# An application of predictive control to the Viikinmäki wastewater treatment plant

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**Abstract:** This paper deals with the development of a multivariable predictive control structure for improving the nitrogen removal of a biological wastewater treatment plant while reducing the operational costs. A simple dynamic matrix control algorithm is utilised as predictive controller and applied to a full-scale municipal wastewater treatment plant for controlling nitrogen concentrations at the end of the biological process. The complex calibrated model of the process is implemented in a commercial simulator that acts as a real-time testing platform for the proposed control structure, and allows the identification of the multivariable inputoutput model for the predictive control. Simulation results show the potentialities of the chosen predictive control, which allows the reduction of ammonia peaks in the effluent and at the same time permits a reduction of the energy consumption costs.

*Keywords:* Activated sludge process model, Wastewater treatment, Multivariable control, Dynamic Matrix Control

# 1. INTRODUCTION

During the last decades the increased consciousness on the negative impact of eutrophication has given rise to more and more stringent requirements in terms of water pollution prevention. The tightened treatment regulations are nowadays acting like a driving force for the improvement of wastewater treatment plants (WWTPs) (Olsson, 2012). The plants are in fact evolving toward efficient and safe operations with high-quality effluents, while optimizing operating and management costs. A major requirement for achieving these objectives relies on real-time automation technologies which would allow for an efficient monitoring and supervision of the process units and to implement advanced control strategies such as model predictive control (MPC) algorithms.

MPC has become an attractive control strategy for a considerable number of WWTP applications over the last decades, starting from the seminal works of Hoen et al. (1996), Steffens and Lant (1999) to Weijers (2000), Rosen et al. (2002), Sotomayor and Garcia (2002), Alex et al. (2002), Marsili-Libelli and Giunti (2002), Corriou and Pons (2004), Vrečko et al. (2004), Stare et al. (2007), Ekman (2008), and more recently Ostace et al. (2011), Vrečko et al. (2011) and O'Brien et al. (2011). This interest

is mainly due to the ability of the MPC algorithms of dealing with multivariate constrained control problems in an optimal way, through the use of simple and generally linear models. The main advantage is that MPC enables easy control of multivariable processes and handles constraints of the manipulated signals in a systematic way (Camacho and Bordons, 1999; Maciejowski, 2002).

The purpose of this work is to test the multivariable predictive controller on the activated sludge process (ASP) of a full-scale WWTP and compare its behaviour with the current control strategy implemented in the plant. As predictive controller is applied a Dynamic Matrix Control (DMC), which utilises a linear finite response process model and a constant output additive disturbance model. The choice of a DMC strategy is preferred to a relatively simpler decoupled feedback controllers configuration mainly because considered less sensitive to the optimal choice of controller set-points and advantageous for a wider range of operating conditions.

As testing platform, a structured model of the activated sludge process at the Viikinmäki WWTP is employed. The structured model is constructed and simulated by means of a commercial software, the GPS-X (Hydromantis, version 6.2) which is linked to the Matlab controller in such a way



Fig. 1. A schematic of the activated sludge process of the Viikinmäki wastewater treatment plant.

that a realistic representation of the real-world behaviour might be achieved. Particular consideration is given to the effluent ammonia and nitrate concentration by manipulating the dissolved oxygen set-points and the internal recirculation flow-rate in the bioreactor. The choice of the control structure is mainly due to the fact that (i) the ammonia-based control of the ASP can led to significant savings in the energy cost and in potential improvements in the removing ammonia (Rieger et al., 2012) and (ii) the nitrate effluent concentration needs to be guaranteed.

The paper is organised as follows. After an introductory description of the investigated unit process and the structured model used as simulation platform (Section 2), the main stages of the DMC configuration are presented and discussed (Section 3). The development of the controller is presented in details, starting from the preliminary process identification (Section 3.2) and configuration (Section 3.1), toward the simulation results in Section 4.

# 2. PROCESS DESCRIPTION AND SIMULATION PLATFORM SET-UP

The Viikinmäki wastewater treatment plant (800 000 population equivalent) is the largest WWTP in Finland. The plant is located in Helsinki and it treats an average influent flow of 250 000  $m^3 d^{-1}$ , with peaks of 800 000  $m^3 d^{-1}$ , of which about 85% is domestic and 15% industrial wastewater. The wastewater treatment consists of bar screening, grit removal, pre-aeration, primary sedimentation, activated sludge treatment (eight parallel lines of cascaded bioreactors), secondary sedimentation and a tertiary biological treatment (ten parallel lines of denitrifying post-filtration reactors). The sludge treatment is achieved with four mesophilic digesters and subsequent dewatering systems. Biogas from sludge digestion is used for electricity and heat production. Nitrogen removal is obtained in the activated sludge process and in the denitrifying post-filtration process. A total nitrogen removal of approximately 90% of yearly average is achieved.

The nitrogen removal in Viikinmäki starts in eight activated sludge lines through tapered aeration and secondary sedimentation. Each ASP line consists of cascaded bioreactors, followed by two settlers, Figure 1. The cascade of bioreactors comprises a mixing zone followed by a nonaerated zone  $(Z_1)$ , five aerated zones  $(Z_2 \text{ to } Z_6)$  and a degasing zone. After the bioreactors, activated sludge is settled at the bottom of the sequential settlers from where part is returned to maintain the concentration of activated sludge in the reactors.

The ASP lines are well monitored with hardware sensors providing on-line measurements for the most important process variables. The influent from the primary sedimentation to the ASP lines is characterized continuously by its flow-rate, ammonia  $(NH_4-N)$  and suspended solids (SS). Flow-rates of sludge recirculation from the secondary sedimentation and internal recirculation (calculated from the pump frequency values), are also reported continuously. The dissolved oxygen (DO) concentration is measured in zones  $Z_2$  to  $Z_6$  and SS concentration is analysed only in zone  $Z_6$ . The  $NH_4-N$  and nitrate-nitrogen  $(NO_3-N)$ concentrations are measured after degasing. It is worth noticing that Figure 1 point out only the on-line measurements considered relevant for the present study.

The current automation systems involves the control of the number of aerated zones, which is realized according to  $NH_4$ -N measurements and the time-delays set for switching the aeration modes. The anoxic volume can be varied by controlling the air flow in zones  $Z_1$  to  $Z_3$ , so that the number of the anoxic zones is adjustable according to treatment needs. Air flow-rate is also controlled in zones  $Z_4$  to  $Z_6$ , by means of a feedback controller with a DO target value set at  $3.5 mgL^{-1}$ .

From a modeling point of view, as the main skeleton for representing the biological reactors in GPS-X environment, the Activated Sludge Model No. 3, ASM3 (Henze et al., 2000) is utilised, whereas the secondary settlers are represented as non-reactive basins by means of the socalled Takács model (Takács et al., 1991). To calibrate the models and characterise the influent wastewater entering the ASP, a set of process measurements from the plant has been collected. The data correspond to one-year continuous operation (Jan 1, 2009 - Dec 31, 2009), recorded as hourly averages from the data acquisition system and twice-three times per week as flow proportional composite samples of wastewater from the laboratory. Table 1 summarizes the overall set of available process variables relevant to the task.

Table 1. Measurements considered in the activated sludge process model set-up.

Name	Description	Unit	
On-line data			
I-Q	Wastewater flow-rate to the ASP	$m^{3}d^{-1}$	
Z2-DO	Dissolved Oxygen in zone 2	$mgL^{-1}$	
Z3-DO	Dissolved Oxygen in zone 3	$mgL^{-1}$	
Z4-DO	Dissolved Oxygen in zone 4	$mgL^{-1}$	
Z5-DO	Dissolved Oxygen in zone 5	$mgL^{-1}$	
Z6-DO	Dissolved Oxygen in zone 6	$mgL^{-1}$	
Z6-SS	Suspended Solids in zone 6	$gL^{-1}$	
D- $NH4$	Ammonia after degas	$mgL^{-1}$	
D-NO3	Nitrate after degas	$mgL^{-1}$	
QA	Internal recycle flow-rate	$m^{3}d^{-1}$	
S1- $QR$	Return activated sludge from settler 1	$m^{3}d^{-1}$	
S2- $QR$	Return activated sludge from settler 2	$m^{3}d^{-1}$	
QW	Wasted sludge flow-rate	$m^{3}d^{-1}$	
T	Liquid temperature	$^{\circ}C$	
Laboratory analysis			
I- $COD$	Chemical Oxygen Demand to the ASP	$mgL^{-1}$	
I- $SS$	Suspended Solids to the ASP	$mgL^{-1}$	
I- $TN$	Total Nitrogen to the ASP	$mgL^{-1}$	
I- $NH4$	Ammonia to the ASP	$mgL^{-1}$	
I- $ALK$	Alkalinity to the ASP	$mgL^{-1}$	

The study started with a preliminary cleaning of the online data: unfeasible zeros and constant process values associated with saturated measurements have been removed. Possible instrument faults in the ASP analysers are cleaned off by means of a moving-window principal component analysis with a window size of one month. For reconstructing to certain extends the continuos behavior of the available scattered laboratory data for the influent concentrations, cubic spline interpolation is applied to get data with hourly sampling average.

The calibration procedure involved the definition of the stoichiometric and temperature dependent kinetic parameters of the ASM3 model and the settling parameters of the Takács model. It has been approached considering firstly the parameters given by Fred (2005), followed by ad-hoc adjustments to match the observed data at the bioreactor effluent.



(a) Ammonia after degas

(b) Nitrate after degas

Fig. 2. Winter scenario: Simulation results comparison for ammonia (a) and nitrate (b) after the degas.

The resulting calibrated model allows representing the main ASP effluent concentrations and it is considered here as the testing platform for the multivariate controller. Comparison examples between the simulated and measured ammonia and nitrate concentrations after the degasing basin are reported in Figure 2 and 3 for winter and spring, respectively. The dynamics of the simulated concentrations are appropriate and the discrepancy considered negligible.



Fig. 3. Spring scenario: Simulation results comparison for ammonia (a) and nitrate (b) after the degas.

# 3. DYNAMIC MATRIX CONTROL DEVELOPMENT

A model predictive controller in its basic formulation of dynamic matrix control is developed in the present work to control the effluent ammonia and nitrate concentrations at the exit of the bioreactor while reducing the operative costs. The controller has been implemented in Matlab and linked to the GPS-X model of the full-scale ASP at the Viikinmäki WWTP (Figure 4).



Fig. 4. Link between the process (GPS-X) and the controller (Matlab) in the simulation loop.

The basic idea of every MPC algorithms is to calculate at each control step a control sequence that minimises a certain objective function. The control sequence is calculated based on a simplified model of the process and measured outputs.

### 3.1 DMC configuration

Denoting with  $H_p$  the prediction horizon and with  $H_u$  the control horizon, the following objective function (Stare et al., 2007) is implemented in the DMC for a *m*-input and *n*-output system:

$$J = [\mathbf{e}(k+1) - \mathbf{A}\Delta\mathbf{u}(k)]^{T} [\mathbf{e}(k+1) - \mathbf{A}\Delta\mathbf{u}(k)] + [\Delta\mathbf{u}(k)]^{T} \mathbf{R}_{\Delta\mathbf{u}} [\Delta\mathbf{u}(k)] + (1) [\mathbf{u}(k) - \mathbf{u}_{\mathbf{0}}]^{T} \mathbf{R}_{\mathbf{u}} [\mathbf{u}(k) - \mathbf{u}_{\mathbf{0}}].$$

k denotes the sampling instants,  $\mathbf{e}(k+1)$  is the  $n \times H_p$ -dimensional projected error vector representing the difference between the desired input trajectory,  $\mathbf{r}(k+1)$ , and current output prediction in absence of further control action,  $\mathbf{y}^0(k)$ . The projected error is corrected by the measured outputs  $\mathbf{d}(k)$  available at the sampling instant. The vector  $\Delta \mathbf{u}(k)$  represents the  $m \times H_u$ -dimensional vector of the future control moves and it is multiplied

by the dynamic matrix  $\mathbf{A}$ , while  $\mathbf{u}(k)$  is the  $m \times H_u$ dimensional input vector. The weighting matrixes  $\mathbf{R}_{\Delta_u}$ and  $\mathbf{R}_{\mathbf{u}}$  penalise, respectively, changes in the control signal in order to avoid excessive effort on the manipulated variables and deviations of the input vector from the desired steady-state values  $\mathbf{u}_0$ .

In this study, the controlled and manipulated variables are, respectively, given by:

$$y = \begin{bmatrix} D-NH4\\ D-NO3 \end{bmatrix}; u = \begin{bmatrix} Z2-DO^{sp}\\ Z3-DO^{sp}\\ Z4-DO^{sp}\\ Z5-DO^{sp}\\ Z6-DO^{sp}\\ QA \end{bmatrix}$$
(2)

The schematic representation of the DMC configuration is given in Figure 5 where the dissolved oxygen concentrations in each bioreactor tank ( $Z_2$  to  $Z_6$ ) are controlled by simple PI feedback controllers that manipulate the airflow rate in each zone.



Fig. 5. DMC configuration for the activated sludge process of the Viikinmäki wastewater treatment plant.

# 3.2 Predictive model identification

A linear predictive model is developed and implemented in the DMC algorithm (Ogunnaike and Ray, 1994). The model is obtained by analysing the response of ammonia and nitrate concentrations after the degasing basin when a step function of different amplitudes is applied to the manipulated variables (i.e., dissolved oxygen setpoints and internal recirculation flow-rate) in the GPS-X environment. Mainly a simple first-order plus delay model has been used to describe each input-output relationship. being the only exception the step response of D-NH4 to a step in QA which has been better represented by second order with lead term plus time delay model. The model parameters have been then estimated at the different operating conditions and a set of process models is considered to represent the input-output responses at different conditions. A nominal model is subsequently assumed by simply considering the mean values of the estimated parameters. The error mismatch can be recovered because the ammonia and nitrate concentrations are measured.

#### 3.3 Tuning and implementation

The parameters related to the DMC development, such as prediction and control horizon, sampling time and weights, are found by analysing the dynamic response of the process, considering the frequency of the inputs variations and by tuning. In particular, a sampling time of 30 min is used and the prediction horizon is set to be equal to 2.5 hr (that is 5 time steps), which allows a small condition number of the matrix **A** (Garriga and Soroush, 2010). The number of control moves is set to be equal to 4 giving a good compromise between the dimension of the dynamic matrix and control efforts. The control signal remains constant during the prediction horizon and only the first control move is applied at each sampling time.

To prevent higher oxygen profile in the first bioreactor zones ( $Z_2$  and  $Z_3$ ) when the ammonia effluent does not exceed the limit values and in order to avoid excessive variation of QA from the value set in the full-scale plant, the weighting matrix  $\mathbf{R}_{\mathbf{u}}$  is set to be equal to diag[0.03 0.1 0.08 0.008 0.2 0.05].

The changes of the manipulated variables have been also penalised with a weighting matrix  $\mathbf{R}_{\Delta u}$ , which has been set to be equal to diag[0.06 0.1 0.2 0.07 0.2 0.15]. The penalisation implies the reduction on the oscillations of the control signals and minimization of energy consumption.

The desired steady-state values  $\mathbf{u}_0$  for the manipulated variables are defined in such a way that a suitable profile of dissolved oxygen inside the bioreactor is assured and the internal recycle flow-rate is the close to the value normally used at the Viikinmäki WWTP:  $\mathbf{u}_0 = [0.02 \ 0.2 \ 1.5 \ 2 \ 1 \ 43200]$ . It is worth noticing that the dissolved oxygen set-point is constrained at a maximum equal to 2.5  $mgL^{-1}$ , in fact higher values do not improve the ammonia removal efficiency, but they only increase the aeration energy consumption (Rieger et al., 2012).

Eventually, the parameters of the inner PI controllers are tuned manually based on the process step responses, in particular the proportional gain is set equally to 25 $m^3d^{-1}(mgL^{-1})^{-1}$  for every PI, as well as the integral time constant equal to 1.2 min, with a controller sampling time of 2 min.

## 4. RESULTS AND DISCUSSION

The performance of the proposed controller has been tested in terms of disturbance rejection, considering the dynamic influent data of the Viikinmäki WWTP.

The results are presented in Table 4 as comparison between the original control configuration at the Viikinmäki WWTP without the DMC and proposed control structure with the DMC. The results refer to one-year simulation with respect to the ammonia and nitrate concentrations, as well as to the overall energy and aeration costs of the activated sludge process. The original control has been implemented by providing to the GPS-X model as setpoints of dissolved oxygen and internal recycle flow-rate as they were acquired from the plant. It must be noticed that the overall costs are calculated by GPS-X as sum of the aeration and pumping energy consumptions in the activated sludge process as a whole, assuming a fixed energy price of  $0.1 \in /KWh$ . This is not related to the real consumptions at the Viikinmäki WWTP but it is used here as a mere comparison term.

An average reduction of 7% in the effluent ammonia concentration is achieved, shrinking the total and aeration costs of, respectively, 2% and 4%, for each of the eight ASP lines at the Viikinmäki WWTP. On average, the effluent nitrate concentration is cut by 1%.

 Table 2. Comparison of the DMC performance over one year simulation.

	Without DMC	With DMC
Aeration cost		
Average $[ \in d^{-1} ]$	151	145
Maximum $[ \in d^{-1} ]$	294	254
Minimum $[ \in d^{-1} ]$	69	47
Total cost		
Average $[ \in d^{-1} ]$	509	498
Maximum $[ \in d^{-1} ]$	856	782
Minimum $[ \in d^{-1} ]$	302	259
Ammonia		
Average $[mgL^{-1}]$	3.7	3.4
Maximum $[mgL^{-1}]$	17.8	15.3
Minimum $[mgL^{-1}]$	0.01	0.04
Nitrate		
Average $[mgL^{-1}]$	12.9	12.7
Maximum $[mgL^{-1}]$	24.2	19.5
Minimum $[mgL^{-1}]$	3.1	4.8

Figure 6 shows the dynamic comparison of the simulation behaviour for a period of one week during winter and spring in 2009. The controlled concentrations of ammonia and nitrate at the end of the bioreactor are compared with the simulation results obtained with the original configuration. In both conditions, the results indicate that the DMC controller is capable to reduce the ammonia peaks, while keeping the nitrate concentration roughly constant associated with a reduction of the total costs.



(c) Winter scenario - Nitrate (d) Spring scenario - Nitrate

Fig. 6. Comparison of ammonia and nitrate concentrations after degas during winter, (a) and (c), and spring (b) and (d).

Figure 7 and Figure 8 report the load of the manipulated variables, for the same winter and spring periods in 2009. It is possible to notice the effect of the controller on the ammonia peaks, which are slight reduced with an increase of the dissolved oxygen concentration in  $Z_2$  during the winter Figure 7(b) and spring Figure 8(b) scenarios, while



Fig. 7. Winter Scenario: Internal recycle flow-rate (a) and dissolved oxygen set-points in the second (b), fourth (c) and sixth (d) zone of the bioreactor.

the internal recirculation flowrate, Figures 7(a) and 8(a), is following the desired trajectory.



Fig. 8. Spring Scenario: Internal recycle flow-rate (a) and dissolved oxygen set-points in second (b), fourth (c) and sixth (d) zone of the bioreactor.

# 5. CONCLUSION

This paper presented a preliminary study for the development of a multivariable predictive controller for improving the nitrogen removal of a biological wastewater treatment plant while reducing the operative costs. The complex calibrated model of the activated sludge process is implemented in a commercial simulator that acts like a realtime testing platform for the proposed control structure. A simple dynamic matrix control algorithm is utilized for controlling nitrogen concentrations at the end of the biological process in a full-scale municipal wastewater treatment plant. The DMC performance is compared with the current control strategy implemented in the plant, which is already performing in a rather satisfactory way.

Based on the simulation results, the presented control structure has shown the potentialities of the dynamic matrix controller. It can be seen that overall the DMC reduces the dissolved oxygen set-points and it is able to cope with the imposed set-point variations by moving the system along the desired trajectory. On the other hand, a careful identification that takes into account the inherent nonlinearities of the activated sludge process is a key step in the development of the predictive controller and needs to be investigated. In fact, major improvements are expected by overcoming the limitations given by the linear predictive model. In addition, the objective function could be further improved by taking into account optimally defined values of  $\mathbf{u}_0$  and by introducing constraints for the effluent violations. This is particularly necessary, from an economical point of view, for the nitrate concentration. Effluent nitrate removal is assured in the subsequent postdenitrification unit at the Viikinmäki WWTP by dosing methanol, which represents a significant cost for the plant.

Summarizing, the application of the proposed control structure has demonstrated the benefits for the WWTP control. It has been shown that stricter regulations can be effectively enforced thought the use of robust multivariable controllers, which are able to improve the process performances allowing a reduction of the operative costs.

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