Preprints of the 12th IFAC Symposium on Computer Applications in Biotechnology
The International Federation of Automatic Control
16-18, 2013, December. Mumbai, India

Design of fractional order controller for Biochemical reactor

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Abstract: This paper presents a simple procedure to design fractional order controller based on synthesis method for the biochemical reactor. The biochemical reactor process exhibits high degree of non linearity. The process parameters will be be varying during the process of fermentation. Hence an attempt is made to design robust PI controller for the biochemical reactor to achieve high steady state productivity.

Keywords: PI controllers, Biochemical reactor, Fractional order controller.

1. INTRODUCTION

Nowadays, the fractional order PID controller finds wide applications in areas such as servo press control system [Fan et al., 2008], Network congestion control [Wenkui et al., 2011], Wind turbine system [Ricchiuto et al., 2011], Three level inverter [Tehrani et al., 2011], Two tank heater mixer setup [Madakyaru et al., 2009], Gas turbine plant [Vinopraba et al., 2012]. In the literature, the fractional order controller is applied to nonlinear process such as two degree of freedom robot manipulator, twin tank model [Delvari et. al., 2010]. Several literatures are available for analytical method of tuning fractional order controller. In time domain, model based fractional order controller is proposed for time delay process [Dazi et al., 2010]. The fractional order model predictive control algorithm has been designed and it has been shown that FO PID Dynamic Matrix control has faster tracking speed than the integer order PID Dynamic Matrix Control [Guo et al., 2010]. In frequency domain, the fractional order controller is tuned for specifications such as phase margin, gain cross over frequency and robustness to gain variation for different process transfer functions in [Li et al., 2010 and Yeroglu and Tan, 2011].

The control of the biochemical process is considered to be challenging task because the process parameters vary during the operation. In the literature, nonlinear controllers such as nonlinear Internal model controller [Henson and Seborg, 1991], fuzzy based model predictive controller [Venkateshwarlu and Naidu, 2003], nonlinear feed forward controller [Jyothi and Chidambaram, 2001], nonlinear self tuning regulator [Radhakrishnan et al., 1999] have been implemented in biochemical reactor. An attempt is made in this paper to design fractional order PI controller based on synthesis method for biochemical reactor.

The direct synthesis method is an easy and straight forward way to design the controller based on the the desired closed loop specifications. Usually the closed loop performance is specified in terms of closed loop poles. The controller can be derived directly from the first order and second order models and this method need not have a standard PI or PID structure.

2. BIOCHEMICAL REACTOR

The control of the biochemical reactor is a challenging task because the dynamics of the process vary during the course of fermentation.

2.1 Process Description

From the literature [Henson and Seborg, 1991], the model of the biochemical reactor is given by

$$\dot{X} = -DX + \mu X \tag{1}$$

$$\dot{S} = D(S_f - S) - \frac{1}{Y_{X/S}}\mu X \tag{2}$$

$$\dot{P} = -DP + (\alpha\mu + \beta)X \tag{3}$$

where X is the cell (biomass) concentration, S is the substrate concentration, P is the product concentration, D is the dilution rate, S_f is the feed substrate concentration, μ is the specific growth rate, $Y_{X/S}$ is the cell mass yield, α and β are the kinetic yield parameters of the product. The specific growth rate exhibits both substrate and product inhibition

$$\mu = \frac{\mu_m \left(1 - \frac{P}{P_m}\right) S}{K_m + S + \frac{S^2}{K_i}} \tag{4}$$

where μ_m is the maximum specific growth rate, P_m is the product saturation constant, K_m is the substrate saturation constant, K_i is the substrate inhibition constant. The control and instrumentation diagram of the biochemical reactor with X as controlled output and D as the manipulated variable is shown in Figure 1.

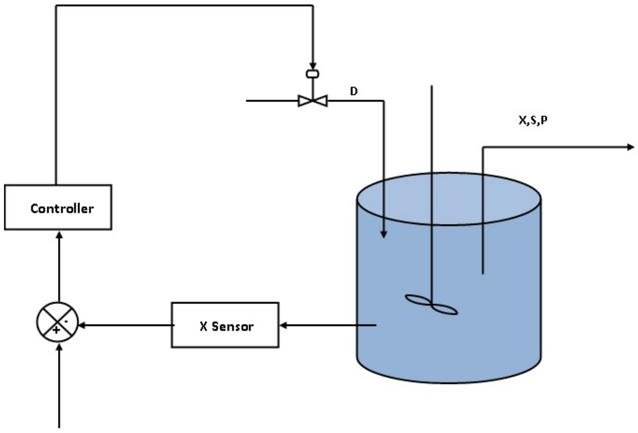

Fig. 1. Biochemical reactor.

The nominal parameters of the process are given in Table 1. The model parameters μ_m and $Y_{X/S}$ are sensitive to the changes in the operating conditions of the process. Hence, these two parameters are considered as unmeasured disturbance or uncertainties [Henson and Seborg, 1991].

Table 1. Nominal parameters of biochemical reactor

Variable	Nominal value
X	6 g/L
S	5 g/L
P	19.14 g/L
S_f	20 g/L
D	$0.202h^{-1}$
$Y_{X/S}$	0.4 g/g
α_x	2.2 g/g
β_x	$0.2h^{-1}$
μ_m	$0.48h^{-1}$
P_m	50 g/L
K_m	1.2 g/L
K_{i1}	22g/L

2.2 Modelling of the process

The control objective of the system is to maximixize the steady state productivity $\bar{Q}$, which is defined as the amount of biomass produced per unit time.

$$\bar{Q} = \bar{D}\bar{X} \tag{5}$$

where overbar represents the steady state value. The productivity is maximum for the dilution rate of 0.202 h^{-1} as shown in Figure 2. At the maximum productivity, ± 10 % step change is given in the dilution rate and the change in the biomass concentration is shown Figure 3. Using the step response data, the averaged first order model of the biochemical reactor around the operating point is found to be

$$G(s) = \frac{-41.8070}{7.38s + 1} \tag{6}$$

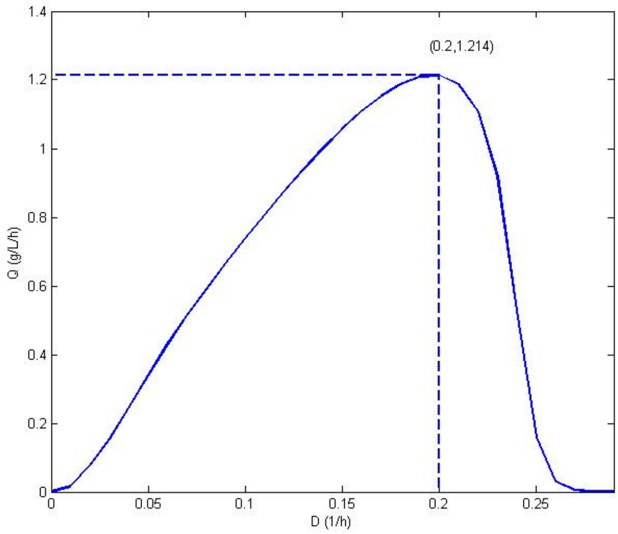

Fig. 2. Effect of the dilution rate on the productivity

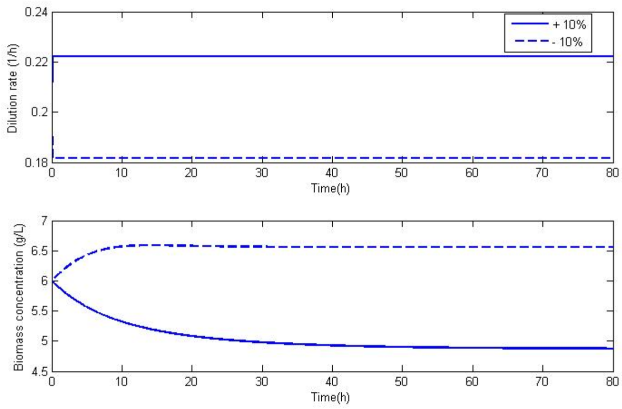

Fig. 3. Open loop for ±10% change in dilution rate

3. CONTROLLER DESIGN

Consider PI controller with fractional order filter of structure

$$G_c(s) = K_c\left(1 + \frac{1}{T_i s}\right)\left(\frac{1}{s^\alpha + 1}\right) \tag{7}$$

The characteristic equation of the closed loop system is given by

$$1 + G_c(s)G(s) = 0 \tag{8}$$

Substituting equations (6) and (7) in equation (8)

$$1 + \left(\frac{-41.8070}{7.38s + 1}\right)K_c\left(1 + \frac{1}{T_i s}\right)\left(\frac{1}{s^\alpha + 1}\right) = 0 \tag{9}$$

By synthesis method, substituting T_i=τ=7.38, equation (9) can be modifed as

$$1 + \left(\frac{-41.8070}{7.38s}\right) K_c \left(\frac{1}{s^\alpha + 1}\right) = 0 \quad (10)$$

Equation (10) can be rewritten as

$$7.38s^{\alpha+1} + 7.38s - 41.8070K_c = 0 \quad (11)$$

The desired specification is closed loop time constant is equal to the open loop time constant. On substituting $s = -\frac{1}{7.38}$, equation (11) becomes

$$(-1)\left(\frac{-1}{7.38}\right)^\alpha - 1 - 41.8070K_c = 0 \quad (12)$$

3.1 Integer order controller

For the integer order controller, α is equal to 1. On substituting the α value and solving the equation (12), the K_c is found to be -0.0211. The peak sensitivity of the closed loop system is found to be 1.0819.

3.2 Fractional order controller

For the different values of α varying from 0.1 to 3, the values of K_c is found using *fsolve* command in MATLAB. The reasonable value of peak sensitivity is given by 1.4 to 2 [Astrom and Hagglund, 1995]. An optimal value of K_c is found to be -0.0249, whose peak sensitivity is 1.407.

4. RESULTS AND DISCUSSIONS

The performance of the designed controller is compared with the performance of the IMC based PI controller designed by Henson and Seborg, 1991. The PI controller with FO filter is implemented using *ninteger* toolbox in MATLAB with CRONE approximation technique. The servo response of the system is shown in Figures 4 and 5. Figure 4 shows the positive step (set point) change in the biomass concentration from 6 g/L to 7 g/L and the corresponding dilution rate is shown. The negative step change of 6 g/L to 5 g/L is shown in Figure 5 and the respective dilution rate is also shown. The servo responses shows that the controller with FO filter has a lower settling time.

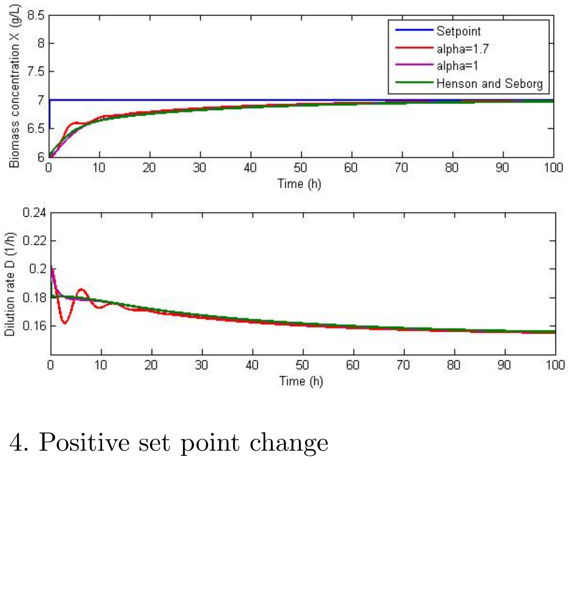

Fig. 4. Positive set point change

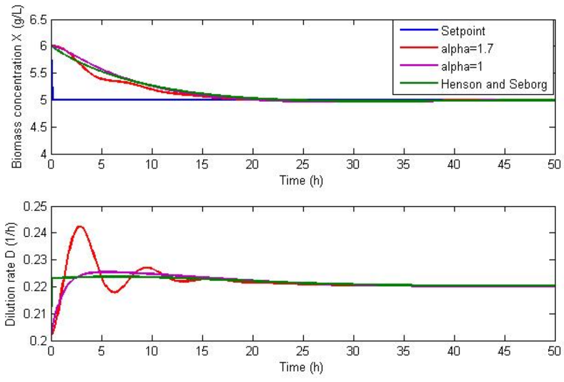

Fig. 5. Negative set point change

The regulatory response of the system is shown in Figures 6 and 7. The maximum growth rate μ_m is changed from $0.48h^{-1}$ to $0.44h^{-1}$ and the corresponding biomass concentration variation and dilution rate variation are shown in Figures 6. The cell mass yield $Y_{X/S}$ is changed from 0.4 g/g to 0.32 g/g and the corresponding biomass concentration variation and dilution rate variations are shown in Figures 7. From the figures, the fractional order controller provides better robust response than the integer order controller.

The closed loop time domain specifications of the system for the servo response is shown in Table 2. On comparing with the integer order controller, the PI controller with fractional order filter has less settling time t_s and less rise time t_r. The performance indices such as Integral Square Error (ISE), Integral Absolute Error (IAE), Integral Time Absolute Error (ITAE) of the designed controller are given in the Table 3 for the servo response. The results show that the PI controller with FO filter provides less performance indices compared to other controllers.

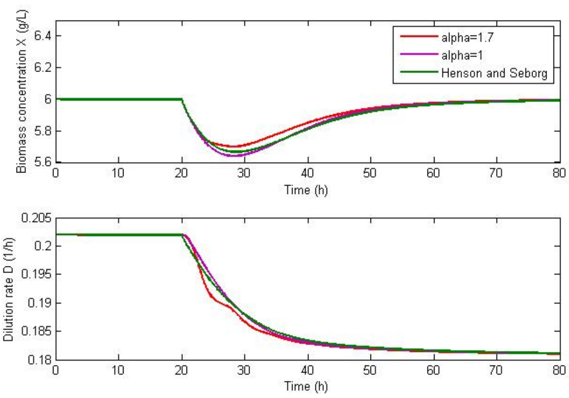

Fig. 6. μ_m disturbance

Table 2 Time domain specifications

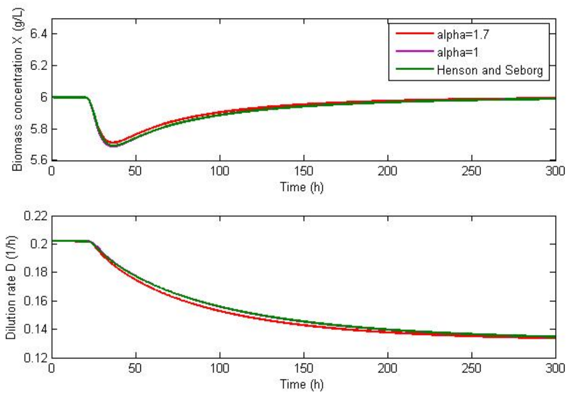

Fig. 7. $Y_{X/S}$ disturbance

Controller	Set point change	t_s (in hours)	t_r (in hours)
$\alpha = 1$	+ve	72	47
	-ve	16.9	413
$\alpha = 1.7$	+ve	74	40
	-ve	15.9	11.4
Henson and Seborg	+ve	64.1	48
	-ve	18.5	14.7

Table 3 Performance Indices

Controller	ISE	IAE	ITAE
$\alpha = 1$	4.012	24.58	2458
$\alpha = 1.7$	3.2011	20.95	1958
Henson and Seborg	3.945	24.56	2481

5. CONCLUSION

The PI controller with fractional order filter is designed using synthesis method for the nonlinear process, biochemical reactor. Results show that the fractional order controller provides better performance than the integer order controller when the process parameters varies.

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