8th International IFAC Symposium on
Dynamics and Control of Process Systems
Preprints Vol.2, June 6-8, 2007, Cancún, Mexico

IDENTIFICATION OF NON-UNIFORMLY SAMPLED MULTIRATE SYSTEMS USING ORTHONORMAL BASIS FILTERS

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Abstract: In this work, we propose a novel scheme for identifying fast-rate deterministic and disturbance model from irregularly sampled multirate input-output data. Generalized Orthogonal Basis Functions (GOBF) are used for representing model dynamics. In the first step, we identify MISO output error (OE) type models using irregularly sampled outputs and inputs changing at the fast rate. To capture the dynamics of unmeasured disturbances, we propose to develop AR type models with time varying coefficients using the irregularly sampled model residuals generated in the first step. The efficacy of the proposed identification technique is demonstrated by simulation studies on Shell control problem.

Keywords: Irregular sampling, Multi-rate systems, Orthonormal Basis Filters, Inferential estimation

INTRODUCTION

Multirate sampled data system, in which different variables are sampled at different rates, are common in the chemical process industries. This is because it is difficult to acquire measurement at a high frequency for quality variables such as composition, density and molecular weight distribution. On the other hand, process variables such as temperature, pressure and flowrate can be readily sampled at a high frequency. For satisfactory control of such processes, it is required that the quality variable also be estimated, directly or indirectly, at higher frequency. One prominent approach for estimation of slow sampled variable is based on Kalman Filter, which requires first principle model of system. Deriving and using first principle model may become prohibitively difficult for large scale chemical systems. Alternately, it is possible to identify dynamic models from input-output data and subsequently these models can be used for designing a fast rate estimator. There has been substantial research in the field of inferential estimation where frequently available process variable are used to generate estimation of slowly sampled process variables, over last three decades. (Joseph and Brosilow, 1978; Stephanopoulos and San, 1984; Gudi et al., 1995). Above methods have considered only uniformly sampled multirate systems and their extension to more relevant and important irregularly sampled case is not obvious, particularly when black box model is used.

In recent years, there has been growing interest among researchers in area of identifying fast rate model from multirate sampled data. Lu and Fisher (1989), presented a recursive least square estimation of parameters in an output error model structure for multirate systems. They proposed to update model parameters only when the output measurements are available, otherwise the same model is used to estimate output at the intermediate time points. Li et al. (2000), used lifting technique to convert multirate systems to single rate systems which, subsequently, is identified using subspace method. The identified lifted model is then converted to fast single rate model. In this approach, the only case where the output is sampled at a slower rate than inputs was considered. Wang et al. (2004), proposed a modified scheme for computation of fast rate model from lifted model to overcome the above limitation.

The problem of identifying optimal models, when some of variables are irregularly sampled, has been studied in statistics using Expectation Maximization (EM) approach. Shumway and Stofferm (1982), considered the application of EM algorithm for state space identification in presence of missing data for the case with no manipulated inputs, i.e. the time series model identification problem. Tanaka and Katayama (1990), outlined a procedure to identify time series state space model in presence of outlier, which are treated as missing data. Isaksson (1993), used modified form of Kalman filtering for reconstruction of missing value and then EM algorithm is used to identify ARX model. Raghavan et al. (2005), presented EM based approach to identify state space model for systems with irregularly sampled outputs. The EM algorithm generates maximum likelihood estimate of the state space model. However, the main difficulty with the EM algorithm is that it requires good initial guess for the deterministic and stochastic model to ensure convergence.

Multirate systems, in which inputs are manipulated regularly and output are measured irregularly at sampling rate which is an integer multiple of the input sampling rate, are considered in this work. We propose a novel scheme for identifying fast-rate deterministic and disturbance model from irregularly sampled multirate input-output data. In recent years, there has been a growing interest in the use of Orthogonal Basis Functions (OBFs) for representing process dynamics (Ninness and Gustafsson, 1997). Recently, Patwardhan and Shah (2005) have proposed a GOBF based two tier scheme for identification of state space models using fast rate data. We show that their approach can be extended to extract a fast rate OE type model from the irregularly sampled multirate data. In the first step, we identify MISO output error (OE) type models using irregularly sampled outputs and inputs changing at the fast rate. To capture the dynamics of unmeasured disturbances, we propose to develop AR type models with time varying coefficients using the irregularly sampled model residuals generated in the first step. Recently, Srinivasrao et al. (2007) have proposed a similar scheme for identifying nonlinear time series models from irregularly sampled multirate data. The efficacy of the proposed identification technique is demonstrated by simulation studies on Shell control problem.

This paper is organized as follow. In next section, we briefly introduce GOBF parameterization of transfer function and then parameter estimation for deterministic model and disturbance model is explained. In section 3, results of case studies on shell control problem, Tennessee Eastman problem and four tank setup is presented. The section 4 ends this report with conclusion.

1. MODELLING AND IDENTIFICATION OF MULTIRATE SYSTEM

The information available from the plant is the sampled sequence of input ($\mathbf{u}$) and output ($\mathbf{y}$) vectors. It is assumed that

- Sampling rates for all the measurements are integer multiples of some time period called 'shortest time unit' (T).
- All the actuators are manipulated at a frequency corresponding to the 'shortest time unit' (T).
- The process under consideration is a fading memory system and does not have any unstable/integrating modes.

Thus, the manipulated inputs are changed at $\{t_k = kT : k = 0, 1, 2, ...\}$ where sampling instants $\{t_k\}$ are called as *minor sampling instants*. The measurements of i^{th} output are assumed to be available only at sampling instants given by the sub-sequence $\{k_{i,0}, k_{i,1}, k_{i,2,}...\}$, called as *major sampling instants*, such that the difference $k_{i,l} - k_{i,l-1} = q_{i,l}$ (> 1) where $q_{i,l}$ is an integer. When $q_{i,l}$ is constant and independent of l, the output data is a regularly sampled multirate system, else, the system is an irregularly sampled system.

In this work, given a MIMO process consisting of r outputs and m inputs, we propose to develop r MISO deterministic OE type models as well as r SISO/MISO stochastic models.

Given input sequence $\{\mathbf{u}(k) : k = 0, 1, 2, ...N\}$ and the corresponding irregularly sampled output data $\{\mathbf{y}_i(k_{i,l}) : k_{i,l} = k_{i,0}, k_{i,1}, k_{i,2}, ..., k_{i,n}\}$ collected from a plant where $k_{i,l}$ represents sampling instants for i^{th} process output, we propose to identify OE type of fast rate model of the form,

$$\hat{\mathbf{y}}_i(k) = \sum_{j=1}^{n_u} \mathbf{G}_{ij}(q)\mathbf{u}_j(k) \tag{1}$$

$$\mathbf{y}_i(k_{i,l}) = \hat{\mathbf{y}}_i(k_{i,l}) + \hat{\boldsymbol{\upsilon}}_i(k_{i,l}) \tag{2}$$

where $\hat{\mathbf{y}}_i(k)$ is noise free component of the process output and $\{\widehat{\boldsymbol{v}}_i(k_{i,l})\}$ is a zero mean colored noise in the output. It may be noted that $\{\widehat{\boldsymbol{v}}_i(k_{i,l})\}$ is available at irregular sampling instants and it is difficult to use conventional fixed sampling time models for modelling such a stochastic process. To capture dynamics of the stochastic process $\{\widehat{\boldsymbol{v}}_i(k_{i,l})\}$, we develop an AR type model with time varying coefficients.

1.1 Deterministic Model

Under the assumption that $\mathbf{G}_{ij}(q)$ in Eq.(1) is strictly proper and stable, it can be approximated using discrete time GOBF as,

$$\mathbf{G}_{ij}(q) \simeq \sum_{p=1}^{n_{ij}} \mathbf{c}_{ijp} F_{ijp}(q) \tag{3}$$

where $F_{ijp}(q)$ represents orthonormal basis filter. The resulting MISO model can be expressed in the following state space form (Patwardhan and Shah, 2005)

$$\mathbf{X}_u^{(i)}(k+1) = \mathbf{\Phi}_u^{(i)}(\boldsymbol{\xi}_u^{(i)})\mathbf{X}_u^{(i)}(k) + \mathbf{\Gamma}_u^{(i)}(\boldsymbol{\xi}_u^{(i)})\mathbf{u}(k) \tag{4}$$

$$\widehat{\mathbf{y}}_i(k) = \boldsymbol{\theta}_u^{(i)}\,\mathbf{X}_u^{(i)}(k) \tag{5}$$

where $\boldsymbol{\xi}_u^{(i)}$ is vector of poles of discrete time GOBF, $\widehat{\mathbf{y}}_i(k)$ is estimated output at the fast rate and $\boldsymbol{\theta}_u^{(i)}$ is a vector consisting of expansion coefficients, $\{\mathbf{c}_{ijp}\}$. In the multirate scenario under consideration, the measured output can be expressed as follows

$$\mathbf{y}_i(k_{i,l}) = \hat{\mathbf{y}}_i(k_{i,l}) + \widehat{\boldsymbol{v}}_i(k_{i,l}) \tag{6}$$

Unknown parameters in Eq.(4), $\boldsymbol{\xi}_u^{(i)}$ and $\boldsymbol{\theta}_u^{(i)}$, are estimated by minimizing 2-norm of the residuals, $\widehat{\boldsymbol{v}}_i(k_{i,l}) = \mathbf{y}_i(k_{i,l}) - \hat{\mathbf{y}}_i(k_{i,l})$. Following the two tier optimization scheme proposed by Patwardhan and Shah (2005), the parameter estimation problem for the i^{th} MISO model can be stated as,

$$(\widehat{\boldsymbol{\xi}}_u^{(i)}, \widehat{\boldsymbol{\theta}}_u^{(i)}) = \arg\min_{\boldsymbol{\xi}_u^{(i)}} \left[\frac{1}{N_{y_i}} \sum_{k=k_{i,1}}^{N_{y_i}} \widehat{\boldsymbol{v}}_i(k, \boldsymbol{\xi}_u^{(i)}, \widehat{\boldsymbol{\theta}}_u^{(i)})^2 \right]$$

$$\text{Subject to} \tag{7}$$

$$\widehat{\boldsymbol{\theta}}_u^{(i)} = \overline{E}\left[\mathbf{X}_u^{(i)}(k_{i,l})\mathbf{X}_u^{(i)}(k_{i,l})^T\right]^{-1} \tag{8}$$

$$\times \overline{E}\left[\mathbf{X}_u^{(i)}(k_{i,l})\mathbf{y}_i(k_{i,l})\right] \tag{9}$$

$$\left|\boldsymbol{\xi}_{u,p}^{(i)}\right| < 1 \qquad \text{for } p = 1, ...n \tag{10}$$

The proposed decomposition strategy to estimate only the GOBF poles by nonlinear iterative search and the GOBF expansion coefficients analytically is based on the fact that every guess of GOBF poles generates a valid orthonormal basis for the set of all stable transfer functions. Approximate knowledge of dominant poles of the system under consideration can be used to initialize the above nonlinear optimization problem.

1.2 Unmeasured Disturbance Model

The next step is to estimate a model for the unmeasured disturbances. To begin with, we develop a SISO model and latter show how it can be extended to deal with multivariable scenario.

Consider signal $\{\widehat{\boldsymbol{v}}_i(k_{i,l})\}$, which is available at irregularly sampling intervals. We propose a discrete AR type model with time varying coefficient matrices as follows

$$\mathbf{x}_v^{(i)}(k_{i,l+1}) = \mathbf{\Phi}_v^{(i)}(k_{i,l+1}, k_{i,l})\ \mathbf{x}_v^{(i)}(k_{i,l}) \tag{11}$$

$$+\mathbf{\Gamma}_v^{(i)}(k_{i,l+1}, k_{i,l})\ \widehat{\boldsymbol{v}}_i(k_{i,l}) \tag{12}$$

$$\widehat{\boldsymbol{v}}_i(k_{i,l+1}) = \boldsymbol{\theta}_v^{(i)}\,\mathbf{x}_v^{(i)}(k_{i,l+1}) + e(k_{i,l+1}) \tag{13}$$

where

$$\mathbf{\Phi}_v^{(i)}(k_{i,l+1}, k_{i,l}) = \exp\left[-\mathbf{A}_v^{(i)}T\,(k_{i,l+1} - k_{i,l})\right]$$

$$\mathbf{\Gamma}_v^{(i)}(k_{i,l+1}, k_{i,l}) = \int_{k_{i,l}T}^{k_{i,l+1}T} \exp\left[-\mathbf{A}_v^{(i)}\tau\right]\mathbf{B}_v^{(i)}d\tau$$

Here, matrices $\mathbf{A}_v^{(i)}$ and $\mathbf{B}_v^{(i)}$ can be viewed as coefficient matrices of a continuous state space model, which are discretized depending on the interval between two successive sampling instants. We propose to parameterize $\mathbf{A}_v^{(i)}$ and $\mathbf{B}_v^{(i)}$ using GOBF of the form

$$F_j(s, \mathbf{a}_v^{(i)}) = \frac{\sqrt{-2\,\mathrm{Re}(\mathbf{a}_{v,j}^{(i)})}}{(s - \mathbf{a}_{v,j}^{(i)})} \prod_{p=1}^{j-1} \frac{(s + \mathbf{a}_{v,p}^{(i)*})}{(s - \mathbf{a}_{v,p}^{(i)})} \tag{14}$$

where $\mathbf{a}_v$ represents a vector of poles of in the left half plane of the complex s-plane. With the proposed parameterization, matrices $\mathbf{A}_v^{(i)}$ and $\mathbf{B}_v^{(i)}$ can be expressed as follows

$$\mathbf{A}_v(\mathbf{a}_v) = [\mathbf{\Xi}_1(\mathbf{a}_v)]^{-1}\,\mathbf{\Xi}_2(\mathbf{a}_v)$$

$$\mathbf{B}(\mathbf{a}_v) = [\mathbf{\Xi}_1(\mathbf{a}_v)]^{-1}\left[\boldsymbol{\beta}_1\ 0\ 0\ \ldots\ 0\right]^T$$

$$\mathbf{\Xi}_1(\mathbf{a}_v) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ -\dfrac{\boldsymbol{\beta}_2}{\boldsymbol{\beta}_1} & 1 & 0 & 0 & 0 \\ 0 & -\dfrac{\boldsymbol{\beta}_3}{\boldsymbol{\beta}_2} & 1 & \ldots & \ldots \\ \ldots & \ldots & \ldots & \ldots & 0 \\ 0 & 0 & 0 & -\dfrac{\boldsymbol{\beta}_n}{\boldsymbol{\beta}_{n-1}} & 1 \end{bmatrix}$$

$$\mathbf{\Xi}_2(\mathbf{a}_v) = \begin{bmatrix} \mathbf{a}_{v,1} & 0 & 0 & 0 \\ \left(\dfrac{\boldsymbol{\beta}_2}{\boldsymbol{\beta}_1}\right)\mathbf{a}_{v,1} & 0 & 0 & 0 \\ 0 & ... & \ldots & \ldots \\ \ldots & \ldots & \ldots & 0 \\ 0 & \ldots & \left(\dfrac{\boldsymbol{\beta}_n}{\boldsymbol{\beta}_{n-1}}\right)\mathbf{a}_{v,n-1} & \mathbf{a}_{v,n} \end{bmatrix}$$

where $\boldsymbol{\beta}_j = \sqrt{-2\,\mathrm{Re}(\mathbf{a}_{v,j})}$. The parameters of the i^{th} SISO / MISO can be computed by solving following optimization problem.

$$(\widehat{\mathbf{a}}_v^{(i)}, \widehat{\boldsymbol{\theta}}_{\boldsymbol{\upsilon}}^{(i)}) = \arg\min_{\mathbf{a}_{\boldsymbol{\upsilon}}^{(i)}} \left[\frac{1}{N_{y_i}} \sum_{k=k_{i,1}}^{Ny_i} \widehat{\boldsymbol{\upsilon}}_i(k, \mathbf{a}_v^{(i)}, \widehat{\boldsymbol{\theta}}_{\boldsymbol{\upsilon}}^{(i)})^2 \right]$$

$$\text{Subject to} \tag{15}$$

$$\widehat{\boldsymbol{\theta}}_{\boldsymbol{\upsilon}}^{(i)} = \overline{E}\left[\mathbf{X}_{\boldsymbol{\upsilon}}^{(i)}(k_{i,l})\mathbf{X}_{\boldsymbol{\upsilon}}^{(i)}(k_{i,l})^T\right]^{-1} \tag{16}$$

$$\times \overline{E}\left[\mathbf{X}_{\boldsymbol{\upsilon}}^{(i)}(k_{i,l})\widehat{\boldsymbol{\upsilon}}_i(k_{i,l})\right] \tag{17}$$

$$\mathrm{Re}\left(\mathbf{a}_{\boldsymbol{\upsilon},p}^{(i)}\right) < 0 \qquad \text{for } p = 1, ...n \tag{18}$$

It may be noted that the constraints on pole location are different here when compared to deterministic component modeling.

1.3 Inter-sample Inferential Estimation

The main reason for developing fast-rate models is inferential estimation of irregularly sampled variable at the fast rate. As described above, we can identify r fast-rate MISO deterministic model and r AR type SISO / MISO multirate stochastic observers from multirate input output data. These observers can be used for inter-sample predictions i.e. when measurements are not available. For carrying inter-sample predictions, the deterministic and stochastic components have to be used in a different manner. As the identified model for deterministic component is discrete time fast rate model with sampling time same as of input, it can be used recursively at every instant k to estimate state.

$$\widehat{\mathbf{X}}_u^{(i)}(k|k-1) = \boldsymbol{\Phi}_u^{(i)}\widehat{\mathbf{X}}_u^{(i)}(k-1|k-2) \tag{19}$$

$$+\Gamma_u^{(i)}\mathbf{u}(k-1) \tag{20}$$

$$k_{i,l} < k \leq k_{i,l+1} \tag{21}$$

Estimation of stochastic component requires use of time varying matrices as innovation, $\{\boldsymbol{\upsilon}_i(\mathbf{k}_{i,l})\}$, is available only at major sampling instants. Last available innovation is used to estimate stochastic contribution at inter-sample instances i.e. innovation at major sampling period $k_{i,l}$ is used to estimate stochastic component at instants $k_{i,l}+1$ to $k_{i,l+1}$. For example, when the SISO unmeasured disturbance model is given by Eq.(11), inter-sample predictions can be generated as,

$$\widehat{\mathbf{x}}_v^{(i)}(k|k_{i,l}) = \boldsymbol{\Phi}_v^{(i)}\,(k, k_{i,l})\,\widehat{\mathbf{x}}_v^{(i)}(k_{i,l}) \tag{22}$$

$$+\boldsymbol{\Gamma}_v^{(i)}\,(k, k_{i,l})\,\boldsymbol{\upsilon}_i(k_{i,l}) \tag{23}$$

$$\widehat{\mathbf{y}}_i(k|k_{i,l}) = \boldsymbol{\theta}_u^{(i)}\left[\widehat{\mathbf{X}}_u^{(i)}(k|k-1)\right] \tag{24}$$

$$+\boldsymbol{\theta}_v^{(i)}\left[\widehat{\mathbf{x}}_v^{(i)}(k|k_{i,l})\right] \tag{25}$$

$$\text{for } k_{i,l} < k \leq k_{i,l+1} \tag{26}$$

2. ILLUSTRATIVE EXAMPLE

In this section, we present simulation studies on the benchmark Shell control problem (heavy oil fractionator system).

To assess the quality of identified model following two criteria are used.

- **Percentage Prediction Error (PPE)**

$$PPE = \frac{\sum\limits_{k=k_{i,1}}^{N_y} [\mathbf{y}_i(k) - \hat{\mathbf{y}}_i(k)]^2}{\sum\limits_{k=k_{i,1}}^{N_y} [\mathbf{y}_i(k) - \overline{\mathbf{y}}_i]^2} \times 100 \tag{27}$$

  In above expression, $\bar{\mathbf{y}}_i$ represents the mean value of the slow sampled measured outputs data. $\hat{\mathbf{y}}_i(k)$ represents the predictions generated by the model while $\mathbf{y}_i(k)$ is measured output. As PPE is computed only using data obtained at major sampling instants, it can be computed for simulation as well as experimental data.

- **Percentage Estimation Error (PEE)**

$$PEE = \frac{\sum\limits_{k=1}^{N} [\widetilde{\mathbf{y}}_i(k) - \hat{\mathbf{y}}_i(k)]^2}{\sum\limits_{k=1}^{N} [\widetilde{\mathbf{y}}_i(k) - \overline{\widetilde{\mathbf{y}}}_i]^2} \times 100 \tag{28}$$

  In the above expression, $\widetilde{y}_i(k)$ represents noise free fast rate measurement of outputs (i.e. at minor sampling instants) obtained from simulations. This index can be computed only for simulated studies.

To carry out simulation studies, we use the benchmark problem for control of a heavy oil fractionator system, which is characterized by large time delays in each input output pair, proposed at Shell process control workshop (Prett and Morari, 1987). The heavy oil fractionator has three product draws, three side circulating loops and a gaseous feed stream. The system consists of seven measured outputs, three manipulated inputs $(\mathbf{u})$ and two unmeasured disturbances $(\mathbf{d})$. Since the controlled outputs of interest from viewpoint of multi-rate sampling are top end point and side end point, in this work we consider a subsystem consisting of only these two outputs. The continuous time transfer function model for this sub-system is as follows

$$\widehat{\mathbf{y}}(s) = G_p(s)\mathbf{u}(s) + G_d(s)\mathbf{d}(s) \tag{29}$$

Transfer function for this sub-system can be found in Patwardhan and Shah, (2006). A discrete version of this system, obtained using sampling interval of 1 min, is used to simulate the plant dynamics. The stationary unmeasured disturbances $\mathbf{d}(z)$ are assumed to be generated by the following stochastic process

$$\mathbf{d}(z) = \frac{z}{z - 0.95}\mathbf{I}\mathbf{w}(z) \tag{30}$$

where $\mathbf{w} \in R^2$ is a zero mean normally distributed white noise process with $\sigma_{w1} = \sigma_{w2} = 0.0075$. In addition, the measured outputs are assumed to be corrupted with measurement noise

$$\mathbf{y}(k) = \widehat{\mathbf{y}}(k) + \boldsymbol{\upsilon}(k) \tag{31}$$

where $\boldsymbol{\upsilon} \in R^3$ represents zero mean normally distributed white noise process with $\sigma_{\upsilon i} = 0.005$ for $i = 1, 2, 3$.

In order to carry out system identification, a low frequency (in the range $\begin{bmatrix} 0 & 0.01\pi \end{bmatrix}$) random binary signals with amplitude 0.075 were simultaneously introduced in all the manipulated inputs. Top end point and side end point are resampled irregularly so that their sampling periods are uniformly distributed between (a) Case A: 11 to 20 min. and (b) Case B: 11 to 30 min.

To begin with, two MISO OE models are estimated without considering any time delay. These identified models are then used to estimated time delay matrix as discussed in Shah and Patwardhan (2005). The estimated time delay matrix and true time delay matrix are as follow,

$$\boldsymbol{\tau}_d(estimated) = \begin{bmatrix} 29 & 27 & 29 \\ 19 & 14 & 17 \end{bmatrix}$$

$$\boldsymbol{\tau}_d(true) = \begin{bmatrix} 27 & 28 & 27 \\ 18 & 14 & 15 \end{bmatrix}$$

Estimated values of time delay are fairly close to true values of time delay and the error is of the order of at most two sampling period.

In second step, the estimated time delays are used to introduce zeros at origin in the GOBF model and MISO OE models are re-identified. Two basis filters with distinct poles are used between each input-output pair while developing the OE models. The irregularly sampled model residuals are then used to develop SISO disturbance models using one basis filter between each pair.

Figure. (1) presents model validation results for case with MR ratio 10-30. The corresponding manipulated inputs are given in Figure (2). The PPE and PEE values obtained using validation data set are reported in Table. (1). Comparison of PPE and PEE values shows that the inter-sample predictions are significantly improved when the noise models are used. It can seen that the improvement of provided by noise model deteriorates when the MR ratio increases. This is because the autocorrelation decreases as the MR ratio is increased. Comparison of the frequency responses of the identified fast rate OE model with that of the true process is presented in Figure (3) while the comparison of step responses is presented in Figure (4). From this figure, it can be observed

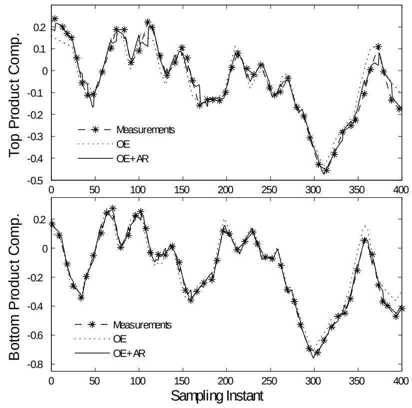

Fig. 1. Model validation for MR ratio 10-30: Comparision of measured and predicted outputs.

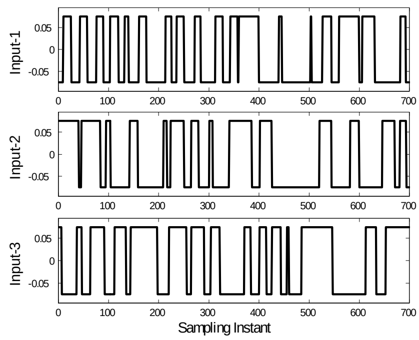

Fig. 2. Model Validation: Variation of manipulated inputs

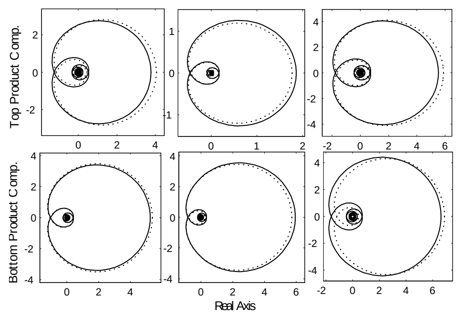

Fig. 3. Comparision of Nyquist plot of identified model and actual process for shell problem (MR ratio = 10-30)

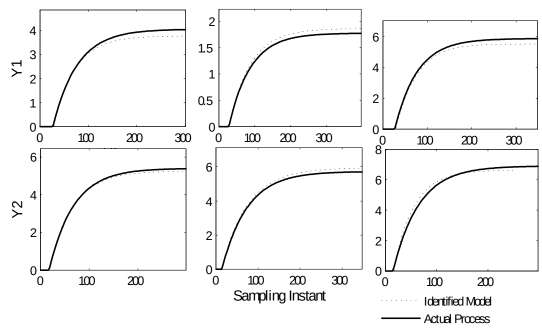

Fig. 4. Model Validation: Comparison of process and model step responses

Table 1. Shell Control Problem: PEE values

MR ratio	Model	PPE		PEE	
		$\mathbf{y}_1$	$\mathbf{y}_2$	$\mathbf{y}_1$	$\mathbf{y}_2$
11-20	OE	6.43	5.05	26.06	22.91
	OE + AR	1.13	1.96	9.11	10.95
11-30	OE	5.76	5.95	25.53	22.76
	OE + AR	2.36	2.05	12.93	11.41

that OE model identified using multirate data closely represents the true process behavior.

3. SUMMARY AND CONCLUSIONS

Multirate sampled data system, in which different variables are sampled at different rates, are common in the chemical process industries. For satisfactory control of such processes, it is required that slowly measured variable be estimated, directly or indirectly, at higher frequency. In this work we considered multirate systems, in which inputs are manipulated regularly and output are measured irregularly at sampling rate which is integer multiple of input sampling rate. We proposed a novel scheme for identifying fast-rate deterministic and disturbance model from multirate input-output data. Generalized Orthogonal Basis Function (GOBF) are used for parameterizing the deterministic and stochastic models. A two tier optimization scheme is used to identify single rate system with time delays. By this approach, models for the deterministic and the stochastic components are identified separately. Fast rate OE type model is extracted from the irregularly sampled multirate data. The multirate residuals generated in the first step are used to develop an unmeasured disturbance model. We propose a novel auto regressive models with time varying coefficients that can capture dynamics of unmeasured disturbances. Efficacy of the proposed scheme is successfully demonstrated by simulation study of Shell Control problem. From the analysis of the simulation and experimental results, following conclusions can be reached.

- Proposed method of identifying OE model based on GOBF from irregularly sampled multirate data, generates accurate estimates of the deterministic component of the process dynamics.
- The proposed autoregressive type unmeasured disturbance model with time varying coefficient is capable of producing excellent intersample estimates.
- The model quality deteriorates with increase in MR ratio. This can be attributed to increase in variance error.

Thus, the proposed approach provides a viable method for developing deterministic and stochastic model for systems that are sampled irregularly. Also, the proposed models can be used for fast rate inferential estimates in any conventional or advanced control schemes.

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