Output Feedback Passivity-Based Controller for Microalgae Cultivation

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Abstract: Microalgae are microscopic plants existing in aquatic environment. They can be used in the production of high value compounds and they have promising opportunities in energy production, wastewater treatment and fixation of carbon dioxide. In this context, the control of the cultivation requires a special attention in order to obtain high amounts of biomass. The control of microalgae cultivation is approached from passivity-based control perspectives. The proposed controller solves set point tracking and stabilizes the control loop by the passivity properties. Moreover, in order to obtain the process state variables from the measurement output, which are used in the control law, a nonlinear observer with guaranteed stability of estimation error dynamics is proposed. The design of the nonlinear controller and the observer is based on the Droop model, describing the dynamic behaviour of the microalgae process in a practical way. Finally, the performance of the nonlinear observer-based controller is shown by simulation.

Keywords: Microalgae, energy production, passivity-based controller, observer, Droop model.

1. INTRODUCTION

Over the last years, there has been an increasing interest in the culture of microalgae due to its high potential for valuable products (Benemann [1997, 2000], Richmond [2004]). The number of publications and research work to improve their yield and large scale production gives an idea of the interest for this promising field. Indeed, several applications can be highlighted from their cultivation like fixation of CO_2 , wastewater treatment, synthesis of highvalue chemicals (vitamins, health food) and production of biofuel (Benemann [1997], Carlsson et al. [2007], Huntley and Redalje [2007], Mata et al. [2010], Bernard [2011], Halim et al. [2012]). The latter is especially relevant as it appears as a serious alternative to land cultures like corn crops and doesn't make a concurrence to the food market (Chisti [2007], Surisetty et al. [2010], Mairet et al. [2011]). However, lots of challenges have to be dealt with in order to give the microalgae the opportunity to be a common route for energy production. For instance, their cell lipid content involved in the biodiesel production is favored by limiting the nitrogen source of feeding and as a consequence, the biomass production (Mairet et al. [2011]). The extension to large-scale production is not yet fully handled and advanced investigation has to be performed (Li et al. [2007], Moazami et al. [2012]). In addition, to reduce the operating cost, diversification is also indicated as combining wastewater treatment to biofuel production or mitigation systems (Wu et al. [2012]). As a consequence, keeping the culture under control at a desired value represents an important objective using suitable control method (Becerra-Celis et al. [2008b]).

In this context, it is a main preoccupation to investigate control techniques to reach the desired level of performance. Generally, the control of bioprocesses has two major problems. The first one is that only few measurements are available with a reasonable sampling period and precision. Furthermore, the presence of living organisms interacting under various biochemical phenomena leads to interdependent complex mathematical models. This is why nonlinear control techniques are favored and designed (Becerra-Celis et al. [2008b]).

In order to circumvent the problem of measuring devices, observers (also called state estimators or software sensors) can be used. They are dynamical systems which are used to estimate important process variables by means of accessible measured variables. Their design and their application in process control have been an active research area over the past decades, especially in bioprocess applications (Doyle [1998], Oisiovici and Cruz [2000], Dochain [2003], Khodadadi and Jazayeri-Rad [2011]).

In the case of microalgae, the design of state estimators has been an active research area over the past decade. For instance, Bernard et al. [2001] designed a high gain observer based on the Droop model for the monitoring of phytoplankton. They used the proposed software sensor to real experimental set up and they showed the validity and efficiency of the observer. Related to this work, Goffaux et al. [2009b] applied interval observers for this culture. Moreover, in Abdollahi and Dubljevic [2012], a movinghorizon observer is used with a model predictive controller to estimate the state process variables for the lipid production in fed-batch mode. However, these methodologies don't take into account the global convergence of the estimation error. In this study, another observer is considered to estimate the state variables. Based on the Lipschitz properties of the chosen microalgae growth model, an observer is designed and a proof of the stability of the error dynamics is given.

In order to compensate the nonlinearity and the complexity of the microalgae process, Becerra et al. (Becerra-Celis et al., 2008b]) used an input-output linearization technique to keep the biomass at a constant value for continuous cultivation. Also, in another work, they applied a nonlinear model predictive control to the culture in order to maintain the culture at an optimal population density in a constant high biomass density mode (Becerra-Celis et al. [2008a]). Furthermore, in (Abdollahi and Dubljevic [2012]), an interior point optimization and a model predictive control with moving - horizon observer are used to maximize and regulate the lipid production in a real time fed-batch microalgae cultivation. The estimator and the controller design are based on a set of linearized models in the microalgae growth process. In this work, a passivitybased control is designed as it has shown its usefulness in solving control problems for a large class of nonlinear systems. Its main features can be summarized by a simple physical interpretation, stability of the control loop, and an ease of implementation.

The article is organized as follows. In Section 2, the description of the microalgae model called Droop model is presented. We address the procedure to design the passivity-based controller as well as the nonlinear observer in Sections 3 and 4. In Section 5, simulation results are illustrated.

2. MODEL DESCRIPTION

The Droop model (Droop [1968], Lemesle and Mailleret [2008]) is a simple and widely used model which can represent the natural behavior of microalgae culture. It includes three state variables: the biomass concentration X, the internal quota Q_N , which is defined as the quantity of nitrogen per unit of biomass and the substrate concentration S. The time-varying evolution equations of the Droop model are given by:

$$\dot{X}(t) = -D(t)X(t) + \mu(Q_N)X(t)
\dot{Q}_N(t) = \rho(S) - \mu(Q_N)Q_N(t)
\dot{S}(t) = D(t)(S_{in} - S(t)) - \rho(S)X(t)$$
(1)

with $\rho(S) = \rho_m \frac{S(t)}{S(t)+K_s}$ as the specific substrate uptake rate and $\mu(Q_N) = \bar{\mu} \left(1 - \frac{K_Q}{Q_N(t)}\right)$ as the specific growth rate. In these relationships, *D* represents the dilution rate as a process input, S_{in} the input substrate concentration. In the expression of the uptake rate, K_s and ρ_m represent a half-saturation constant for the substrate and the maximum uptake rate respectively. $\bar{\mu}$ is the theoretical maximum growth rate, obtained for an infinite internal quota and K_Q the minimum internal quota allowing growth.

The parameters of the model are given in Table 1 (Goffaux et al. [2009a]).

In this study, the biomass concentration X is considered as the measured output and the dilution rate D is the control input. This choice comes from a practical point of view as biomass sensors based on optical properties are quite popular. Moreover, the dilution rate, defined by the inflow rate per unit of volume, is directly related to the quantity of nutrients supplied in the bioreactor.

2.1 Properties of the Droop model

There are two important properties for the Droop model. First, the trajectories of the Droop model are bounded and $K_Q \leq Q_N \leq Q_{N_{max}}$. Second, the Droop model is uniformly input observable with y = X if $X \neq 0$. The proof of these properties can be found in Bernard and Gouzé [1995].

3. CONTROLLER DESIGN

3.1 Passivity-based control

We consider a system given by the following nonlinear equations:

$$\dot{x} = f(x) + g(x)u,$$

$$y = h(x),$$
(2)

where $x \in \mathcal{X} \in \mathbb{R}^{n_x}$ is the state vector, $u \in U \in \mathbb{R}$ is the scalar control input and $y \in Y \in \mathbb{R}$ is the output scalar function of the system. The vector fields f(x) and g(x) are assumed to be smooth vector fields on \mathcal{X} . Moreover, a storage function $V(x) \in \mathbb{R}$ is defined such that the following condition locally holds in the region of operation of the system:

 $L_q V \neq 0,$

where $L_g V \in \mathbb{R}$ is the Lie derivative of function V along a vector field $g: L_g V = g^T(x) \frac{\partial V}{\partial x}$. Moreover, $\frac{\partial V}{\partial x} \in \mathbb{R}^{n_x}$ with $\frac{\partial V}{\partial x} = \left[\frac{\partial V}{\partial x_1}, \dots, \frac{\partial V}{\partial x_{n_x}}\right]^T$. Finally, the output y = h(x)is considered locally non-zero in this region. The time derivative of V(x) is then given by:

$$\dot{V} = L_f V + u L_g V = L_g V (u + \frac{L_f V}{L_g V}), \qquad (3)$$

The following invertible input coordinate transformation can be used to make the closed-loop system lossless (Fossas et al. [2004]): the entering energy to the system is equal to the stored energy in the system, i.e. such as $\dot{V} = y\nu$:

$$u = -\frac{L_f V}{L_g V} + \nu \frac{h}{L_g V},\tag{4}$$

So, the closed-loop system is written as:

$$\dot{x} = f(x) - g \frac{L_f V}{L_g V} + g \frac{h}{L_g V} \nu.$$
(5)

After some mathematical manipulations, the closed-loop system can be represented by :

$$\dot{x} = \Phi(x)\frac{\partial V}{\partial x} + \Gamma(x)\nu,$$

$$y = \Gamma^{T}(x)\frac{\partial V}{\partial x},$$
(6)

Table 1. Parameters of the Droop model

Parameter	Unit	Value
S_{in}	$\mu mol/L$	100
K_s	$\mu mol/L$	0.105
$ar{\mu}$	1/d	2
K_Q	$\mu mol/\mu m^3$	1.8
$ ho_m$	$\mu mol/\mu m^3/d$	9.3

where $\Phi(x)$ is a skew-symmetric matrix. The expressions of $\Phi(x)$ and $\Gamma(x)$ are given by:

$$\Phi(x) = \frac{1}{L_g V} [f(x)g^T(x) - g(x)f^T(x)],$$

$$\Gamma(x) = g(x)\frac{h(x)}{L_g V(x)}.$$

Moreover, V(x) can be considered as a quadratic energy storage function, whose gradient satisfies the following linear property:

$$\frac{\partial V(x)}{\partial x} - \frac{\partial V(x_d)}{\partial x_d} = \frac{\partial V(e)}{\partial e},$$

with $e = x - x_d$ being a tracking error and x_d is an auxiliary vector which is to be designed. Consequently, let $V(e) = V(x - x_d)$ be a positive definite storage function of e. Then,

$$\dot{V} = \frac{\partial V(e)}{\partial e^T} [\Phi(x) \frac{\partial V}{\partial x} + \Gamma(x)\nu - \dot{x_d}]$$

= $\frac{\partial V(e)}{\partial e^T} [\Phi(x) \frac{\partial V(e)}{\partial e} + \Gamma(x)\nu + \Phi(x) \frac{\partial V(x_d)}{\partial x_d} - \dot{x_d}].$ (7)

If x_d has the following time-varying dynamics with output equation:

$$\dot{x_d} = \Phi(x)\frac{\partial V(x_d)}{\partial x_d} + R_I(e)\frac{\partial V(e)}{\partial e} + \Gamma(x)\nu,$$

$$y_d = \Gamma^T(x_d)\frac{\partial V(x_d)}{\partial x_d},$$
(8)

with $R_I(e)$ which is strictly positive definite symmetric matrix, the following condition will be obtained:

$$\dot{V}(e) = \frac{\partial V(e)}{\partial e^T} [\Phi(x) - R_I(e)] \frac{\partial V(e)}{\partial e}, \qquad (9)$$
$$\dot{V}(e) = -\frac{\partial V(e)}{\partial e^T} R_I(e) \frac{\partial V(e)}{\partial e} < 0.$$

as $\frac{\partial V(e)}{\partial e^T} \Phi(x) \frac{\partial V(e)}{\partial e} = 0$. Hence, the tracking error asymptotically converges to zero and the state vector x tracks the controlled state x_d . One can remark that the evolution of the controlled state is depending on the unknown state. To handle this situation, a state observer is introduced in section 4.

3.2 Application to the microalgae process

At first, model Equations (1) are written in the form (2), with the dilution rate D as the control input u.

$$f(x) = \begin{pmatrix} \mu X \\ \rho - \mu Q_N \\ -\rho X \end{pmatrix}, \ g(x) = \begin{pmatrix} -X \\ 0 \\ S_{in} - S \end{pmatrix}, \ h(x) = X.$$

We consider the following storage function V(x) given by:

$$V(x) = \frac{1}{2}(S^2 + X^2 + Q_N^2)$$
(10)

The storage function directional derivative $L_g V$ along the control input vector field, g(x), is given by:

$$L_{q}V = S(S_{in} - S) - X^{2}$$
(11)

Based on the physical properties of the process state variables, Equation (11) never goes to zero. After some manipulations using input transformation (4), the system can be transformed in the form of Equation (6). In this form, the matrices Φ and Γ have the following forms:

$$\Phi(x) = \frac{1}{S(S_{in} - S) - X^2} \begin{pmatrix} 0 \\ -X(\rho - \mu Q_N) \\ \rho X^2 - \mu X(S_{in} - S) \end{pmatrix}$$
$$\begin{pmatrix} X(\rho - \mu Q_N) & -\rho X^2 + \mu X(S_{in} - S) \\ 0 & (\rho - \mu Q_N)(S_{in} - S) \\ (-\rho + \mu Q_N)(S_{in} - S) & 0 \end{pmatrix},$$
$$\Gamma(x) = \frac{X}{S(S_{in} - S) - X^2} \begin{pmatrix} -X \\ 0 \\ S_{in} - S \end{pmatrix}.$$

The following input coordinate transformation can make the system passive with respect to the proposed storage function V(x):

$$u = D = \frac{S\rho X - \mu X^2 - Q_N(\rho - \mu Q_N)}{S(S_{in} - S) - X^2} + \nu \frac{X}{S(S_{in} - S) - X^2}.$$
(12)

If $X_d = 0$ is considered and if R_I is a diagonal matrix, then:

$$\nu = \frac{R_1(X - X_d)(S(S_{in} - S) - X^2) + XQ_d(\rho - \mu Q_N)}{X^2} + \frac{(-\rho X^2 + \mu X(S_{in} - S))S_d}{X^2}.$$
(13)

with Q_d and S_d given by the solution of:

$$\dot{Q_d} = \frac{-X(\rho - \mu Q_N)X_d + (\rho - \mu Q_N)(S_{in} - S)S_d}{(S(S_{in} - S) - X^2)} + R_2(Q_N - Q_d),$$

$$\dot{S}_{d} = \frac{(\rho X^{2} - \mu X(S_{in} - S))X_{d} - (\rho - \mu Q_{N})(S_{in} - S)Q_{d}}{(S(S_{in} - S) - X^{2})} + R_{3}(S - S_{d}) + \frac{X(S_{in} - S)}{S(S_{in} - S) - X^{2}}\nu.$$

(14)

Finally, the final control input law, which is able to stabilize the control loop, is:

$$u = \frac{R_1(X - X_d)(S(S_{in} - S) - X^2) + XQ_d(\rho - \mu Q_N)}{XS(S_{in} - S) - X^3} + \frac{(-\rho X^2 + \mu X(S_{in} - S))S_d + S\rho X - Q_N(\rho - \mu Q_N)}{S(S_{in} - S) - X^2} + \frac{-\mu X^2}{S(S_{in} - S) - X^2}.$$

As it can be seen, all the derived equations depend on of the process state variables. Therefore, it is necessary to know the proper value of the variables. In order to estimate the process variables, a nonlinear observer is introduced in the next section.

4. OBSERVER DESIGN

There are several observer design approaches such as the high gain observer (Biagiola and Figueroa [2004]), the moving-horizon observer (Rao and Rawlings [2002]), the extended Kalman filter (Khodadadi and Jazayeri-Rad [2011]) and the extended Luenberger observer (Quintero-Marmol et al. [1991]). However, one of the difficulties in designing an observer is the proof of the global convergence of the estimation error. In this work, based on the properties of the Droop model, a nonlinear observer with guaranteed convergence of estimation error dynamics is designed. The complete explanation for the design procedure is presented in the next section.

4.1 Reformulation of the Droop model

After mathematical manipulations on the Droop model, it is possible to obtain the nonlinear model, combination of a linear and a nonlinear part satisfying the observability condition. (i.e the pair (A, C) is observable).

$$\dot{x} = A(D)x + \phi(x, D), \qquad (16)$$

$$y = CX, \qquad (16)$$

$$A(D) = \begin{pmatrix} \bar{\mu} - D & \bar{\mu} & 0 \\ 0 & -\bar{\mu} & -\rho_m \\ -\rho_m & 0 & -D \end{pmatrix}, \ C = (1, 0, 0), \qquad (16)$$

$$\phi(x, D) = \begin{pmatrix} -\bar{\mu}Q - \frac{\bar{\mu}K_QX}{Q} \\ K_Q\bar{\mu} + S\rho_m + \rho \\ DS_{in} + X(\rho_m - \rho) \end{pmatrix}.$$

As it can be seen in (16), the nonlinear part $\phi(x, D)$ in terms of the state variables is continuous and differentiable. In addition, based on the first property of the Droop model, $\phi(x, D)$ is bounded. So, it can be concluded that $\phi(x, D)$ satisfies Lipschitz property.

4.2 Nonlinear observer design for system with Lipschitz property

The nonlinear System (16) can be written by the following structure with u = D:

$$\dot{x} = A(u)x + \phi(x, u), y = Cx,$$
(17)

where $\phi(x, u)$ is a nonlinear vector field with a Lipschitz constant γ , the vector $x \in \mathbb{R}^n$ stands for the state variables and the input u represents the manipulated variable. The measured output is represented by vector y. The pair (A, C) is considered observable.

The observer is considered to be of the form:

$$\dot{\hat{x}} = A(u)\hat{x} + \phi(\hat{x}, u) + L(y - C\hat{x}).$$
 (18)

By subtracting Equation (18) from Equation (17), the estimation error dynamics $(e = x - \hat{x})$ is given by

$$\dot{e} = (A(u) - LC)e + [\phi(x, u) - \phi(\hat{x}, u)], \qquad (19)$$

where L refers to the observer gain and it can be determined by pole placement technique.

4.3 Stability analysis of the estimation error dynamics

Based on the Lipschitz property for nonlinear function, the following inequality globally would be valid:

$$\| \phi(x,u) - \phi(\hat{x},u) \| \le \gamma \| (x - \hat{x}) \|$$
 (20)

The following Lyapunov function is considered for the error dynamics (19)

$$V(e) = e^T P e, \ P = P^T > 0.$$

The time derivative of Lyapunov function is
$$\dot{V}(e) = e^{\dot{T}} P e + e^T P \dot{e},$$

$$\dot{V}(e) = [(A(u) - LC)e + \phi(x, u) - \phi(\hat{x}, u)]^T P e$$

$$+ e^T P[(A(u) - LC)e + \phi(x, u) - \phi(\hat{x}, u)],$$

$$\dot{V}(e) = e^T (A(u) - LC)^T P e + (\phi(x, u) - \phi(\hat{x}, u)^T P e$$

$$+ e^T P(A(u) - LC)e + e^T P(\phi(x, u) - \phi(\hat{x}, u)),$$

$$\dot{V}(e) = e^T ((A(u) - LC)^T P + P(A(u) - LC))e$$

$$+ 2e^T P(\phi(x, u) - \phi(\hat{x}, u)).$$

Based on the Lipschitz property of the nonlinear part, the latter can be written

$$2e^{T}P(\phi(x,u) - \phi(\hat{x},u)) \leq 2 || Pe |||| \phi(x,u) - \phi(\hat{x},u) || \leq 2\gamma || Pe |||| e ||.$$
(21)

Using the following mathematics inequality

$$2 \parallel Pe \parallel \parallel e \parallel \gamma \leq \gamma^2 e^T PPe + e^T e,$$

the derivative of Lyapunov function is

$$\dot{V} \le e^T (A(u) - LC)^T + (A(u) - LC)e + \gamma^2 e^T PPe + e^T Pe$$
(22)

$$= e^{T} ((A(u) - LC)^{T}P + P(A(u) - LC) + \gamma^{2}PP + I)e.$$

So, if the inequality (23) is satisfied, the error dynamics would be stable.

$$(A(u) - LC)^{T}P + P(A(u) - LC) + \gamma^{2}PP + I < 0.$$
(23)

Remark 1. One can note that the Riccati Equation (23) is a function of a time-varying control input, the dilution rate. In the following theorem, the existence of a solution for a Riccati equation is presented.

Theorem 2. (Zhu and Pagil [2005]) Consider the following Riccati equation

$$\mathcal{A}^T P + P \mathcal{A} + P R P + Q = 0,$$

If $P \geq 0$ is a solution of the Riccati equation, then the following conditions need to be true

$$\lambda_{\min}(R)tr(Q) - n\lambda_{\min}^2(\mathcal{S}) < 0, \qquad (24)$$

$$\lambda_{min}(\mathcal{S}) < 0, \qquad \mathcal{S} = \frac{(\mathcal{A} + \mathcal{A}^T)}{2}, \qquad (25)$$

where tr(X) and $\lambda_{min}(X)$ are the trace and the smallest eigenvalue of matrix X respectively.

Remark 3. If \mathcal{A} is Hurwitz, then $\lambda_{min}(\mathcal{S}) < 0$.

Consequently, the following procedure can be considered. Based on the pole placement technique, a gain matrix L is computed satisfying some dynamics of the linear part of the estimation error. Then, the related Riccati equation is solved greeting to Theorem2 in order to find suitable matrices Q and P. If no matrix can be obtained, then another gain matrix L should be determined.

In order to solve the resulting Riccati equation, the identity matrix is chosen for matrix R and matrix Q. Furthermore, gain matrix L is such that $\mathcal{A} = A(u) - LC$ is Hurwitz and one has $\lambda_{min}(\mathcal{S}) < 0$. Consequently, Equation (25) is satisfied. Moreover, Equation (24) can be rewritten by the following inequality:

$$1 < \lambda_{\min}^2(\mathcal{S}). \tag{26}$$

In conclusion, knowing L and P, Equation (26) and Equation (23) have to be satisfied. If not, L is modified by trial and error.

5. SIMULATION RESULTS

In order to show the effectiveness of the passivity based output feedback controller, a PI controller is designed around the operating point and compared with the non-linear observer-based controller. $R_1 = 20$, $R_2 = 10$, and $R_3 = 10$, are selected parameters for the passive controller. The eigenvalues for the linear part of the estimation error dynamics are $\lambda_1 = -8$, $\lambda_2 = -6$ and $\lambda_3 = -4$. The tuning parameters for the PI controller are $K_C = -0.037$, $K_I = -0.057$. Simulation results show that the performance is almost the same for both strategies. However, when the operating point changes, the PI controller is unable to control the process properly and the performance of the nonlinear controller is better than the linear controller as expected (Figs[1-2]).



Fig. 1. Control of biomass concentration

In Figs [3-4], the observer performance to estimate the process variables are presented. As it can be seen, it has a fast rate of convergence and after a short period of time the estimated values converge to real value. Related to robustness issues, some advanced extensions should be performed to design a solution taking into account model uncertainties. In this context, a sensitivity analysis can show that $\bar{\mu}$ and ρ_m have bigger impacts than the other parameters. Indeed, K_Q and K_s only influence the transient phase. Consequently, one can show that a 10% error on the unmeasured variables Q and S is obtained with 10% uncertainty on $\bar{\mu}$ and 3% uncertainty on ρ_m respectively. Finally, the next steps should be to develop a coupled scheme observer/controller taking into account some parameter uncertainties.



Fig. 2. Control Action (dilution rate)



Fig. 3. Estimation of the internal quota



Fig. 4. Estimation of the substrate

6. CONCLUSION

For microalgae cultivation, the regulation of the biomass concentration is studied by considering an observer-based controller. An output feedback passivity-based controller was designed to control the biomass cultivation inside the bioreactor. However, it requires the whole state vector for evaluation of the control law. The controller is combined with a nonlinear observer which was used to estimate the process variables. Based on the property of the Droop model, the stability of estimation error dynamics was demonstrated. Finally, the proposed observer and controller were coupled and simulation results showed that the controller-observer scheme has a good performance over a wide range of operation.

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