approximation of the dead time term e^{-sL} is avoided.

The paper is organised as follows. The design method is discussed in Section 2, and its effectiveness is demonstrated through examples in Section 3. Conclusion is given in Section 4.

2. THE DESIGN METHOD

A plant is considered which is described by the following transfer function,

$$G_P(s) = \frac{N(s)}{D(s)}e^{-sL} = G_1(s)e^{-sL} \qquad (1)$$

where, $N(s)/D(s)$ is a rational transfer function and L is the time delay of the plant. The poles of $G_1(s)$ are considered to be in the left hand side of s-plane.

The PID controller in the parallel form is implemented as given by

$$G_C^{PID}(s) = K_P + \frac{K_I}{s} + K_D s \qquad (2)$$

where, K_P, K_I and K_D are the proportional, integral and derivative constants of the controller that are to be determined by the proposed design method.

The closed-loop block diagram is shown in Figure 1 and Figure 2 where, $G_P(s)$ is the process, $G_M(s)$ is the model of the process, $G_C^{IMC}(s)$ the IMC controller, r is the input, e is the error, u is the controller output, d is the disturbance, x is the plant input and y is the output to the plant.

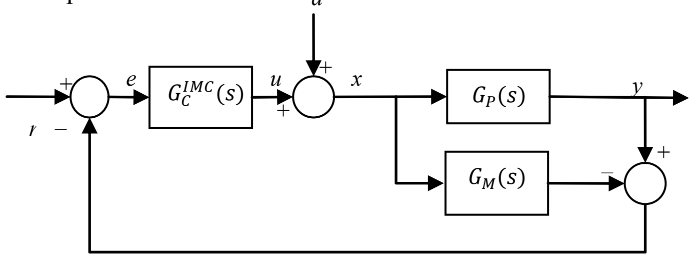

Figure 1: Block diagram of IMC control schemes.

The IMC configuration in Figure 1 is simplified to have the standard unity negative feedback control with the controller $G_C(s)$ as shown in Figure 2. The ideal feedback controller can be expressed in terms of IMC controller and the model of the process as:

$$G_C(s) = \frac{G_C^{IMC}(s)}{1 - G_M(s)G_C^{IMC}(s)} \tag{3}$$

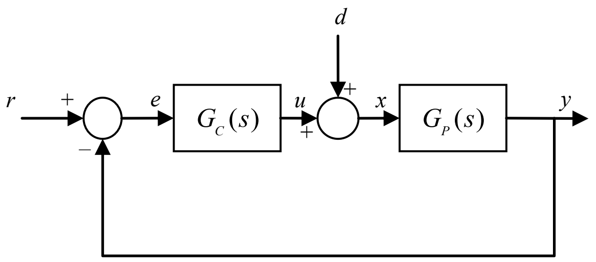

Figure 2: Closed-loop configuration considered.

The set-point response in IMC control scheme is given below

$$\frac{Y(s)}{R(s)} = \frac{G_P(s)G_C^{IMC}(s)}{1 + G_C^{IMC}(s)[G_P(s) - G_M(s)]} \tag{4}$$

For the perfect model of the plant i.e., $G_P(s) = G_M(s)$ the Equation (4) becomes

$$\frac{Y(s)}{R(s)} = G_P(s)G_C^{IMC}(s) \tag{5}$$

According to the IMC design method, the process model $G_M(s)$ is factored into two parts as:

$$G_M(s) = G^+(s)G^-(s) \tag{6}$$

where, $G^+(s)$ and $G^-(s)$ are the parts of the model that are to be inverted and not inverted by the controller, respectively. The non-invertible part includes the dead-time and right half plane zeros.

The desired closed-loop response is considered in terms of the following transfer function.

$$\frac{Y(s)}{R(s)} = \frac{G^-(s)}{\lambda s + 1} = \frac{e^{-sL}}{\lambda s + 1} \tag{7}$$

where, λ is the desired closed-loop time constant.
To achieve the desired closed-loop response the IMC controller will be

$$G_C^{IMC}(s) = \frac{1}{G^+(s)} f = \frac{1}{G^+(s)(\lambda s + 1)} \tag{8}$$

where, $f = \frac{1}{\lambda s + 1}$ is the filter in the IMC control scheme.

The controller $G_C(s)$ may be written as

$$G_C(s) = \frac{G_C^{IMC}(s)}{1 - G_M(s)G_C^{IMC}(s)} = \frac{1/G^+(s)}{(\lambda s + 1) - e^{-sL}} \tag{9}$$

It may be seen from Equation (9) that the controller $G_C(s)$ has a structure different than the PID controller and also includes a term e^{-sL}. Thus it is required that the $G_C(s)$ should be approximated to $G_C^{PID}(s)$ as

$$G_C^{PID}(s) \cong G_C(s) \tag{10}$$

$$\text{or, } G_{CR}^{PID}(\omega) + jG_{CI}^{PID}(\omega) \cong G_{CR}(\omega) + jG_{CI}(\omega) \tag{11}$$

where,

$$G_C^{PID}(s)\big|_{s=j\omega} = G_{CR}^{PID}(\omega) + jG_{CI}^{PID}(\omega)$$

$$G_C(s)\big|_{s=j\omega} = G_{CR}(\omega) + jG_{CI}(\omega)$$

and $G_{CR}^{PID}(\omega), G_{CI}^{PID}(\omega), G_{CR}(\omega)$ and $G_{CI}(\omega)$ are real functions of ω.
Separating the real and imaginary parts in Equation (11), one may write:

$$\begin{aligned} G_{CR}^{PID}(\omega) &\cong G_{CR}(\omega) \text{ and} \\ G_{CI}^{PID}(\omega) &\cong G_{CI}(\omega) \end{aligned} \tag{12}$$

In order to force the equivalence of two real functions, $G_{CR}(\omega)$ and $G_{CI}(\omega)$ with their approximants $G_{CR}^{PID}(\omega)$ and $G_{CI}^{PID}(\omega)$, respectively, one may equate appropriate number of initial few terms of the corresponding Taylor's series expansions about $\omega = 0$. Thus, to accomplish approximate matching of the L.H.S functions in Equation (12) with the corresponding functions on the R.H.S., the initial N derivatives of the corresponding functions are equated at $\omega = 0$ to give

$$\frac{d^k}{d\omega^k}\left[G_{CR}^{PID}(\omega)\right]\bigg|_{\omega=0} = \frac{d^k}{d\omega^k}\left[G_{CR}(\omega)\right]\bigg|_{\omega=0} \tag{13}$$

$$\frac{d^k}{d\omega^k}\left[G_{CI}^{PID}(\omega)\right]\bigg|_{\omega=0} = \frac{d^k}{d\omega^k}\left[G_{CI}(\omega)\right]\bigg|_{\omega=0} \tag{14}$$

where $k \in [0, N-1]$

Now, using the mathematical preliminaries given in (Pan & Pal, 1995), $G_{CR}^{PID}(\omega)$ approximately matches $G_{CR}(\omega)$ if

$$G_{CR}^{PID}(\omega)\big|_{\omega=\omega_k} = G_{CR}(\omega)\big|_{\omega=\omega_k}; \quad k \in [0, N-1] \tag{15}$$

where ω_k is the small positive values around 0. Similarly,

$$G_{CI}^{PID}(\omega)\big|_{\omega=\omega_k} = G_{CI}(\omega)\big|_{\omega=\omega_k}; \quad k \in [0, N-1] \tag{16}$$

It is clear from Equations (15) and (16) that N values of ω give $2N$ linear equations with the unknown parameters. For 3 numbers of unknowns of the PID controller N is at least 2. Thus, the parameters of the PID controller can be found by solving the equations (15) and (16). Here, it is to be noted that the method is general in the sense that the controller structure and the order can be chosen arbitrarily.

However, theoretically for frequency response the range of ω is from 0 to ∞ and for this infinite range, choosing the frequency around $\omega = 0$ i.e., of 'low value' should be consistent with the effective range of frequency response. Here, the 'low frequency' values are selected with the following concept.

If τ is the dominant time constant of the plant, 10 times of $2\pi/\tau$ which is $20\pi/\tau$ can be assumed as the effective range of dominant frequency response of the plant. Hence, the low frequency values, for the purpose of matching, can be selected at around 0.01 times or like of the effective range. Such frequency points for matching give good result for the most of the plants.

By putting such a low frequency values ω_0 and ω_1 in Equation (15) and separating the real and imaginary parts, we get the following two linear algebraic equations.

$$G_{CR}^{PID}(\omega)\Big|_{\omega=\omega_0} = K_{P1} = G_{CR}(\omega)\Big|_{\omega=\omega_0} \qquad (17)$$

$$G_{CR}^{PID}(\omega)\Big|_{\omega=\omega_1} = K_{P2} = G_{CR}(\omega)\Big|_{\omega=\omega_1} \qquad (18)$$

It is observed from various examples that, the solutions of Equations (17) and (18) are almost equal i.e., $K_{P1} \approx K_{P2}$ and we may take the value of K_P as any one of K_{P1} or K_{P2}.

Similarly, Equation (16) will give two linear algebraic equations as

$$G_{CI}^{PID}(\omega)\Big|_{\omega=\omega_0} = \frac{K_I}{\omega_0} + K_D\omega_0 = G_{CI}(\omega)\Big|_{\omega=\omega_0} \qquad (19)$$

$$G_{CI}^{PID}(\omega)\Big|_{\omega=\omega_1} = \frac{K_I}{\omega_1} + K_D\omega_1 = G_{CI}(\omega)\Big|_{\omega=\omega_1} \qquad (20)$$

The values of K_I and K_D are determined by solving the Equations (19) and (20). Thus, all the parameters of the PID controller are evaluated.

3. Examples

Example 1:

The following FOPDT process (Panda, 2008) has been considered.

$$G_P(s) = \frac{1}{s+1}e^{-0.25s}$$

Table 1: Parameters of the PID controller considering different frequency point for matching for Example 1.

S. No.	ω_0	ω_1	K_P	K_I	K_D
1	0.01	0.02	1.55	1.48	0.146
2	0.03	0.04	1.55	1.48	0.064
3	0.5	0.6	1.55	1.48	0.065
4	1.2	1.4	1.55	1.48	0.066
5	2.8	3.0	1.57	1.48	0.066

The PID controllers are designed considering the filter as $f = \frac{1}{0.425s+1}$ and for various pairs of low frequency points as shown in Table 1. In the table it is observed that the performances of the PID controller evaluated at low frequency points are almost same as long as the frequency values selected for matching are sufficiently small. Finally, the PID controller with $\omega_0 = 0.01$ rad/sec and $\omega_1 = 0.02$ rad/sec is chosen as given below:

$$G_C^{PID}(s) = 1.55 + \frac{1.481}{s} + 0.1467s$$

The process output and the controller output for set-point as well as load-disturbance with unit step input and disturbance are shown in Figure 3 and 4. The performance comparison of the proposed method with Panda (Panda, 2008) and IMC-MAC (Lee et al., 1998) is shown in Table 2. The proposed design method with $\lambda = 0.425$ (Proposed 1) gives comparable performance with the other methods while further improvement in the performance is achieved with $\lambda = 0.25$ (Proposed 2). The set-point response of the proposed method is very much attractive as compared to other methods as depicted by the figures and table. The robustness of the proposed controller is studied by considering +20% change in the gain, time constant and the time delay of the plant and the corresponding step response is shown in Figure 5.

Table 2: Performance comparison for Example 1. (M_P-peak overshoot and T_s-settling time with 2% criteria)

Method	K_P	K_I	K_D	λ	Set-point Response		Load disturbance response		IAE (for 30 sec)
					M_P (%)	T_s (sec)	Max process output	Settling time (sec)	
Proposed 1	1.55	1.48	0.146	0.425	1	1.7	0.34	4.3	1.408
Proposed 2	2.12	2.00	0.160	0.25	2	2.0	0.30	3.8	1.072
Panda	1.58	1.48	0.103	0.425	0.5	1.7	0.34	4.4	1.371
IMC-MAC	1.55	1.48	0.065	0.425	0.5	1.7	0.35	4.5	1.353

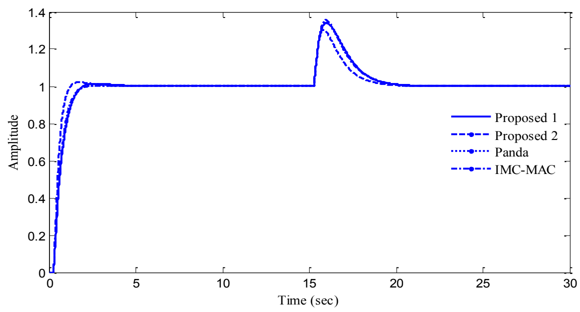

Figure 3: Process output for Example 1.

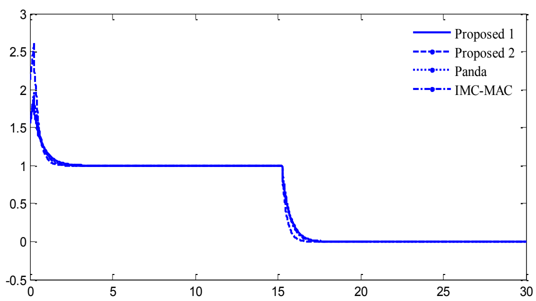

Figure 4: Controller output for Example 1.

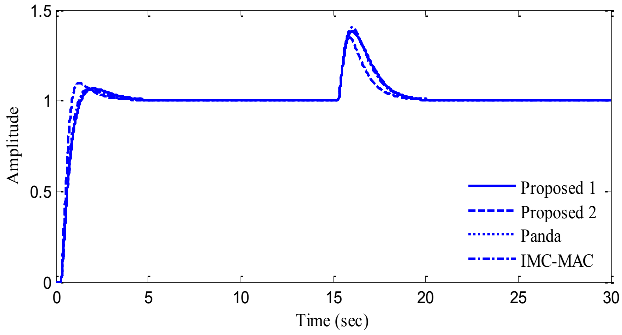

Figure 5: Process output for Example 1 with +20% change in gain, time constant and time delay of the plant.

Example 2:

A second order plus dead time (SOPDT) process is considered (Panda, 2008) as

$$G_P(s) = \frac{2}{(10s+1)(5s+1)} e^{-1s}$$

The proposed design procedure is followed and frequency points for matching are taken as $\omega_0 = 0.01$ rad/sec and $\omega_1 = 0.02$ rad/sec which results in PID controller as

$$G_C^{PID}(s) = 3.149 + \frac{0.207}{s} + 11.12s$$

The process output and the controller output by the proposed method along with that by the methods of Panda (Panda, 2008) and IMC-MAC (Lee et al., 1998) are shown in Figure 6 and 7, respectively. The various performance parameters for the comparison of the design methods are tabulated in Table 3. From Figure 6, 7 and Table 3 it is observed that the performance of the proposed method is better than the other methods. The robustness of the proposed controller is studied by considering +20% change in the gain, the dominant time constant and the time delay of the plant and the corresponding step response is shown in Figure 8.

Table 3: Performance comparison for Example 2.

Method	K_P	K_I	K_D	λ	Set-point Response		Load disturbance response		IAE (for 180 sec)
					M_P (%)	T_s (sec)	Max process output	T_s (sec)	
Proposed	3.14	0.207	11.12	1.41	12	42.5	0.22	41.4	12.31
Panda	1.93	0.130	6.123	1.41	11.5	41.5	0.36	47	15.92
IMC-MAC	1.91	0.130	5.777	1.41	11.8	40.4	0.36	46.5	15.87

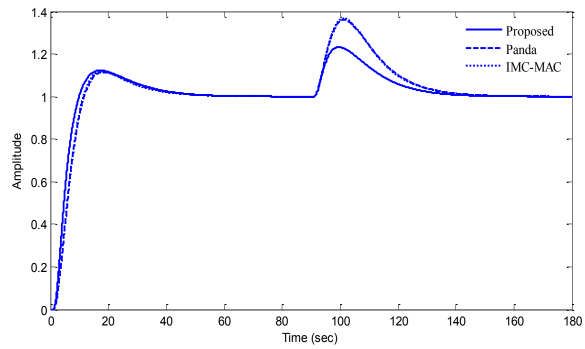

Figure 6: Process output for Example 2.

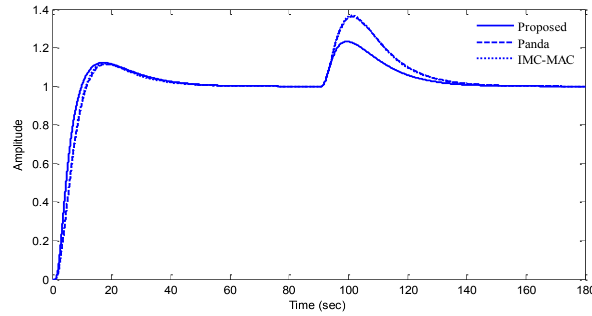

Figure 7: Controller output for Example 2.

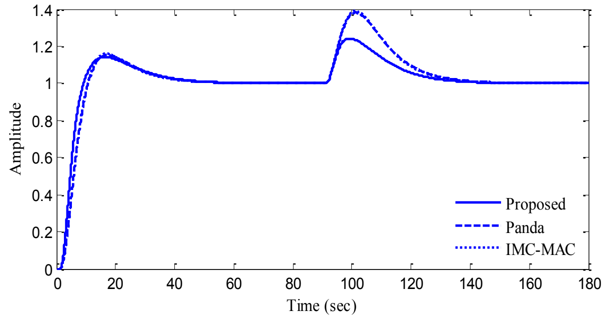

Figure 8: Process output for Example 2 with +20% change in gain, dominant time constant and time delay of the plant.

Example 3:

A third order oscillatory system with time delay is considered (Wang et al., 1999) as given by:

$$G_P(s) = \frac{1}{(s^2+2s+3)(s+3)} e^{-0.3s}$$

The PID controller is designed considering the filter as $f = 1/(1.3s+1)$ and frequency points for matching are taken as $\omega_0 = 0.1$ rad/sec and $\omega_1 = 0.2$ rad/sec which yields the controller as:

$$G_C^{PID}(s) = 5.78 + \frac{5.62}{s} + 3.66s$$

The step response of the proposed controller along with that of Wang et al (Wang et al., 1999) and Ho et al (Ho et al., 1995) for set-point as well as load disturbance is shown in Figure 9 and 10 and performance comparison is tabulated in Table 4. It is observed from the figures and the table that the proposed controller perform better in comparison with the other methods. To show the robustness of the proposed controller a +20% change in the gain and the time delay is considered and corresponding step response is shown in Figure 11.

Table 4: Performance comparison for Example 3.

Method	K_P	K_I	K_D	Set-point Response		Load disturbance response		IAE (for 30 sec)
				M_P (%)	T_s (sec)	Max process output	T_s (sec)	
Proposed	5.78	5.62	3.66	0.5	4.2	0.076	3.8	1.84
Wang et al	3.88	5.38	2.15	3.2	4.4	0.085	3.9	1.96
Ho et al	5.06	5.92	1.08	11.4	7.1	0.087	3.5	1.98

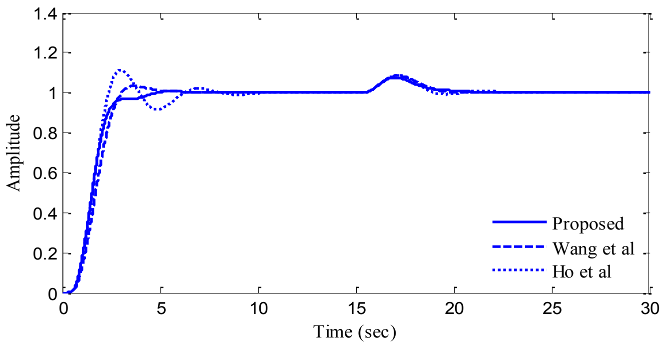

Figure 9: Process output for Example 3.

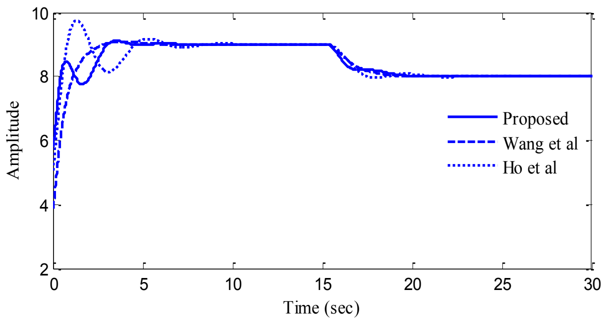

Figure 10: Controller output for Example 3.

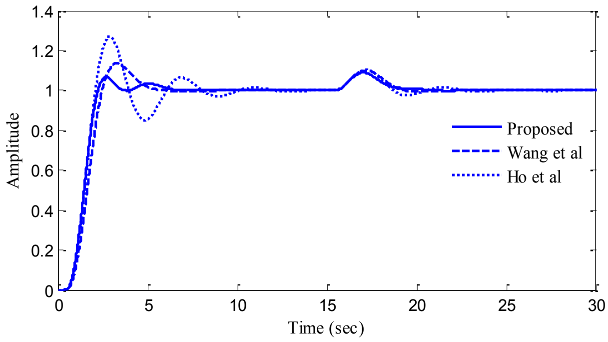

Figure 11: Process output for Example 3 with +20% change in gain and the time delay of the system.

Example 4:

A second order time delay system with inverse response is considered (Jeng & Lin, 2012) as:

$$G_p(s) = \frac{2(-3s+1)}{(2s+1)(s+1)} e^{-0.5s}$$

A PID controller is designed with frequency points for matching are taken as $\omega_0 = 0.01$ rad/sec and $\omega_1 = 0.02$ rad/sec which gives the PID controller as:

$$G_C^{PID}(s) = 0.25 + \frac{0.076}{s} + 0.213s$$

Table 5: Performance comparison for Example 4.

Method	K_P	K_I	K_D	λ	Set-point Response		Load disturbance response		IAE (for 90 sec)
					M_P (%)	T_s (sec)	Max process output	T_s (sec)	
Proposed	0.25	0.076	0.213	3	0	11.7	2.37	19.1	25.54
Jeng and Lin	0.19	0.063	0.128	4.39	0	17.6	2.17	23.5	28.88
Chien et al	0.15	0.078	0.156	-	14.5	22.5	2.14	30.5	29.89

The performance of the proposed controller is compared with that of the Jeng and Lin (Jeng & Lin, 2012) and Chien et al (Chien et al., 2003). The step responses for set-point as well as the load-disturbance response are shown in Figure 12 and 13 and various performance parameters are shown in Table 5. It is observed from the figures and the table that the proposed controller has better performance. To show the robustness of the proposed controller a +20% change in system the gain and the time delay is considered and corresponding step response for set-point as well as load disturbance response is shown in Figure 14.

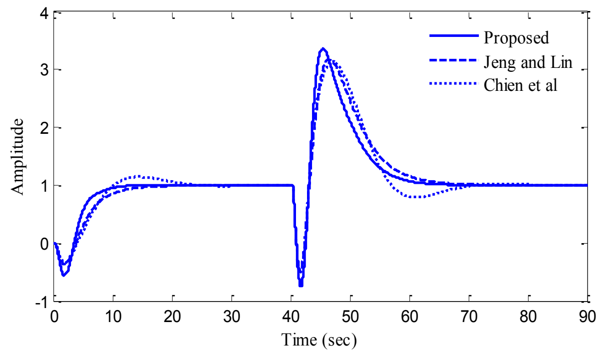

Figure 12: Process output for Example 4.

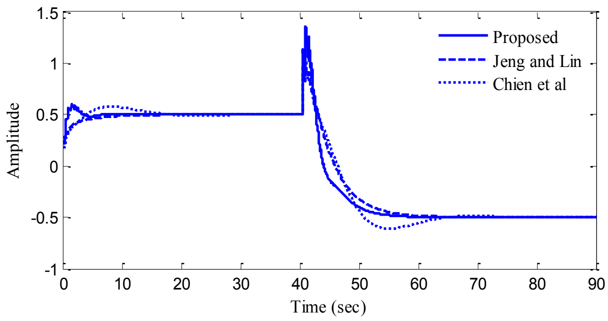

Figure 13: Controller output for Example 4.

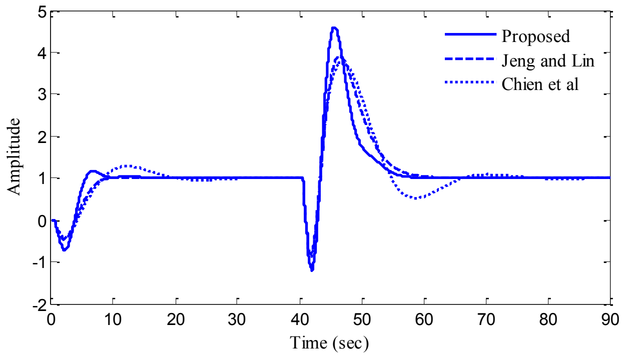

Figure 14: Process output for Example 4 with +20% change in the gain and the time delay of the plant.

4. Conclusion

A simple IMC based PID controller design has been proposed in this paper. The feedback controller equivalent to internal model control is obtained and further simplified to PID controller by an approximate frequency response matching method. Two low frequency points are required for matching the frequency response and a criterion has been provided for choosing such two frequency points that does not require elaborate frequency response analysis. The method is involved with solution of linear algebraic equations and approximation of the delay term e^{-sL} is not required. The method is mathematically simple and the computational burden is very small.

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