Preprints of the 10th IFAC International Symposium on Dynamics and Control of Process Systems
The International Federation of Automatic Control
December 18-20, 2013. Mumbai, India

Lagrangian Relaxation Based Production Optimization of Tight-formation Wells ⋆

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Abstract: Dry and semi-dry tight formation gas wells normally share the characteristic production profile defined by an initial high production, with an early steep decline and subsequent low pseudo steady-state gas rates. Small volumes of co-produced liquids will, even for dry gas wells, eventually bring the wells into the state of liquid loading, causing erratic unpredictable production rates deteriorating the performance of the wells. This state of the wells can be prevented by performing short shut-ins when the gas rate falls below the minimum rate needed to avoid liquid loading. Multi-well shut-ins may however lead to very high and low peak rates, possibly causing problems for the capacity of shared surface systems or lower and upper bounds on the total rate in a production plan. This paper presents a Lagrangian relaxation based scheme for scheduling of shut-in times for late-life tight formation gas wells with a shared gathering system. The proposed scheme includes a QP formulation for solving the Lagrangian dual, together with an aggregated construction and improvement heuristic for generating primal feasible solutions from the solution of the Lagrangian. We include several test examples to demonstrate the efficiency of the proposed decomposable scheme.

Keywords: Optimization, Scheduling, Decomposition, Plantwide control.

1. INTRODUCTION

Natural gas produced from unconventional, tight formations such as tight-sand and shales constitutes a significant part of the gas supply in North America [EIA, 2011] as well as in the Middle East and the North Africa. The exploitation of these resources is land-based, and requires a large number of wells to achieve sustainable recovery rates. The properties of these wells vary, depending on the the formation permeability, the well depth, the completion of the wells, and in particular on the presence of gas condensate or liquids in the formation. Tight formation wells still share a number characteristics [Kennedy et al., 2012], an early peak or plateau rate followed by a steep decline and a long transient decline to low pseudo steady-state production rates.

One of the major production challenges of mature tight-formation wells is related to liquid loading. This state is reached when the wells' pressure is insufficient to lift co-produced liquids to the surface [Whitson et al., 2012], causing liquid accumulation in the wellbore which increases the backpressure in the well. The accumulated liquid may be due to low saturations of water in the formation, condensates or oil, or left-over water from the hydraulic fracturing. Almost all gas wells eventually reach this state, which can be observed as low erratic unstable production rates [Al Ahmadi et al., 2010]. Common remedies for liquid loading gas wells include installing a plunger-lift or gas-lift system, or a downhole pumping unit. A different approach however, involving no installation of additional equipment, is to perform short, regular shut-ins [Whitson et al., 2012, Knudsen et al., 2012]. For low permeability wells, regular short shut-ins can be performed with almost no loss of ultimate recovery; the lower the permeability, the less is the loss of cumulative production [Whitson et al., 2012].

An illustration of a land-based gas well gathering system is given in Fig. 1, with multiple wells feeding gas into a shared compressor. Each well has its own wellhead choke, and it is quite common for each well to have a small separator for gas-water separation. Liquid or condensate rich wells would include additional separation, with storage of condensates in on-site tanks. The pressure drop from the wellhead choke to the compressor inlet is normally very small, and the wells essentially operate on a constant wellhead pressure set by the line or the compressor inlet pressure. Note that the framework in this paper is aimed on dry and semi-dry gas wells.

Production planning for the type of gas wells considered is often subject to lower and upper bounds. The bounds may both be soft, as bounds on the total rate specified by the day-to-day gas price and demands, or they can be hard bounds limited by rate handling capacities in the surface equipment. Examples on hard bounds are maximum rates in the surface lines, a lower bound on the separator inlet pressures, and maximum and minimum throughput rates in the compressors. The field development of tight

⋆ This work was supported by the Center for Integrated Operations in the Petroleum Industry, Trondheim, Norway.

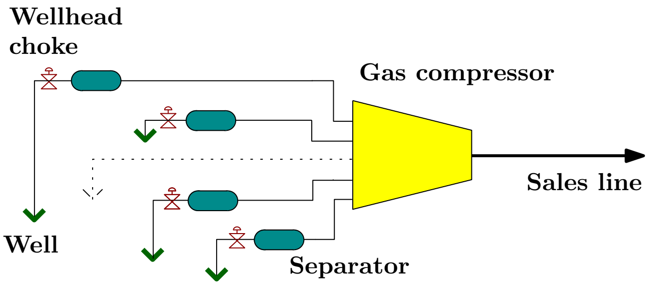

Fig. 1. Illustration of surface gathering systems.

formation assets is performed in a stepwise order: the wells are always drilled and completed sequentially over time, among other based on the success of the initial wells, while the surface equipment and infrastructure is installed in an early phase. The gas compressors are hence designed and installed with capacity for a given number of planned wells based on their expected decline curves. As the rates from the initially connected wells eventually reach low pseudo steady-state levels, there will be some excess capacity in the compressor, and new wells which would earlier be bypassed are connected to utilize the load capacity of the compressor. Generally, a lower bound on the total gas rate is also common, either due to delivery requirements in a sales contract, or to prevent compressor surge. Note that gas compression can both be performed by the operators of the wells, or the gas can be sold to mid-stream companies which compress the gas before selling it to the customers.

The subsequent peak rate after well shut-ins may be high with a steep decline, and consequently lead to particularly high and low total-rates in systems with many wells if the shut-ins are poorly scheduled. Special care must therefore be taken to prevent possible violation of total-rate bounds in a production plan or in the capacity of the compressor. This may be cast into a feasibility problem for the total rate with respect to upper and lower bounds. On the other hand, however, there is often an incentive to maximize production within the upper and lower bounds which gives rise to an optimization problem.

The problem of finding optimal shut-in times for a set of tight formation wells that maximize the production within the aforementioned bounds can be formulated as a mixed integer linear program (MILP) by using simplified well models. For this problem the use of Lagrangian relaxation provides an efficient solution technique. By dualizing complicating linking constraints, the Lagrangian problem can be easily solved as a set of independent subproblems, while its optimal value provides a best bound. Still, the technique requires an efficient method for finding optimal Lagrangian multipliers, as well as a heuristic for finding primal feasible solutions based on the solution from solving the dualized problem. In this paper, we present methods for both the latter problems.

This paper is part of an on-going research effort to increase production and recovery of oil and gas systems by applying process systems techniques [Foss, 2012]. The rest of the paper is organized as follows: in section 2 we review the single well and reservoir model together with the primal MILP formulation. Section 3 includes the main contribution and describes the Lagrangian relaxation scheme, with numerical results and concluding remarks in section 4–5.

2. PROBLEM FORMULATION

2.1 Model review

Hydraulically fractured shale and tight gas reservoirs are mainly modeled using either dual porosity system (see e.g. Al Ahmadi et al. [2010]), or as fully discretized single-porosity dual-permeability models [Cipolla et al., 2010]. The former, idealized modeling scheme is often used to derive static production forecasting tools by assuming steady-state operations, while the latter scheme typically leads to complex, numerically demanding models. To obtain a numerically simple model that captures dominating physics during shut-in operations, a single well and reservoir proxy model for dry shale-gas wells was derived in Knudsen et al. [2012]. Using a single layer, cylindrical geometry, a radially dependent permeability $k(r)$ and the integral transformation from pressure p to pseudopressure $m(p)$ [Al-Hussainy et al., 1966],

$$m(p) = 2\int_0^p \frac{p'}{\mu(p')Z(p')}dp', \tag{1}$$

the reservoir proxy model is formulated as the PDE

$$\phi\mu(p)c(p)\frac{\partial m}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(k(r)r\frac{\partial m}{\partial r}\right), \tag{2}$$

where ϕ is the porosity, $\mu(p)$ is the gas viscosity, $Z(p)$ is the gas compressibility factor and $c(p)$ is the total compressibility. The PDE (2) is defined with boundary conditions and initial conditions, using the usual Neumann boundary conditions with a producing well in the center and no-flow conditions at the outer boundary. The resulting initial/boundary-value problem is discretized in space using central difference approximations, and in time using the backward Euler scheme, or equivalently, a first order collocation on finite elements, see Knudsen et al. [2012] for details. In the design of tight-formation gas wells, a maximum rate is specified based on the surface equipment together with long-term strategic planning of the wells. Moreover, a minimum wellhead pressure is required with respect to the given line pressure. The well rate will initially, or after a shut-in, deliver a peak or plateau rate for some time until the wellhead pressure is equal to the line pressure and the rate starts to decline. Combing these expected features, a simple aggregated well and wellbore model was given in Knudsen et al. [2012], leading to the discrete time proxy model

$$Am_{k+1} = m_k + Bq_{k+1}, \tag{3a}$$

$$m_0 = m_{\text{init}}, \tag{3b}$$

$$q_k = \min\left\{q^{\max},\ \beta\left(m_{1,k} - m(e^{\text{S}}p_{\text{w}})\right)\right\}, \tag{3c}$$

where $m \in \mathbb{R}^4$ is a vector containing the pseudopressure in each grid block, q is the gas rate, $q^{\max}$ is the specified maximum rate, p_{w} is a *constant* wellhead pressure equal to the line pressure, and β and S are constants. A is a tridiagonal matrix. The model was tuned to match a high fidelity reservoir simulator for the frequency range of interest in this study, see Knudsen and Foss [2013].

A critical gas rate q_{gc} can be specified as a lower bound on q_k in order to ensure continuous removal of liquids in the wellbore. The rate q_{gc} is normally calculated by the model suggested in Turner et al. [1969]. By ensuring that the well is either shut-in or producing above q_{gc}, the wells are kept free from liquid accumulation, while we limit the valid region of our model and avoid the need to use a multiphase model for low liquid systems as dry shale and tight-gas wells.

2.2 MILP formulation

An MILP formulation for tracking a reference rate was derived in Knudsen and Foss [2013], based on the disjunction that a well must either be shut-in or producing above the critical rate q_{gc}. An exact linear reformulation of the nonsmooth well and wellbore model (3c) was derived to obtain the MILP model. The model can be applied to the problem described above with only few modifications. Let $j \in \mathcal{J}$ be the well index, and $k \in \mathcal{K}$ be the time index. Furthermore, let τ_1 and τ_2 be the minimum number of timesteps a well must be shut-in and producing, respectively, between each production and shut-in cycle. A minimum shut-in time is often required to achieve sufficient pressure build-up during shut-ins. Minimum production times are typically required from an operational point of view to avoid too frequent shut-ins with possible wear and tear of equipment, and to keep the well up-time high. The bounds of various forms described in section 1 will be treated as hard bounds on the total rate. With these considerations, we can define the following *primal* MILP model:

$$Z = \max \sum_{k\in\mathcal{K}\setminus K} \sum_{j\in\mathcal{J}} q_{jk}\,\Delta k, \tag{4a}$$

s.t.

$$\sum_{j\in\mathcal{J}} q_{jk} \le q_{\mathrm{tot}}^{\mathrm{up}}, \qquad \forall k\in\mathcal{K} \tag{4b}$$

$$\sum_{j\in\mathcal{J}} q_{jk} \ge q_{\mathrm{tot}}^{\mathrm{low}}, \qquad \forall k\in\mathcal{K} \tag{4c}$$

$$A_j m_{jk+1} = m_{jk} + B_j q_{jk+1}, \qquad \forall j\in\mathcal{J}, k\in\mathcal{K}\setminus K \tag{4d}$$

$$m_{j0} = m_{j,\mathrm{init}}, \qquad \forall j\in\mathcal{J} \tag{4e}$$

$$q_{jk} \ge y_{jk}^1 q_{\mathrm{gc}}, \qquad \forall j\in\mathcal{J}, k\in\mathcal{K} \tag{4f}$$

$$\tilde{q}_{jk} = \beta\left(m_{jk1} - m_{\mathrm{wf}}\right), \qquad \forall j\in\mathcal{J}, k\in\mathcal{K} \tag{4g}$$

$$q_{jk} \le y_{jk}^1 q_j^{\max}, \qquad \forall j\in\mathcal{J}, k\in\mathcal{K} \tag{4h}$$

$$q_{jk} \le \tilde{q}_{jk}, \qquad \forall j\in\mathcal{J}, k\in\mathcal{K} \tag{4i}$$

$$q_{jk} \ge q_j^{\max}\left(y_{jk}^1 + y_{jk}^2 - 1\right), \qquad \forall j\in\mathcal{J}, k\in\mathcal{K} \tag{4j}$$

$$q_{jk} \ge \tilde{q}_{jk} - y_{jk}^2 \tilde{q}_j^{\mathrm{M}}, \qquad \forall j\in\mathcal{J}, k\in\mathcal{K} \tag{4k}$$

$$y_{jk}^1 + y_{jk}^2 \ge 1, \qquad \forall j\in\mathcal{J}, k\in\mathcal{K} \tag{4l}$$

with the minimum shut-in and production time given by the constraints [Takriti et al., 2000]

$$y_{jk-1}^1 - y_{jk}^1 \le 1 - y_{j\rho}^1, \quad \forall j\in\mathcal{J}, k\in\mathcal{K}, \quad \rho\in[k+1, \min\{k+\tau_1-1, K\}] \tag{4m}$$

$$y_{jk}^1 - y_{jk-1}^1 \le y_{j\rho}^1, \quad \forall j\in\mathcal{J}, k\in\mathcal{K}, \quad \rho\in[k+1, \min\{k+\tau_2-1, K\}] \tag{4n}$$

where $y_{jk}^1, y_{jk}^2 \in \{0,1\}$, $\tilde{q}_j^{\mathrm{M}} := \beta_j\left(m^{\max} - m_{\mathrm{wf}}\right)$ is a big-M type parameter, Δk is the timestep and $m_{\mathrm{wf}} := m(e^{S_j} p_{\mathrm{w}})$. Initial conditions for y_{jk}^1, i.e. for $k = -1$, are given with (4m)–(4n). Note that we assume that the inlet pressure to the compressor is kept constant.

3. LAGRANGIAN RELAXATION

The $\mathcal{J}$ wells in the MILP formulation (4) are loosely coupled in the sense that the only constraints linking them together are the lower and upper total-rate constraints (4b) and (4c). A viable solution approach is hence to use a decomposition technique. Let $\nu_k \ge 0$ and $\lambda_k \ge 0$ be Lagrangian multipliers associated with the constraints (4b) and (4c), respectively. By dualizing these linking constraints, we obtain the Lagrangian relaxation

$$Z_{\mathrm{LR}}(\lambda,\nu) = \max \sum_{k\in\mathcal{K}\setminus K}\sum_{j\in\mathcal{J}} q_{jk}\,\Delta k + \sum_{k\in\mathcal{K}} \lambda_k\Big(\sum_{j\in\mathcal{J}} q_{jk} - q_{\mathrm{tot}}^{\mathrm{low}}\Big) + \sum_{k\in\mathcal{K}} \nu_k\Big(q_{\mathrm{tot}}^{\mathrm{up}} - \sum_{j\in\mathcal{J}} q_{jk}\Big). \tag{5}$$

The Lagrangian relaxation (5) decomposes the primal problem (4) into $|\mathcal{J}|$ independent subproblems,

$$Z_{\mathrm{LR}}^j(\lambda,\nu) = \max \sum_{k\in\mathcal{K}\setminus K} \left(\Delta k + \lambda_k - \nu_k\right) q_{jk} + \left(\lambda_K - \nu_K\right) q_{jK} \tag{6}$$

$$\text{s.t. eq. (4d)–(4n) for given } j\in\mathcal{J}.$$

The Lagrangian relaxation provides an upper bound on the primal optimal value Z. By duality theory, then

$$Z_{\mathrm{LB}} \le Z \le Z_{\mathrm{LR}}, \tag{7}$$

for any $\lambda, \nu \ge 0$, where Z_{LB} is the objective value of any primal feasible solution. Due to the inherent nonconvexity of the binary variables y_{jk}^1, y_{jk}^2, we may have a nonzero duality gap. The least upper bound of the Lagrangian relaxation is called the Lagrangian dual,

$$Z_{\mathrm{D}} = \min_{\lambda,\,\nu} Z_{\mathrm{LR}}. \tag{8}$$

Solving (4) by using Lagrangian relaxation is an iterative technique consisting of three major parts, illustrated in Fig. 2. Each of these parts must fulfill certain requirements for the method to be efficient: the relaxed problem (5) must be significantly easier to solve than the original problem, the Lagrangian dual (8) must be solvable in an efficient and stable manner, and based on the solution of (5) and (8) it should be possible to generate good primal *feasible* solutions using some heuristic. Finally, the duality gap should be sufficiently small when terminating the algorithm.

The chosen Lagrangian relaxation (5) clearly renders a relaxed problem that is easy to solve, as it leads to separable subproblems (6). These problems only involve maximizing the gas rate q_{jk} with given costs ν_k and λ_k generated by solving the Lagrangian dual. The dual variables represent the cost of violating the constraints (4b)-(4c). Hence, different values of ν_k and λ_k lead to different coefficients in objective function (6) of the Lagrangian subproblems, which possibly leads to different shut-in schedules y_{jk}^1 that maximizes production for each well while avoiding liquid loading.

3.1 A Lagrangian heuristic

The solution obtained by solving the Lagrangian (5) is generally infeasible with respect to the dualized constraints (4b)-(4c), i.e. it is primal infeasible. A commonly applied technique to generate primal feasible solutions is to fix the binary variables from the solution of the Lagrangian and solve the resulting LP. This is not tractable for (4), since the binary variables y_{jk}^1 represents the actual degrees

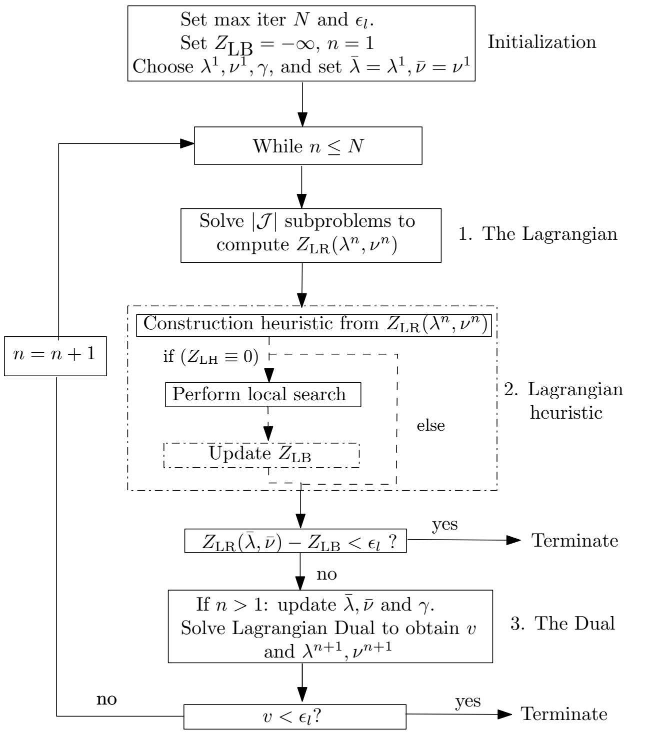

Fig. 2. Lagrangian relaxation scheme.

of freedom. We can, however, still utilize the solution of the Lagrangian (5) to heuristically generate primal feasible solutions. The following general scheme is inspired by Fischetti and Lodi [2008]. Let $\mathcal{V}^{\text{u}} \subseteq \mathcal{K}$ and $\mathcal{V}^{\text{l}} \subseteq \mathcal{K}$ be the sets of indices in which the total rate $\sum_{j \in \mathcal{J}} q_{jk}^n$ obtained by solving the Lagrangian in iteration n violates the constraints (4b) and (4c), respectively. Quite often the solution from the Lagrangian is almost primal feasible. Since its objective value provides an upper bound on Z, we would like to generate feasible solutions as close as possible to the solution of the Lagrangian. In analogy with the phase I problem in the primal Simplex algorithm, we introduce artificial variables $\sigma_k^+, \sigma_k^- \geq 0$ and define the following construction heuristic, using the solution of the Lagrangian (5) as the *starting* point for the MILP:

$$Z_{\text{LH}} = \min \sum_{k \in \mathcal{V}^{\text{u}}} \sigma_k^+ + \sum_{k \in \mathcal{V}^{\text{l}}} \sigma_k^- \tag{9a}$$

s.t.

$$\sum_{j \in \mathcal{J}} q_{jk} - \sigma_k^+ \leq q_{\text{tot}}^{\text{up}}, \qquad \forall k \in \mathcal{V}^{\text{u}} \tag{9b}$$

$$\sum_{j \in \mathcal{J}} q_{jk} + \sigma_k^- \geq q_{\text{tot}}^{\text{low}}, \qquad \forall k \in \mathcal{V}^{\text{l}} \tag{9c}$$

$$\sum_{k \in \mathcal{K} \setminus K} \sum_{j \in \mathcal{J}} q_{jk} \Delta k \geq (1 + \epsilon)\, Z_{\text{LB}} \tag{9d}$$

$$\begin{aligned} \text{eq.}\quad & (4\text{b}), && \forall k \in \mathcal{K} \setminus \mathcal{V}^{\text{u}} \\ & (4\text{c}), && \forall k \in \mathcal{K} \setminus \mathcal{V}^{\text{l}} \\ & (4\text{d}) \text{ - } (4\text{n}), && \forall j \in \mathcal{J}, k \in \mathcal{K} \end{aligned}$$

where Z_{LB} is the best known primal feasible solution, and with $\epsilon \in [0, 0.01]$. A drawback of adding the objective cut (9d) is that the starting point from the solution of the Lagrangian (5) for given λ^n, ν^n may in some cases be infeasible for (9). This is more likely in later iterations. On the other hand, the similar problem definition without including the objective cut may produce very poor feasible solutions, since the original objective is completely discarded in the modified objective (9a). We implement an adaptive reduction of ϵ, in which we set $\epsilon^{n+1} = 0.5\epsilon^n$ each time the construction heuristic terminates with $Z_{\text{LH}} \neq 0$ within the allocated CPU time.

The solution from (9) is primal feasible if $Z_{\text{LH}} \equiv 0$. Whenever this occurs, we switch to an improvement heuristic using Local branching [Fischetti and Lodi, 2003]. Given a primal (integer) feasible solution $\bar{y}_{jk}^1, \bar{y}_{jk}^2$, we then solve the primal problem MILP (4) with the additional local branching constraint

$$\begin{aligned} &\sum_{(j,k) \in \left\{(\mathcal{J} \times \mathcal{K}) : \bar{y}_{jk}^1 = 1\right\}} (1 - y_{jk}^1) + \sum_{(j,k) \in \left\{(\mathcal{J} \times \mathcal{K}) : \bar{y}_{jk}^1 = 0\right\}} y_{jk}^1 + \\ &\sum_{(j,k) \in \left\{(\mathcal{J} \times \mathcal{K}) : \bar{y}_{jk}^2 = 1\right\}} (1 - y_{jk}^2) + \sum_{(j,k) \in \left\{(\mathcal{J} \times \mathcal{K}) : \bar{y}_{jk}^2 = 0\right\}} y_{jk}^2 \leq r, \end{aligned} \tag{10}$$

where the parameter r is the neighborhood radius. Starting from the feasible solution $\bar{y}_{jk}^1, \bar{y}_{jk}^2$, we hence search in a limited neighborhood of the current solution for solutions that improve Z by allowing at most r binary variables to switch from one to zero, given by the first term in both lines, and from zero to one, given by the second term. From the recommendations in Fischetti and Lodi [2003], we set $r = 20$ and impose a maximum CPU time of 30 seconds for the improvement heuristic. If no improved solution is found within the provided time, we shrink the neighborhood by dividing the radius by two, continuing until r is less than 5 or an improved solution is found. Using a local search to improve an feasible solution obtained by solving (9) may be particularly useful for the current shut-in scheduling problem, as there may be several almost similar shut-in patterns that provide almost the same objective value. Note that the proposed heuristics for obtaining primal feasible solutions may be combined and modified in many different ways, such as in which of the two heuristics the objective cut is added, possibly using other improvements heuristics, or combining the two into one heuristic.

3.2 Solving the Lagrangian Dual

Solution techniques for solving the Lagrangian dual can generally be divided into methods based on subgradients and methods based on cutting planes. The subgradient method is easy to implement and widely used, but may require extensive tuning of the stepsize to obtain good practical convergence. Further, it lacks a true termination criteria. The basic cutting plane method has better theoretical convergence properties, but may suffer from oscillations (instability), causing slow convergence for problems with many dual variables. The problem is also unbounded for initial iterations. To cope with these limitations, many authors have suggested improvements of the cutting plane method, see Frangioni [2005] for a thorough review.

The instabilities of the cutting plane method can be reduced by adding proximity control in terms of a trust region or a stabilization term in the objective function. Many versions of this approach exist, see Hiriart-Urruty and Lemarechal [1996]. To solve (8), we adopt a cutting plane method with penalty stabilization from Hiriart-Urruty and Lemarechal [1996]. Let λ^1, ν^1 be initial multipliers, and let q^n_{jk}, $n = 1, 2, ..$ be the solutions obtained by solving the Lagrangian for given multipliers λ^n and ν^n. Furthermore, let $\bar{\lambda}, \bar{\nu} \geq 0$ be the *prox* center or the *stability* center, initially set equal to λ^1 and ν^1, and let $Z_{\text{LR}}(\bar{\lambda}, \bar{\nu})$ be the objective value of the Lagrangian for the prox center. To update the Lagrangian multipliers, we then solve the quadratic program (QP)

$$Z_{\text{D}} = \min_{\eta,\lambda,\nu} \eta + \frac{1}{2}\gamma \left(||\lambda - \bar{\lambda}||^2 + ||\nu - \bar{\nu}||^2\right) \tag{11a}$$

s.t.

$$\eta \geq \sum_{k\in\mathcal{K}\setminus K} \sum_{j\in\mathcal{J}} q^n_{jk}\, \Delta k \tag{11b}$$
$$+ \sum_{k\in\mathcal{K}} \lambda_k \Big(\sum_{j\in\mathcal{J}} q^n_{jk} - q^{\text{low}}_{\text{tot}} \Big)$$
$$+ \sum_{k\in\mathcal{K}} \nu_k \Big(q^{\text{up}}_{\text{tot}} - \sum_{j\in\mathcal{J}} q^n_{jk} \Big), \qquad n = 1, 2, ...$$

where $||\cdot||$ is the Euclidean norm and γ is a scalar penalty parameter. In the next iteration $n+1$, a new cut is added to (11) after solving the Lagrangian for λ^{n+1}, ν^{n+1}. Moreover, η defines a lower bound on Z_{D}, while the subsequent objective value of the Lagrangian $Z_{\text{LR}}(\lambda^{n+1}, \nu^{n+1})$ defines an upper bound on Z_{D} [Frangioni, 2005]. Comparing the predicted reduction of the gap $v := Z_{\text{LR}}(\bar{\lambda}, \bar{\nu}) - \eta \geq 0$ with the actual decrease of Z_{LR}, we define a step to be *serious* if

$$Z_{\text{LR}}(\lambda^{n+1}, \nu^{n+1}) \leq Z_{\text{LR}}(\bar{\lambda}, \bar{\nu}) - m_1 v, \tag{12}$$

for $m_1 \in (0, 1)$ in which the prox center is updated, i.e. $\bar{\lambda} = \lambda^{n+1}$ and $\bar{\nu} = \nu^{n+1}$, and the best upper bound is set to $Z_{\text{LR}}(\lambda^{n+1}, \nu^{n+1})$. Otherwise, the current step of (11) is declared a null-step, in which the prox center is left unchanged. The weight γ on the penalty term is updated by using the safeguarded quadratic interpolation suggested by Kiwiel [1990]. The gap v serves as a second termination criteria to the duality gap: if $v < \epsilon_l$ for some small tolerance $\epsilon_l \geq 0$, then (11) terminates with optimal multipliers $\bar{\lambda}, \bar{\nu}$.

4. COMPUTATIONAL RESULTS

The proposed Lagrangian relaxation scheme for (4) is tested on numerical examples using different number of wells $|\mathcal{J}|$ and planning horizons $|\mathcal{K}|$. We use the same realization of the single well model (3) as in Knudsen and Foss [2013], which was tuned and cross-validated against a high-fidelity multi-fracture reference model. Each well is assumed to have been operational for different lengths of time and hence have different initial conditions. We use a one-day fixed timestep, and set $\tau_1 = \tau_2 = 2$ days. The lower and upper bound on the total rate, $q^{\text{low}}_{\text{tot}}$ and $q^{\text{up}}_{\text{tot}}$, are set relative to the number of wells $|\mathcal{J}|$, since these are in practice designed according to the field and well specifications. We set $\lambda^1 = \nu^1 = 0$, initial penalty $\gamma = 10$, $m_1 = 0.05$, $\epsilon_l = 0.001$ and maximum iterations $N = 50$.

The algorithm in Fig. 2 is implemented in GAMS v23.7.3, using CPLEX v12.3 as MILP solver running up to six threads in deterministic parallel mode. All computations are performed on a Dell laptop with Intel Core i7 CPU and 8 GB of RAM. We allow a maximum CPU time of 240 seconds for the construction heuristic. Any possible infeasibility by the supplied starting point in the construction heuristic is attempted fixed by a root node repair heuristic in CPLEX. Since the construction heuristic is the most demanding in terms of computation time, we switch directly to local branching using the best feasible point and a diversification strategy with $r = r + \lceil r/2 \rceil$ whenever the construction heuristic fails to converge to $Z_{\text{LH}} \equiv 0$ for four consecutive steps.

Table 1 summarizes the results for 6, 10, 16 and 25 wells, with planning horizons from 14 to 56 days. The reported solution times are real (clock) times. We compare the results with solving (4) as one MILP using the same CPLEX settings but with 60 minutes maximum computation time. For the problems with 14 days planning horizon, the fullspace approach is clearly superior for the 6 well example, while for the example with 16 wells, the Lagrangian scheme terminates with almost the same duality gap as the fullspace approach but in one-third of the computation time. The Lagrangian scheme improves consistently compared to the fullspace approach for larger problem sizes, both in the number of wells and the length of the planning horizon. While the Lagrangian scheme terminates with relatively small duality gaps for all the large test instances in the lower part of Table 1, the fullspace approach struggles to find any feasible solutions for many of these sets. The performance of the proposed Lagrangian scheme is less efficient, however, for long planning horizons, which is seen for the example with 56 days planning horizon, where the algorithm is terminated after 3 hours, but still with a duality gap less than 3%. In the last row in Table 1, marked with a *, we show average results for the example with 16 wells and a 28 days planning horizon, using 8 different realizations in terms of different initial well conditions. The results are of the same magnitude as the corresponding results in row 6 in the table with the single realization.

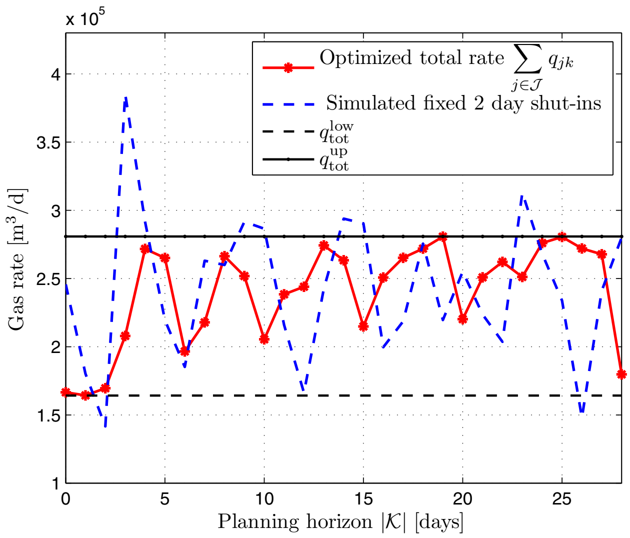

Fig. 3. The optimized total rate compared to an approach using fixed shut-in times.

In Fig. 3, the optimized total rate for the example with 16 wells and a 28 days planning horizon is compared with an approach where each well is shut-in with a fixed shut-in time of two days each time the well rate falls below the critical rate q_{gc}. Comparing the curves in Fig. 3, we

Table 1. Computational results and problem sizes.

		Lagrangian scheme			Fullspace		Problem size		
$\|\mathcal{J}\|$	K	Duality gap [%]	Time [min]	#iter	Duality gap [%]	Time [min]	Binary var.	Cont. var	Constraints
6	14	6.7	23.2	50	2.0	60	180	541	1165
10	14	2.5	20.5	50	1.5	60	300	901	1921
16	14	0.9	23.3	50	0.7	60	480	1441	3055
6	28	3.6	20.3	30	2.5	60	348	1045	2285
10	28	2.1	21.3	21	5.2	60	580	1741	3769
16	28	1.1	20.4	23	1.1	60	728	2785	5995
16	42	1.4	49.5	24	100	60	1376	4172	8935
16	56	2.5	180	47	100	60	1824	5473	11875
25	28	0.3	15.5	15	1.6	60	1450	4351	9334
25	42	0.8	68.7	18	100	60	2150	6451	13912
$\bar{16}^*$	$\bar{28}^*$	1.8	24.5	24	($15^\dagger$/1.5)	(30/60)	-	-	-

† One of the test sets has 100% duality gap at 30 min.

see how the optimized shut-in times avoid the very high and low "peak" rates, which clearly would cause feasibility problems for the lower and upper total rate constraints (4). The relative difference in cumulative production between the optimized shut-in times and the example with a fixed shut-in time is negligible, even though the optimized scheme generates long shut-in times for some of the wells. This shows that an approach using optimized, possibly longer, shut-in times is a viable production scheme.

None of the examples in Table 1 terminate with duality gaps less than ϵ_l, i.e. the first termination in Fig. 2. Although both the duality gap and the cutting-plane associated gap v are consistently reduced in the iterations, the major challenge is to reduce the upper bound, i.e. to solve the Lagrangian dual (8). Limited testing with other methods for solving the dual, including trust-region cutting plane methods and the basic subgradient method, indicates worse performance than the proximal QP (11). The tightness of the upper bound provided by solving the Lagrangian (in the case of maximization) may depend on the chosen relaxation. For the MILP (4), a different relaxation can be obtained by defining duplicate variables of m_{k+1} and applying temporal Lagrangian *decomposition*, though leading to a significant increase in the number of dual variables. Observe that the total lower and upper rate constraints may very well be time-varying. This would be particularly meaningful if varying prices and demands are taken into account in the production planning. There are also several possible formulations other than (4a)–(4c) for the problem considered: one possibility would be to minimize an infeasibility parameter added to the bounds (4b)–(4c). Finally, note that the scheme can easily be parallelized using grid computing to further reduce the computation time.

5. CONCLUDING REMARKS

This paper presents an efficient Lagrangian-based scheme for shut-in optimization of tight formation well and compressor systems. The proposed scheme is fairly general, and can possibly be applied to other applications with capacity type constraints similar to (4b)–(4c).

REFERENCES

H. Al Ahmadi, A. Almarzooq, and R. Wattenbarger. Application of Linear Flow Analysis to Shale Gas Wells - Field Cases. In *SPE Unconventional Gas Conference*, Pittsburgh, Pennsylvania, USA, 2010.

R. Al-Hussainy, H.J. Ramey Jr., and P.B. Crawford. The flow of real gases through porous media. *Journal of Petroleum Technology*, 18(5):624–636, 1966.

C. L. Cipolla, E.P. Lolon, J.C. Erdle, and B. Rubin. Reservoir Modeling in Shale-Gas Reservoirs. *SPE Reservoir Evaluation & Engineering*, 13(4):638–653, 2010.

EIA. Annual Energy Outlook 2011. Technical report, The U.S. Energy Information Administration, USA, 2011.

M. Fischetti and A. Lodi. Local branching. *Mathematical Programming*, 98(1-3):23–47, 2003.

M. Fischetti and A. Lodi. Repairing MIP infeasibility through local branching. *Computers & Operations Research*, 35(5):1436–1445, 2008.

B. Foss. Process control in conventional oil and gas fields–Challenges and opportunities. *Control Engineering Practice*, 20(10):1058–1064, 2012.

A. Frangioni. About Lagrangian Methods in Integer Optimization. *Annals of Operations Research*, 139(1): 163–193, 2005.

J-B Hiriart-Urruty and C. Lemarechal. *Convex Analysis and Minimization Algorithms: part II*. Springer, 1996.

R. L. Kennedy, W. N. Knecht, and D. T. Georgi. Comparisons and Contrasts of Shale Gas and Tight Gas Developments , North American Experience and Trends. In *SPE Saudi Arabia Section Technical Symposium and Exhibition*, Al-Khobar, Saudi Arabia, 2012.

K. C. Kiwiel. Proximity control in bundle methods for convex nondifferentiable minimization. *Mathematical Programming*, 46(3):105–122, 1990.

B. R. Knudsen and B. Foss. Shut-in based production optimization of shale-gas systems. *Computers & Chemical Engineering*, 58:54–67, 2013.

B.R. Knudsen, B. Foss, C. H. Whitson, and A. R Conn. Target-rate Tracking for Shale-gas Multi-well Pads by Scheduled Shut-ins. In *IFAC Symposium on Advanced Control of Chemical Processes*, Singapore, 2012.

S. Takriti, B. Krasenbrink, and Li. S.-Y. Wu. Incorporating fuel constraints and electricity spot prices into the stochastic unit commitment problem. *Operations Research*, 48(2):268–280, 2000.

R.G. Turner, M.G. Hubbard, and A.E. Dukler. Analysis and Prediction of Minimum Flow Rate for the Continuous Removal of Liquids from Gas Wells. *Journal of Petroleum Technology*, 21(11):1475–1482, 1969.

C. H. Whitson, S. D. Rahmawati, and A. Juell. Cyclic Shut-in Eliminates Liquid-Loading in Gas Wells. In *SPE/EAGE European Unconventional Resources Conference and Exhibition*, Vienna, Austria, 2012.