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ON TEST DESIGN FOR SUBSPACE IDENTIFICATION OF MULTIVARIABLE ILL-CONDITIONED SYSTEMS

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Abstract: Ill-conditioned processes often produce data of low quality for model identification in general, and for subspace identification in particular, because data vectors of different outputs are typically close to collinearity, being aligned in the "strong" direction. One of the solution that can be adopted is the use of appropriate input signals (usually called "rotated" inputs), which must excite sufficiently the process in the "weak" direction. In this paper open-loop (uncorrelated and rotated) random signals are compared against closed-loop signals with the aim of finding the most appropriate ones to be used in multivariable subspace identification of ill-conditioned systems. As a result it is shown that closed-loop data give superior models, both in the sense of frequency response and in terms of performance when used to design a model predictive control system.

Keywords: input design, subspace identification, ill-conditioned system, MPC

1. INTRODUCTION AND PREVIOUS WORK

The research on consistent, reliable and efficient identification algorithms is currently focused on two different approaches: Prediction Error methods (PE) and Subspace IDentification methods (SID). Conceptually, PE is the simplest one, and is based on the idea of minimizing the gap between plant and model outputs, called prediction error. SID, instead, involves matrices obtained from output and input data and performs their projection onto subspaces that guarantee particular properties with respect to system noise.

The increasing popularity of Model Predictive Control (MPC) has been leading to utilization of state-space models, particularly suitable in predictive control algorithms, so that SID methods, which were explicitly developed to obtain this kind of process description, gained increasing success in the last decade. SID is a relatively young technique: the first examples useful in practice were presented by Van Overschee and De Moor (1994) with N4SID algorithm, Verhaegen (1994) who developed MOESP, and Larimore (1994) with CVA family methods. Several new subspace approaches were recently developed to enhance numerical stability and efficiency, as well as theoretical properties (Qin *et al.*, 2005). In particular, the algorithm by Huang *et al.* (2005) is used in the present work, with some modifications introduced to improve results in system matrix recovery.

It is well-known that multivariable systems may show "directions" (in the input vector space) in which the (steady-state and/or dynamic) effect of the inputs on the process outputs is much larger than in other directions. In such situations the process is said to be ill conditioned, and frequent examples of ill-conditioned systems are high-purity distillation

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columns. Ill-conditioned processes represent a challenge from a control point of view since conventional decentralized controllers are typically inadequate but model-based controllers may suffer from robustness issues (Pannocchia, 2003). Moreover, ill-conditioned processes may be difficult to deal with from an identification point of view, because traditional uncorrelated open-loop step tests tend to excite the system mostly in high-gain directions (Koung and MacGregor, 1993), and so information in low-gain directions may be poor, often resulting in a model not suitable for control purposes (Koung and MacGregor, 1994).

This work focuses on test design, being it basilar to obtain good models even if, traditionally, it received less attention than other aspects of the "identification problem" (Zhu, 2001). Structured inputs considered in this work can be also called "rotated" from their own construction, because they are designed considering the angles of "strong" and "weak" directions of the process, whose correct value should be derived by a Singular Value Decomposition (SVD) of the gain matrix. In practice, however, the rotation angle must be recovered by a trial and error method (Misra and Nikolaou, 2003). In the present work the use of rotated inputs is criticized for a number of reasons. This approach in fact, besides being time consuming, may lead to poor results when the correct rotation angle is not used. Moreover, structured inputs could be inappropriate when specific SID methods are used, especially for system order recovering, and in general closed-loop identification tests may be more effective.

2. SUBSPACE IDENTIFICATION METHOD

2.1 Basic definitions

Linear discrete time-invariant state-space systems in the following form are considered:

$$\begin{aligned} x_{k+1} &= Ax_k + Bu_k \\ y_k &= Cx_k + e_k \ , \end{aligned} \tag{1}$$

in which $x \in \mathbb{R}^n$ is the state, $u \in \mathbb{R}^m$ is the input, $y \in \mathbb{R}^p$ is the output, $e \in \mathbb{R}^p$ is stochastic noise, $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$ and $C \in \mathbb{R}^{p\times n}$ are system matrices. The following standing assumptions are made.

Assumption 1. The pair (A, B) is stabilizable, the pair (A, C) is observable, the noise e_k is white and statistically independent of past outputs and inputs, i.e. $E\{y_k e_j'\} = 0$ and $E\{u_k e_j'\} = 0$ for all $j > k$.

Given a positive integer r, assumed to satisfy $r > n$, let the vector of "future" (w.r.t. time k) outputs be:

$$\bar{y}_k = \begin{bmatrix} y_k' \ y_{k+1}' \ \cdots \ y_{k+r-1}' \end{bmatrix}' \tag{2}$$

and similarly for "future" inputs ($\bar{u}_k$) and noise ($\bar{e}_k$).

Assumption 2. Data vectors $(u,\ y)$ are collected for $L = M + 2r - 1$ sampling times.

From the model (1), one can obtain:

$$\bar{y}_k = \Gamma_r x_k + H_r \bar{u}_k + \bar{e}_k \ , \tag{3}$$

in which Γ_r is the extended observability matrix and H_r is a lower block-triangular Toeplitz matrix:

$$\Gamma_r = \begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^{r-1} \end{bmatrix}, H_r = \begin{bmatrix} 0 & 0 & \cdots & \cdots & 0 \\ CB & 0 & \cdots & \cdots & 0 \\ CAB & CB & 0 & \cdots & 0 \\ \vdots & & \ddots & \ddots & \vdots \\ CA^{r-2}B & \cdots & \cdots & CB & 0 \end{bmatrix} \tag{4}$$

where 0 denotes the full zero matrix of suitable dimensions (identity matrix is denoted with I).

2.2 Obtaining the system matrices

Projecting input and output data matrices onto a subspace orthogonal to noise, subspace methods "clean up" data from stochastic information, leaving the deterministic part. Each SID method is mostly characterized by the matrix used to perform these projections. In this work a variant of that introduced by Huang *et al.* (2005) is proposed.

Writing (3) for $k = r, r+1, \ldots, r+M-1$ gives:

$$Y_f = \Gamma_r X + H_r U_f + E_f \ , \tag{5}$$

where Y_f (and similarly X_f, U_f and E_f) is

$$Y_f = \begin{bmatrix} \bar{y}_r \ \bar{y}_{r+1} \ \ldots \ \bar{y}_{r+M-1} \end{bmatrix} . \tag{6}$$

Projection onto the space $\mathcal{W}$, orthogonal to noise, is performed post-multiplying both sides of (5) by an appropriate W', designed in such a way that

$$\lim_{M\to\infty} \frac{1}{M} E_f W' = 0 \ . \tag{7}$$

Now (5) is rewritten as follows:

$$\begin{bmatrix} I & -H_r \end{bmatrix} Z_f W' = \Gamma_r X W' \ , \tag{8}$$

in which $Z_f = \begin{bmatrix} Y_f' \ U_f' \end{bmatrix}'$.

Choosing $W = W' = Z_p^+ Z_p$, where Z_p is the "past" data matrix (built as Z_f but considering data shifted r sampling times in the past) and Z_p^+ is the right pseudo-inverse of Z_p (computed via SVD), an orthogonal projection onto the row space of Z_p can be performed [see (Huang *et al.*, 2005) for more details]. This choice of W can be seen as equivalent to that in (Wang and Qin, 2002) with a suitable column weighting (Wang and Qin, 2006).

Now, since all terms in (8) are deterministic, estimates of A and C can be calculated from Γ_r [obtained from (8)] with good results, whereas it was found that estimation of B with the original method in (Wang and Qin, 2002; Huang *et al.*, 2005) produces poor outcomes in a relevant number of cases. An alternative approach is here proposed to recover B (and x_0 as well), which is a sort of PE method applied to (1).

2.2.1. Estimating A and C Pre-multiplying (8) by a matrix $\Gamma_r^\perp$ orthogonal to Γ_r, one can obtain

$$\Gamma_r^\perp \begin{bmatrix} I & -H_r \end{bmatrix} Z_f W' = 0 \,. \tag{9}$$

Let $Z = Z_f W'$, it is clear that $\Gamma_r^\perp \begin{bmatrix} I & -H_r \end{bmatrix} = Z^\perp$; thus, $Z^\perp$ can be computed performing an SVD of Z (i.e. $Z = USV'$) and selecting a matrix U_2, composed by the columns of U that correspond to zero singular values in the S matrix, i.e.

$$Z = \begin{bmatrix} U_1 & U_2 \end{bmatrix} \begin{bmatrix} S_1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_1' \\ V_2' \end{bmatrix} , \tag{10}$$

in which the dimension of the square diagonal matrix S_1, i.e. the rank of Z, should be $mr + n$ (Wang and Qin, 2002, Lemma 1) in absence of noise ($\operatorname{rank} Z > mr + n$ otherwise). The dimension of S_1, and consequently the system's order n, is obtained in this work using a heuristic Principal Component Analysis (PCA) approach as described below. The first mr singular values of Z are considered, and then subsequent $\hat{n}$ singular values are selected according to:

$$\frac{\sigma_{mr+\hat{n}}}{\sum_{j=1}^{\hat{n}} \sigma_{mr+j}} > \rho, \quad \sigma_j \in \operatorname{diag} S_1, \quad 0 \le \hat{n} \le pr \,, \tag{11}$$

in which ρ is a (small) positive scalar. Model order n is considered equal to the largest value of $\hat{n}$ for which (11) holds.

Next, combining (9) and (10) leads to:

$$\Gamma_r^\perp \begin{bmatrix} I & -H_r \end{bmatrix} = TU_2' \,, \tag{12}$$

where T is a nonsingular transformation matrix (usually I). Finally, by partitioning:

$$TU_2' = \begin{bmatrix} P_1' & P_2' \end{bmatrix} \,, \tag{13}$$

in which $P_1 \in \mathbb{R}^{pr \times (pr-n)}$, it results that:

$$\begin{aligned} P_1' \Gamma_r &= 0 \\ -P_1' H_r &= P_2' \,, \end{aligned} \tag{14}$$

which can be solved to find estimates of Γ_r and H_r.

Estimate of A and C are therefore obtained observing that (in a MATLAB notation)

$$C = \Gamma_r(1:p,:) \tag{15a}$$

$$\Gamma_r(1:(r-1)p,:)A = \Gamma_r(p+1:rp,:) \,, \tag{15b}$$

solving equation (15b) for A in a least square sense.

2.2.2. Estimating B and x_0 Given the computed estimates of A and C, let $\hat{y}_{k|B,x_0}$ be the estimate of y_k given an input matrix B and initial state x_0, i.e.

$$\hat{y}_{k|B,x_0} = C \sum_{j=0}^{k-1} A^j B u_j + CA^k x_0 = f_k(\vartheta) \,, \tag{16}$$

where $\vartheta = \begin{bmatrix} (\operatorname{Vec} B)' & x_0' \end{bmatrix}'$, $\operatorname{Vec} B$ is a vector obtained by stacking each column of B on top of the next one, and $f_k(\cdot)$ is suitably defined. Being $f_k(\cdot)$ linear in its argument and $f_k(0) = 0$, it follows:

$$\hat{y}_{k|B,x_0} = \varphi_k \vartheta \,, \tag{17}$$

where $\varphi_k \in \mathbb{R}^{p \times n(m+1)}$ is the Jacobian matrix of f_k (computed from A, C and known inputs). An estimate of B and x_0 comes from least-square problem

$$\min_\vartheta \,(\Psi - \Phi\vartheta)' \,(\Psi - \Phi\vartheta) \tag{18}$$

where

$$\Psi = \begin{bmatrix} y_0' & \dots & y_{L-1}' \end{bmatrix}' , \quad \Phi = \begin{bmatrix} \varphi_0' & \dots & \varphi_{L-1}' \end{bmatrix}' \,. \tag{19}$$

The solution of (18) is given by:

$$\vartheta^* = \Phi^+ \Psi = (\Phi'\Phi)^{-1} \Phi' \Psi \,. \tag{20}$$

It can be demonstrated that estimates of B and x_0 asymptotically converge to their true value if the estimates of A and C are consistent [see (Ljung, 1999; Qin *et al.*, 2005)].

2.3 Open-loop and closed-loop test design

Open-loop (OL) tests are widely used in industrial system identification for their simplicity, even if closed-loop (CL) tests may be preferred for some practical and theoretical reasons, such as they maintain outputs in range, they contains information over the most significant frequencies for control, and they seem to be superior in system model recovery.

However, some problems could be experienced with CL test data, when subspace identification is performed. In fact, if Z_p is used for projection, the space orthogonal to Z may also contain controller model information, and so the process model identification procedure may be erroneous [see (Huang *et al.*, 2005) for details]. A valuable solution could be to adopt a matrix Y_f^c constructed as Y_f using "future" set-point changes vectors $\bar{y}_k^c$ built as in (2), because (7) still holds (being Y_f^c and E_f independent), and is such that the controller model does not show up in the decomposition of Z. By stacking the matrices Y_f^c and Z_p and following the same pattern for projection and matrix recovery, CL data can be handled without problems.

3. ANALYSIS OF INPUT SIGNALS FOR ILL-CONDITIONED PROCESSES

Ill-conditioned systems, as previously said, represent one of the most difficult kind of processes to be identified: models obtained for these processes often suffer from errors in order and gain matrix recovery. System order, in particular, cannot be retrieved properly because identification matrices may have collinear columns (Misra and Nikolaou, 2003). A possible answer to this issue is an appropriate input design that "excites" the unfavorable direction to make its contribution magnitude similar to that of favorable direction. In particular, the so-called "rotated" inputs can be used to overcome order recovering issues in ill-conditioned systems (Koung and MacGregor, 1994; Misra and Nikolaou, 2003), as explained below. A slightly more general investigation, which considers (possibly) non-square multivariable systems of arbitrary dimensions, can be found in (Micchi and Pannocchia, 2006). In the present section, a 2×2 system is considered for simplicity of exposition. Performing an SVD on the steady-state gain matrix

$$y_s = G_s u_s = \begin{bmatrix} \cos(\phi) & -\sin(\phi) \\ \sin(\phi) & \cos(\phi) \end{bmatrix} \cdot \\ \cdot \begin{bmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{bmatrix} \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} u_s \,, \tag{21}$$

where $_s$ subscript indicates steady state. Calling $u_s = \begin{bmatrix} u_1 & u_2 \end{bmatrix}'$, $y_s = \begin{bmatrix} y_1 & y_2 \end{bmatrix}'$, computing and approximating the expression in (21), it follows that

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \approx \sigma_1 \begin{bmatrix} \cos(\phi)(u_1 \cos(\theta) - u_2 \sin(\theta)) \\ \sin(\phi)(u_1 \cos(\theta) - u_2 \sin(\theta)) \end{bmatrix} \,, \tag{22}$$

because $\sigma_1 \gg \sigma_2$ for an ill-conditioned process. This leads directly to

$$y_2 \approx y_1 \tan\phi \,, \tag{23}$$

which shows that outputs are “almost” collinear, always aligned in the favorable direction. It is important to point that the above analysis shows strong correlation of the outputs in steady-state conditions, whereas correlation may be different in transient conditions if the outputs have fairly different dynamics. Nonetheless it is useful to motivate the adoption of “rotated” input signals (Koung and MacGregor, 1994; Misra and Nikolaou, 2003).

In order to evaluate the effects of output collinearity on subspace identification, a rank analysis is here presented. Starting from (3), it is possible to obtain

$$Z_f = \begin{bmatrix} \Gamma_r & H_r \\ 0 & I \end{bmatrix} \begin{bmatrix} X_f \\ U_f \end{bmatrix} + \begin{bmatrix} E_f \\ 0 \end{bmatrix} \,, \tag{24}$$

where E_f represent noise contribution in outputs. The maximum rank of the deterministic part of Z_f is $mr + n$, because

$$\text{rank} \begin{bmatrix} X_f \\ U_f \end{bmatrix} \leq n + mr, \quad \text{rank} \begin{bmatrix} \Gamma_r & H_r \\ 0 & I \end{bmatrix} \leq pr + mr \,, \tag{25}$$

with $pr > n$ (notice that $r > n$). If strong collinearity of outputs is present, an SVD of Z_f leads to $mr + \hat{n}$ non-zero singular values, where $\hat{n} \leq n$, typically excluding weaker directions. In noisy systems, the term on the right of the sum in (24) must not be neglected: the effect of its presence is to increase the rank of Z_f, so it is possible that $\hat{n} > n$, although still without no significant information regarding weaker directions which are “covered” by noise.

To avoid collinearity of output data it is possible to construct an input for which (Koung and MacGregor, 1994; Misra and Nikolaou, 2003)

$$\frac{\xi_2}{\xi_1} = \kappa = \frac{\sigma_1}{\sigma_2} \,, \tag{26}$$

where

$$\xi_1 = (\cos(\theta) u_1 - \sin(\theta) u_2) \tag{27a}$$

$$\xi_2 = (\sin(\theta) u_1 + \cos(\theta) u_2) \,. \tag{27b}$$

Substituting (27a) in (26), it results

$$u_2 = \frac{\kappa \cos(\theta) - \sin(\theta)}{\cos(\theta) + \kappa \sin(\theta)} u_1 \approx \cot(\theta) u_1 \,. \tag{28}$$

Now, using these inputs, no terms can be neglected in equation (29), and so outputs are not collinear

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} \cos(\phi)\sigma_1\xi_1 - \sin(\phi)\sigma_2\xi_2 \\ \sin(\phi)\sigma_1\xi_1 + \cos(\phi)\sigma_2\xi_2 \end{bmatrix} \,. \tag{29}$$

A rotated signal can be constructed, respecting this condition, in three steps:

(1) define a binary signal u_1 with L samples;
(2) select a value for θ;
(3) construct $u_{k,2} = \cot(\theta)\, u_{k,1} + \zeta_k$, where ζ is a random signal (Zhu, 2001) with small amplitude.

Clearly, the dithering signal is introduced to avoid exact collinearity of the inputs.

4. CASE STUDIES

4.1 Introduction

OL identification data are constructed by using Generalized Binary Noise (GBN) signals (Zhu, 2001), either uncorrelated or correlated in the case of “rotated” inputs. CL identification data are constructed in two different ways: with an MPC algorithm (Pannocchia and Rawlings, 2003) based on an “erroneous” model and with a decentralized multivariable PI controller. Output setpoints are generated by GBN signals.

The identified models are evaluated by comparing their frequency response with that of the real process. A monotonically decreasing weight function:

$$\delta(\omega) = \frac{1 - \text{erf}\,(\omega)}{\int_0^\infty (1 - \text{erf}\,(\omega)) d\omega} \tag{30}$$

was selected to increase the importance of low-middle frequencies, which are the most significant in control. Defining $G^r(z)$ and $G^{id}(z)$ as the real and identified transfer function models, respectively, the following scalar parameter is computed to measure the quality of the identified model:

$$\epsilon = \sup_\omega \delta(\omega) \sigma_1 \left(G^{id}(e^{i\omega T_s}) - G^r(e^{i\omega T_s}) \right) \tag{31}$$

with T_s being the sampling time.

The “classical” high-purity LV distillation column studied by Skogestad and Morari (1988), modified with a 10 minute time delay, is chosen as first example of ill-conditioned system (the steady-state condition number is 142). The sampling time is 5 minutes, the outputs are logarithmic distillate purity and logarithmic bottom impurity, and the inputs are reflux and boil-up rates.

Table 1. Skogestad & Morari example: values of identified systems order (M&N: Misra & Nikolaou algorithm; M&N w. a.:Misra & Nikolaou algorithm with incorrect θ)

Type of input	Data collection	SID algorithm	order
random	OL	proposed	8
rotated	OL	proposed	10
rotated	OL	M&N	2
rotated	OL	M&N w. a.	1
random	CL-MPC	proposed	2

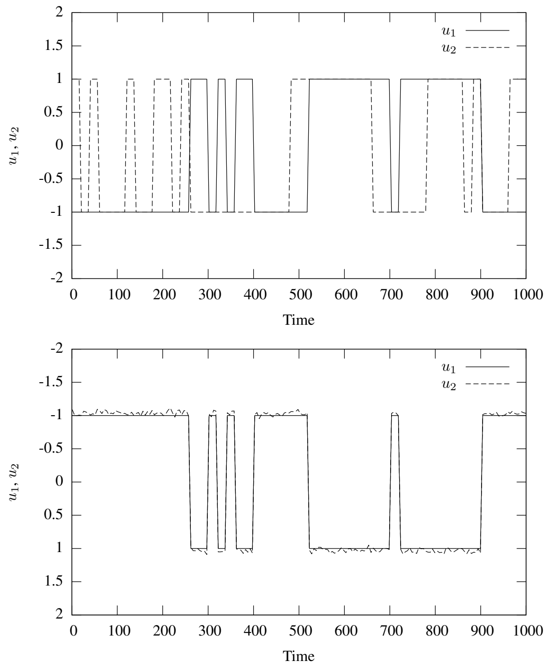

Fig. 1. Uncorrelated (top) and rotated (bottom) GBN inputs

A 3 input, 5 output model (with order 30) of another high-purity LV distillation column was chosen as second (non square) example. The steady-state condition number is 164, outputs are head and bottom product impurity and opening percent of three control valves, while inputs are reflux rate, boil-up rate and feed rate. A 10 minute time delay was considered on head and bottom impurity, and a sampling time of one minute was adopted.

4.2 *Model order recovering*

Order recovering is verified only for the 2×2 example, because it is more significant than in a more complex system such as the 3×5 example. A value of 0.05 for ρ in (11) is assumed to pick out of singular values for system order recovering: results are shown in Table 1. As previously said, rotated inputs generate mistakes when used with algorithms which project also the future input matrix (such as the one adopted here), because of high correlation in rotated inputs (see Figure 1). As a cross-check, using the SID algorithm proposed in (Misra and Nikolaou, 2003), which projects only future outputs matrix, gives the correct order with the same data. In CL scheme, uncorrelated set-point changes are used: it can be seen from Table 1 that they work better of every other kind of inputs.

Table 2. Values of ϵ for different identified models

Type of input	Data collection	2×2 system	3×5 system
random	OL	0.0167	0.1688
rotated	OL	0.0699	0.2399
random	CL-MPC	0.0144	0.0284
random	CL-PI	0.0299	0.0229

4.3 *Quality of identified models and their effects on MPC closed-loop behavior*

Quality of the identified models can be measured using the index ϵ. From Table 2 it is clear that the models identified starting from CL data are to be preferred in terms of frequency response. In Figure 2 closed-loop inputs and outputs (for the Skogestad & Morari example) obtained with MPC regulators based on different models (as specified in the legend) during a set-point change in the unfavorable direction are plotted. It is clear that regulators based on models obtained from OL rotated and CL uncorrelated signals are superior, indeed very close to that based on the true model.

In conclusion, models obtained in CL are to be preferred, also because this kind of inputs does not suffer of problems experienced with correlated OL signals (see 4.4). These advantages obtained over other test design methods can be associated to the fact that random set points force the outputs in many different directions, thus avoiding correlation of outputs over the high gain direction but also avoiding correlation of the inputs.

4.4 *Practical issues in implementing rotated inputs*

Rotated inputs guarantee good performance in order recovery when used with specific identification methods [e.g that in (Misra and Nikolaou, 2003)], but until now it has not been shown their behavior if there are errors in the rotation angle. Referring to the Skogestad & Morari example, it is possible to show that the region in which the correct order is recovered with the Misra and Nikolaou (2003) algorithm is quite narrow (approximately between $\theta = \frac{2\pi}{9}$ and $\theta = \frac{5\pi}{18}$). Effects of errors on rotation angle can be seen in Table 1, where it is shown that even if Misra & Nikolaou's method is used, the system order is not well recovered if θ differs from the correct value ($\pi/3$ instead of $\pi/4$). Since in general the true order is unknown, in practice it is not straightforward to define a simple way of choosing the correct angle, and often several trails may be required. In addition, it is important to consider that, increasing the number of system inputs and outputs, this approach becomes more and more

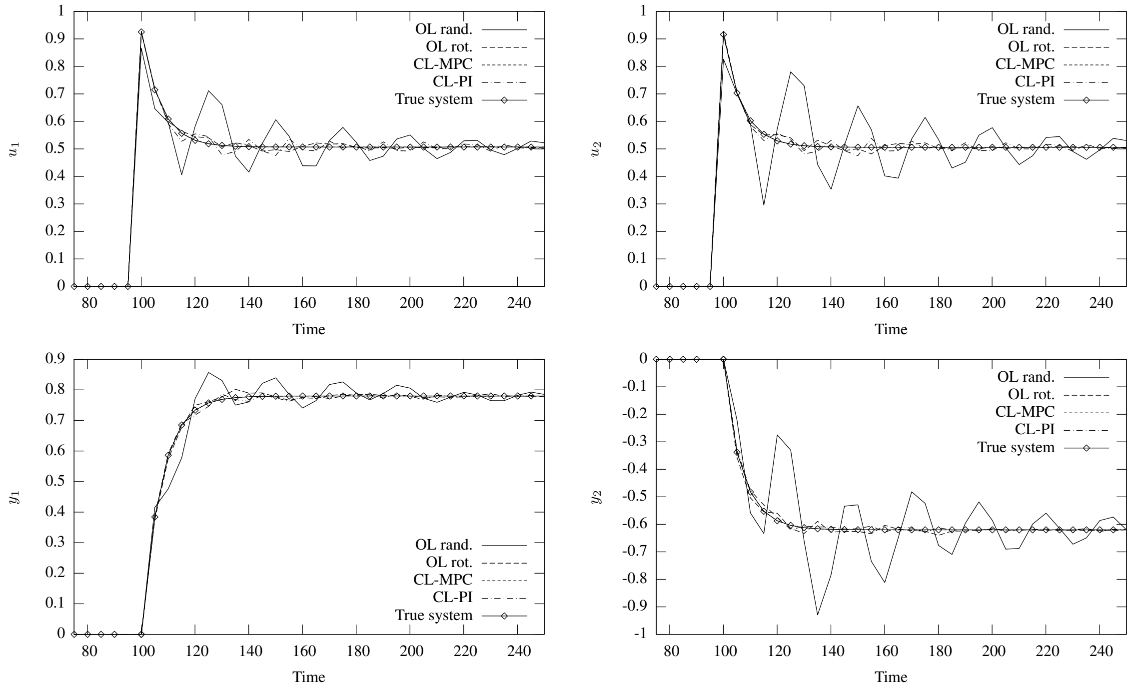

Fig. 2. Skogestad & Morari example: control inputs u_1, u_2 (top); process outputs y_1, y_2 (bottom)

complicated, because of the growing number of "angles" to be selected.

5. CONCLUSIONS

Ill-conditioned processes are difficult to be identified from data because of problems that derive mainly from the lack of information in the "weak" direction. A possible solution, proposed in the literature, is the use of tailored inputs such as "rotated" inputs, that excite the system sufficiently in the weak direction. To construct these inputs, the system gain matrix (which is however unknown) has to be decomposed via SVD, and inputs are designed to generate signals of the same magnitude both in the weak and in the strong direction. This method was applied to two ill-conditioned distillation examples, and was compared with the use of uncorrelated random inputs, both in open-loop and closed-loop operation. Results clearly show that, with OL data, rotated inputs grant better models only when specific subspace identification algorithms are used and if the correct rotation angle is chosen. On the other hand, using CL data obtained from uncorrelated set points offers superior outcomes, especially for design of model-based controllers. This test design approach is also to be preferred, because random set points can be easily generated without necessity of several trials to find the most appropriate rotation angle.

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