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ROBUST H_2 FILTERING FOR CONTINUOUS-TIME STOCHASTIC SYSTEMS WITH UNCERTAINTIES

Kwan Ho Lee * **Biao Huang** *

* *The Department of Chemical and Materials Engineering, University of Alberta, Edmonton, Alberta T6G 2G6, Canada*

Abstract: In this paper, robust H_2 optimal filtering is addressed for continuous-time stochastic systems with polytopic parameter uncertainty. A new robust stability condition is presented. A continuous-time robust H_2 optimal filter is obtained by solving a sufficient linear matrix inequality condition characterizing a solution of a minimum variance filtering problem which takes into account the polytopic type of model uncertainties. The advantage of the proposed approach is illustrated through numerical examples.

Keywords: Continuous-time systems; Uncertain systems; Robust filtering; H_2 filtering; Convex optimization.

1. INTRODUCTION

There has been a large amount of work to search for optimal H_2 filtering, also called minimum variance filtering, with a large spectrum of practical applications in control engineering, signal processing, system failure detection, etc, since the seminal work of Kalman filtering Anderson and Moore (1979). In recent years, enhancing robustness in filtering has attracted much attention because the consideration of uncertainty, which is present in most physical systems almost inevitably, is of a prime importance for applications to real systems. The purpose of robust H_2 filtering is to design a filter such that the worst case mean square estimation error is minimized for all admissible uncertainties under the assumption that the process noise input is a white noise with zero-mean and known covariance. In Xie and Soh (1994); Shaked and de Souza (1995); Sayed (2001); Sun and Packard (2005), the Riccati equation approaches were presented to deal with systems with norm-bounded parameter uncertainty. In Geromel (1999); Geromel and de Oliveira (2001); Geromel et al. (2000); Yang and Hung (2002), the linear matrix inequality (LMI) approaches were applied for systems with norm-bounded parameter uncertainty or convex polytopic type uncertainty.

As is well known, most robust filtering techniques rely on employing a single quadratic Lyapunov function over all uncertainty domain. Obviously there exists conservativeness in this type of design. Several attempts have been made in the past few years toward reducing conservativeness in existing robust filtering algorithms, particularly for uncertain discrete-time systems. In Shaked et al. (2001); Geromel et al. (2002), an LMI approach was applied to improve robustness of optimal H_2 filtering by using a robust stability condition established in de Oliveira et al. (1999), which enables the use of a parameter dependent Lyapunov function and consequently leads to a less conservative design for uncertain discrete-time systems. In Xie et al. (2004), a nonconvex bilinear matrix inequality optimization method with scaling parameters was proposed to solve the optimal H_2 filtering problem by using a robust stability condition established in Peaucelle et al. (2000), which offers extra degree-of-freedom in optimization. In Xie et al. (2004), a parameter search is necessary to find the best values for scaling parameters that lead to the best filter performance.

On the other hand, there have been few attempts for uncertain continuous-time systems. This is because of the difficulty in obtaining new LMI characterizations for robust stability in continuous-time and the problem still remains open. In Tuan et al. (2001), an LMI approach with a parameter dependent Lyapunov function was presented for continuous-time systems with convex polytopic uncertainty by applying an LMI characterization for robust stability found by Projection Lemma. In Barbosa et al. (2005), a nonconvex matrix inequality method with searching parameters was proposed to produce a less conservative result as in Xie et al. (2004). In Gonçalves et al. (2006), a domain search method using the branch and bound algorithm was applied directly in the space of filter parameters over a set of polytopic points in order to avoid conservativeness in robust filtering design at the expense of the computational effort. Note that the nonconvex optimization methods or the domain search methods, which were developed in Xie et al. (2004); Barbosa et al. (2005); Gonçalves et al. (2006) at the expense of the computational cost, are not prohibitive for the design of robust filters with high order; however, in many applications, an LMI solution may be preferred due to the less computational complexity.

In this paper, an LMI solution is proposed for robust H_2 filtering of continuous-time systems with polytopic parameter uncertainty. A new sufficient robust stability condition, which is expressed as LMIs, is presented for uncertain continuous-time stochastic systems. Using the robust stability condition presented in this paper, a new continuous-time robust H_2 filter is obtained by solving a sufficient linear matrix inequality condition characterizing a solution of a minimum variance filtering problem which takes into account convex parameter uncertainty. In case when there is no uncertainty, the proposed robust H_2 filter is reduced to the standard Kalman filter. Numerical comparisons with existing results are given.

2. MAIN RESULTS

2.1 Problem Description

Consider a plant described by

$$\Sigma : \begin{aligned} \dot{x} &= Ax + Bv, \\ y &= Cx + Dv, \end{aligned} \tag{1}$$

where $x \in \mathbf{R}^n$ is the state of the plant to be estimated, $v \in \mathbf{R}^m$ is the white noise input with zero mean and unit covariance matrix, and $y \in \mathbf{R}^p$ is the measured plant output. The system matrices in (1) are assumed to be unknown but belong to a known convex compact set of polytopic type, i.e.,

$$\mathscr{S} \triangleq (A, B, C, D) \in \Omega,$$
$$\Omega \triangleq \left\{ \mathscr{S} | \mathscr{S} = \sum_{i=1}^{r} \tau_i \mathscr{S}_i, \tau \triangleq (\tau_1, \ldots, \tau_r) \in \Gamma \right\} \tag{2}$$

where $\Gamma \triangleq \{(\tau_1, \ldots, \tau_r) | \sum_{i=1}^{r} \tau_i = 1, \tau_i \geq 0\}$.

In this paper, a new robust H_2 filter will be developed for the stochastic system (1) and (2).

2.2 Robust Stability Condition

A robust H_2 performance condition is suggested first for continuous-time systems. Our LMI-based robust condition will play an important role in implementing a robust continuous-time H_2 filter.

Lemma 1. Boyd et al. (1994) Given a system $\Sigma_1 : \dot{x} = Ax + Bv$, $y = Cx$, where $\mathscr{P} \triangleq (A, B, C)$, $\mathscr{P} \in \Omega$, the following statements are equivalent:

(i) A is stable and $\|T(\tau) \triangleq C(sI - A)^{-1}B\|_2^2 < \nu$, $\forall \tau \in \Gamma$.

(ii) $\exists P(\tau) = P^T(\tau), W(\tau) = W^T(\tau)$ such that

$$\begin{pmatrix} A^T P(\tau) + P(\tau)A & P(\tau)B \\ B^T P(\tau) & -\nu I_m \end{pmatrix} < 0, \tag{3}$$

$$\begin{pmatrix} P(\tau) & C^T \\ C & W(\tau) \end{pmatrix} > 0, \tag{4}$$

$$tr(W(\tau)) < 1 \tag{5}$$

for all $\tau \in \Gamma$ such that $\mathscr{P} \in \Omega$. $\diamondsuit$

A sufficient condition for Lemma 1 is proposed based on linear matrix inequalities (LMIs) in the following theorem:

Theorem 1. Given system Σ_1, if there exist matrices $P_i = P_i^T$, Y, and $W_i = W_i^T$ satisfying the following LMIs

$$\begin{pmatrix} \begin{pmatrix} \frac{1}{2}P_i - \frac{1}{2}(I - A_i^T)Y \\ -\frac{1}{2}Y^T(I - A_i) \end{pmatrix} & * & * \\ B_i^T Y & -\nu I_m & * \\ \frac{1}{2}(I + A_i^T)Y & 0 & -\frac{1}{2}P_i \end{pmatrix} < 0, \tag{6}$$

$$\begin{pmatrix} P_i & C_i^T \\ C_i & W_i \end{pmatrix} > 0, \tag{7}$$

$$tr(W_i) < 1 \tag{8}$$

for all $i = 1, 2, \ldots, r$, then the system Σ_1 is robustly stable and $\|T(\tau)\|_2^2 < \nu$ for all $\tau \in \Gamma$. Moreover, for any $\mathscr{P} \in \Omega$, $P(\tau)$ given by

$P(\tau) \triangleq \sum_{i=1}^{r} \tau_i P_i$ is a parameter dependent positive-definite Lyapunov function such that (3)-(5) hold.

Proof: Assume that there exists a solution $\{P_i, Y, W_i\}$ by (6)-(8) for all $i = 1, 2, \ldots, r$. Let $P(\tau) \triangleq \sum_{i=1}^{r} \tau_i P_i$, i.e. $P(\tau)$ is a linear parameter dependent function. Multiplying each LMI in (3) by $\tau_i > 0$ and adding them to get a convex combination for $\tau_i, i = 1, 2, \ldots, r$, we have the inequality (9),

$$\begin{pmatrix} \begin{pmatrix} \frac{1}{2}P(\tau) \\ -\frac{1}{2}(I - \sum_{i=1}^{r} \tau_i A_i^T)Y \\ -\frac{1}{2}Y^T(I - \sum_{i=1}^{r} \tau_i A_i) \end{pmatrix} & * & * \\ \sum_{i=1}^{r} \tau_i B_i^T Y & -\nu I_m & * \\ \frac{1}{2}(I + \sum_{i=1}^{r} \tau_i A_i^T)Y & 0 & -\frac{1}{2}P(\tau) \end{pmatrix} < 0, \tag{9}$$

where $\sum_{i=1}^{r} \tau_i A_i = A(\tau)$ and $\sum_{i=1}^{r} \tau_i B_i = B(\tau)$ according to the definition of the convex set (2). Pre- and post-multiplying (9) by

$$\begin{pmatrix} Y^{-T} & 0 \\ 0 & I \end{pmatrix}, \quad \begin{pmatrix} Y^{-1} & 0 \\ 0 & I \end{pmatrix},$$

respectively, Inequality (9) is converted to

$$\begin{pmatrix} \begin{pmatrix} \frac{1}{2}Y^{-T}P(\tau)Y^{-1} \\ -\frac{1}{2}Y^{-T}(I - A(\tau)^T) \\ -\frac{1}{2}(I - A(\tau))Y \end{pmatrix} & * & * \\ B(\tau)^T & -\nu I_m & * \\ \frac{1}{2}(I + A(\tau)^T) & 0 & -\frac{1}{2}P(\tau) \end{pmatrix} < 0. \tag{10}$$

Applying Schur complement to the above inequality yields

$$\begin{pmatrix} \begin{pmatrix} \frac{1}{2}Y^{-T}P(\tau)Y^{-1} \\ -\frac{1}{2}Y^{-T}(I - A(\tau)^T) \\ -\frac{1}{2}(I - A(\tau))Y^{-1} \\ +\frac{1}{2}(I + A(\tau))P^{-1}(\tau)(I + A(\tau)^T) \end{pmatrix} & * \\ B(\tau)^T & -\nu I_m \end{pmatrix} < 0. \tag{11}$$

It is noted that the inequality $(I - A(\tau)^T - P(\tau)Y^{-1})^T P(\tau)^{-1}(I - A(\tau)^T - P(\tau)Y^{-1}) \geq 0$ holds for all $\tau \in \Gamma$ because $P(\tau) > 0$. Hence it is clear that

$$\begin{aligned}(I-A(\tau))Y^{-1}+Y^{-T}(I-A(\tau))^T-Y^{-T}P(\tau)Y^{-1} \\ \leq (I - A(\tau))P^{-1}(I - A(\tau))^T. \end{aligned} \tag{12}$$

In (11), replacing the terms $\frac{1}{2}Y^{-T}P(\tau)Y^{-1} - \frac{1}{2}Y^{-T}(I - A(\tau)^T) - \frac{1}{2}(I - A(\tau))Y^{-1}$ by its lower bound, it follows that

$$\begin{aligned} 0 &> \begin{pmatrix} \begin{pmatrix} \frac{1}{2}Y^{-T}P(\tau)Y^{-1} \\ -\frac{1}{2}Y^{-T}(I - A(\tau)^T) \\ -\frac{1}{2}(I - A(\tau))Y^{-1} \\ +\frac{1}{2}(I + A(\tau))P^{-1}(\tau)(I + A(\tau)^T) \end{pmatrix} & * \\ B(\tau)^T & -\nu I_m \end{pmatrix} \\ &\geq \begin{pmatrix} \begin{pmatrix} \frac{1}{2}(I + A(\tau))P^{-1}(\tau)(I + A(\tau)^T) \\ -\frac{1}{2}(I - A(\tau))P(\tau)^{-1}(I - A(\tau))^T \end{pmatrix} & * \\ B(\tau)^T & -\nu I_m \end{pmatrix}. \end{aligned}$$

Therefore we have the following inequality:

$$\begin{pmatrix} \begin{pmatrix} \frac{1}{2}(I + A(\tau))P^{-1}(\tau)(I + A(\tau)^T) \\ -\frac{1}{2}(I - A(\tau))P(\tau)^{-1}(I - A(\tau))^T \end{pmatrix} & * \\ B(\tau)^T & -\nu I_m \end{pmatrix} < 0. \tag{13}$$

After manipulations, pre- and post-multiplying (13) by

$$\begin{pmatrix} P(\tau) & 0 \\ 0 & I_m \end{pmatrix}, \tag{14}$$

we have

$$\begin{pmatrix} A^T(\tau)P(\tau) + P(\tau)A(\tau) & P(\tau)B(\tau) \\ B(\tau)^T P(\tau) & -\nu I_m \end{pmatrix} < 0, \tag{15}$$

which is the same as (3). Applying the same steps as the above to (7) yields $C(\tau)P(\tau)C^T(\tau) < W(\tau)$, which is converted to (4) by Schur complement. Finally, (5) is obtained by the following inequality: $1 > \max_{i=1,2,\ldots,r} tr(W_i) > tr(W(\tau)) > tr(C(\tau)P(\tau)C^T(\tau))$, where $W(\tau) \triangleq \sum_{i=1}^{r} \tau_i W_i$. This completes the proof. ■

2.3 Robust H_2 Optimal Filtering

Consider the synthesis of a continuous-time robust filter for system (1)

$$\hat{\Sigma}: \begin{aligned} \dot{\hat{x}} &= \hat{A}\hat{x} + \hat{B}y, \\ \hat{z} &= \hat{C}\hat{x}, \end{aligned} \tag{16}$$

where $\hat{x} \in \mathbf{R}^n$ is an estimate of the state x of the system (1). Define a system matrix $\mathscr{R}$ of the filter $\hat{\Sigma}$ as

$$\mathscr{R} \triangleq \begin{pmatrix} \hat{A} & \hat{B} \\ \hat{C} & 0 \end{pmatrix}. \tag{17}$$

A system state vector to be estimated by (16) is defined as $z = Lx$, where L is known, for example, $L = I_n$ for state estimation. The filtering error dynamics is then given by the following state equations:

$$\Sigma_e: \begin{aligned} \dot{x}_e &= \mathscr{A}x_e + \mathscr{B}v, \\ z_e &= \mathscr{C}x_e, \end{aligned} \tag{18}$$

where $x_e \triangleq \left(x^T \ \hat{x}^T\right)^T$ and $z_e \triangleq z - \hat{z}$ is the estimation error. Σ_e can also be defined by the estimation error transfer function $T_{vz_e}(\tau, s) \triangleq \mathscr{C}(sI - \mathscr{A})^{-1}\mathscr{B}$. The filtering error dynamic system Σ_e is given by

$$\left(\begin{array}{c|c} \mathscr{A} & \mathscr{B} \\ \hline \mathscr{C} & 0 \end{array}\right) = \left(\begin{array}{cc|c} A & 0 & B \\ 0 & 0 & 0 \\ \hline L & 0 & 0 \end{array}\right) + \left(\begin{array}{cc} 0 & 0 \\ I_n & 0 \\ \hline 0 & I_p \end{array}\right) \mathscr{R} \left(\begin{array}{cc|c} 0 & I_n & 0 \\ C & 0 & D \end{array}\right), \tag{19}$$

where $\mathscr{S} \triangleq (A, B, C, D) \in \Omega$ is defined as in (2).

Our aim is to design a robust H_2 filter, also called robust minimum variance filter, by minimizing

$$\max_{\mathscr{S}\in\Omega} \|T_{vz_e}\|_2^2, \tag{20}$$

which is given in the following theorem:

Theorem 2. The H_2-norm of the estimation error transfer function for all admissible systems described in (2) is less than $\sqrt{\nu}$, i.e., $\|T_{vz_e}(\tau)\|_2^2 < \nu$ for all $\tau \in \Gamma$, if there exists a solution $\{\bar{P}_i$, $\bar{A}$, $\bar{B}$, $\bar{C}$, X, Q, R, Z, ν, $\forall i = 1, 2, \ldots, r\}$ of the following LMI Problem 1. Given a solution, a robust H_2 filter is constructed as

$$T_{\mathscr{R}}(s) = \bar{C}U^{-1}(sI - V^{-T}\bar{A}U^{-1})^{-1}V^{-T}\bar{B} \tag{21}$$

by choosing nonsingular matrices U and V such that

$$U^T V = Q. \tag{22}$$

Problem 1.

$$\text{Minimize}_{\{\bar{P}_i, \bar{A}, \bar{B}, \bar{C}, X, Q, R, Z, \nu\}} \ \nu$$

subject to (23), (26),

$$\begin{pmatrix} \bar{P}_i & \begin{pmatrix} L^T \\ -\bar{C}^T \end{pmatrix} \\ \begin{pmatrix} L & -\bar{C} \end{pmatrix} & Z_i \end{pmatrix} > 0, \tag{24}$$

$$tr(Z_i) < 1, \tag{25}$$

for all $i = 1, 2, \ldots, r$.

Proof: Using Theorem 1, we have the following robust synthesis condition for the filtering error dynamics (18).

$$\begin{pmatrix} \begin{pmatrix} \frac{1}{2}P_i \\ -\frac{1}{2}(I - \mathscr{A}_i^T)\Lambda \\ -\frac{1}{2}\Lambda^T(I - \mathscr{A}_i) \end{pmatrix} & \Lambda^T\mathscr{B}_i & \frac{1}{2}\Lambda^T(I + \mathscr{A}_i) \\ \mathscr{B}_i^T\Lambda & -\nu I_m & 0 \\ \frac{1}{2}(I + \mathscr{A}_i^T)\Lambda & 0 & -\frac{1}{2}P_i \end{pmatrix} < 0, \tag{27}$$

$$\begin{pmatrix} P_i & \mathscr{C}_i^T \\ \mathscr{C}_i & Z_i \end{pmatrix} > 0, \tag{28}$$

where $\Lambda \in \mathbf{R}^{2n\times 2n}$ is an instrumental variable as in Theorem 1. Partition Λ, in accordance with the partition of $\mathscr{A}$ in (19), and define Π as follows:

$$\Lambda = \begin{pmatrix} X & Y \\ V & W^{-1} \end{pmatrix}, \quad \Pi = \begin{pmatrix} I_n & 0 \\ 0 & U \end{pmatrix}$$

where $X, W \in \mathbf{R}^n$ and $U \triangleq WV$. Then the following identities hold:

$$\Pi^T Y^T \Pi = \begin{pmatrix} X^T & V^T U \\ U^T Y^T & V^T U \end{pmatrix} \tag{29}$$

$$\Pi^T Y^T \mathscr{A}_i \Pi = \begin{pmatrix} X^T A_i + V^T \hat{B} C_i & V^T \hat{A} U \\ U^T Y^T A_i + V^T \hat{B} C_i & V^T \hat{A} U \end{pmatrix} \tag{30}$$

$$\Pi^T Y^T \mathscr{B}_i = \begin{pmatrix} X^T B_i + V^T \hat{B} D_i \\ U^T Y^T B_i + V^T \hat{B} D_i \end{pmatrix} \tag{31}$$

$$\mathscr{C}\Pi = \begin{pmatrix} L & -\hat{C}U \end{pmatrix} \tag{32}$$

Define Γ_1 and Γ_2 as

$$\Gamma_1 = \begin{pmatrix} \Pi & 0 & 0 \\ 0 & I_m & 0 \\ 0 & 0 & \Pi \end{pmatrix}, \quad \Gamma_2 = \begin{pmatrix} \Pi & 0 \\ 0 & I_p \end{pmatrix}$$

Pre- and post-multiplying (27) by Γ_1^T and Γ_1, respectively, and again performing the congruence transformation Γ_2^T to (28), we have

$$\begin{pmatrix} \begin{pmatrix} \frac{1}{2}\Pi^T P_i \Pi \\ -\frac{1}{2}\Pi^T(I - \mathscr{A}_i^T)\Lambda\Pi \\ -\frac{1}{2}\Pi^T\Lambda^T(I - \mathscr{A}_i)\Pi \end{pmatrix} & * & * \\ \mathscr{B}_i^T\Lambda\Pi & -\nu I_m & * \\ \frac{1}{2}(I + \mathscr{A}_i^T)\Lambda\Pi & 0 & -\frac{1}{2}\Pi^T P_i \Pi \end{pmatrix} < 0,$$

$$\begin{pmatrix} \Pi^T P_i \Pi & \Pi^T \mathscr{C}_i^T \\ \mathscr{C}_i \Pi & W \end{pmatrix} > 0.$$

Now applying the following linearizing changes of variables: $\bar{A} \triangleq V^T\hat{A}U$, $\bar{B} \triangleq V^T\hat{B}$, $\bar{C} \triangleq \hat{C}U$, $Q \triangleq U^T V$, $R \triangleq YU$, and $\bar{P}_i \triangleq \Pi^T P_i \Pi$, to the identities (30)-(32), we obtain the LMI solution in Problem 1. A robust H_2 filter for the stochastic uncertain system (1) is then parameterized as $\hat{A} = V^{-T}\bar{A}U^{-1}$, $\hat{B} = V^{-T}\bar{B}$, $\hat{C} = \bar{C}U^{-1}$. Hence the transfer function (21) is obtained. This completes the proof. ■

3. ILLUSTRATIVE NUMERICAL EXAMPLES

3.1 Example 1

Consider an uncertain two masses-spring system studied in Iwasaki (1996); Geromel and de Oliveira (2001), where the system model is described by

$$\begin{aligned} \dot{x} &= \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -2 & 1 & -c & 0 \\ 2 & -2 & 0 & -2c \end{pmatrix} x + \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} v, \\ y &= \begin{pmatrix} 1 & 0 & 0 & 0 \end{pmatrix} x + \begin{pmatrix} d \end{pmatrix} v, \end{aligned} \tag{33}$$

where c and d are bounded constant uncertain parameters with $0.5 \le c \le 3.5$ and $0.5 \le d \le 1.5$, respectively.

$$\begin{pmatrix} (1,1) & \begin{pmatrix} X^T B_i + \bar{B} D_i \\ R^T B_i + \bar{B} D_i \end{pmatrix} & \frac{1}{2}\begin{pmatrix} X^T + X^T A_i + \bar{B} C_i & Q^T + \bar{A} \\ R^T + R^T A_i + \bar{B} C_i & Q^T + \bar{A} \end{pmatrix} \\ * & \nu I_m & 0 \\ * & * & -\frac{1}{2}\bar{P}_i \end{pmatrix} < 0, \tag{23}$$

$$(1,1) \triangleq \frac{1}{2}\bar{P}_i + \frac{1}{2}\begin{pmatrix} \begin{pmatrix} -(X+X^T) + X^T A_i + A_i^T X \\ +\bar{B} C_i + C_i^T \bar{B}^T \end{pmatrix} & -(R+Q^T) + A_i^T R + C_i^T \bar{B}^T + \bar{A} \\ -(Q+R^T) + R^T A_i + \bar{B} C_i + \bar{A}^T & -(Q+Q^T) + \bar{A} + \bar{A}^T \end{pmatrix}. \tag{26}$$

Table 1. Comparison of robustness of H_2 optimal filters

LMI Methods	ν
Geromel and de Oliveira (2001)	0.4339
Tuan et al. (2001)	0.4045
LMI Problem 1	0.2646

Table 2. Comparison of robustness of H_2 optimal filters

LMI Methods	ν
Geromel and de Oliveira (2001)	∞ (infeasible)
Tuan et al. (2001)	103.5490
LMI Problem 1	17.9417

The design of a robust H_2 optimal filter shall be addressed for the uncertain two masses-spring system (33) with $L = \begin{pmatrix} 0 & 1 & 0 & 0 \end{pmatrix}$ in the sense of minimizing the upper bound ν on the error variance. Using the proposed LMI Problem 1, we obtain a robust H_2 optimal filter

$$T_{\mathscr{R}}(s) = \frac{0.04973s^3 + 0.1783s^2 + 0.575s + 0.7281}{s^4 + 2.889s^3 + 5.429s^2 + 4.275s + 1.648}.$$

guaranteeing $\|T_{vz_e}(\tau)\|_2^2 < \nu = 0.2646, \forall\tau \in \Gamma$.

For comparison purpose, robust optimum filters have also been implemented using the existing LMI methods Geromel and de Oliveira (2001); Geromel (1999); Tuan et al. (2001). Using the robust filtering methods Geromel and de Oliveira (2001); Geromel (1999), we obtain a minimum $\nu = 0.4339$ for the uncertain system (33). Using the recent parameter dependent Lyapunov function approach Tuan et al. (2001), we obtain a minimum of $\nu = 0.4045$ for the parameter uncertainties. Hence, it is clearly shown that the proposed LMI solution produces a less conservative result than that of the existing LMI results of Geromel and de Oliveira (2001); Geromel (1999); Tuan et al. (2001) for this example. The comparative result of robustness of H_2 optimal filters is given in Table 1. The computational cost of the LMI methods is comparable and not compared here due to the page limitation.

3.2 Example 2

Consider an uncertain system model studied in Shaked and de Souza (1995); Geromel (1999); Tuan et al. (2001), where the system model is of the form

$$\begin{aligned} \dot{x} &= \begin{pmatrix} 0 & -1+0.3a \\ 1 & -0.5 \end{pmatrix} x + \begin{pmatrix} -2 & 0 \\ 1 & 0 \end{pmatrix} v, \\ y &= \begin{pmatrix} -100+10b & 1 \end{pmatrix} x + \begin{pmatrix} 0 & 1 \end{pmatrix} v, \end{aligned} \tag{34}$$

where a and b are bounded constant uncertain parameters with $|a| \le \alpha$ and $|b| \le \beta$, respectively. It is noted that the system is stable if and only if $a < \frac{10}{3}$. Thus we consider $\alpha < \frac{10}{3}$.

A robust H_2 optimal filter shall be designed for the uncertain system (34) for a given α, β. Consider system (34) with $L = \begin{pmatrix} 1 & 0 \end{pmatrix}$. Using the proposed LMI Problem 1, we obtain a robust H_2 optimal filter for the uncertain system (34) with $\alpha = \beta = 3$ as

$$T_{\mathscr{R}}(s) = \frac{2.6490 \cdot 10^{-5}s - 1.3540 \cdot 10^{-4}}{s^2 + 0.6044s + 0.3240}.$$

guaranteeing $\|T_{vz_e}(\tau)\|_2^2 < \nu = 17.9417, \forall\tau \in \Gamma$.

We also implemented robust optimal filters for the uncertain systems (34) using the existing LMI methods of Geromel and de Oliveira (2001); Geromel (1999); Tuan et al. (2001). Using the existing robust filtering methods of Geromel and de Oliveira (2001); Geromel (1999), in contrast, no feasible robust optimal filter is obtained for the uncertain system (34). Using the recent parameter dependent Lyapunov function approach of Tuan et al. (2001), we obtain a minimum $\nu = 103.5490$ for the parameter uncertainties. Hence, it is clearly shown that the proposed LMI method significantly reduces conservativeness in the existing LMI methods Geromel and de Oliveira (2001); Geromel (1999); Tuan et al. (2001) in this example. The comparative result of robustness of H_2 optimal filters is given in Table 2.

Let $\alpha = \beta = \gamma$ and for different values of γ, a robust H_2 optimal filter shall be designed for the uncertain system (34). Fig. 1 shows achievable optima ν obtained from different LMI approaches in Geromel and de Oliveira (2001); Geromel (1999); Tuan et al. (2001). It is shown that the proposed LMI solution for this example is superior to that of Tuan et al. (2001) for all values of γ.

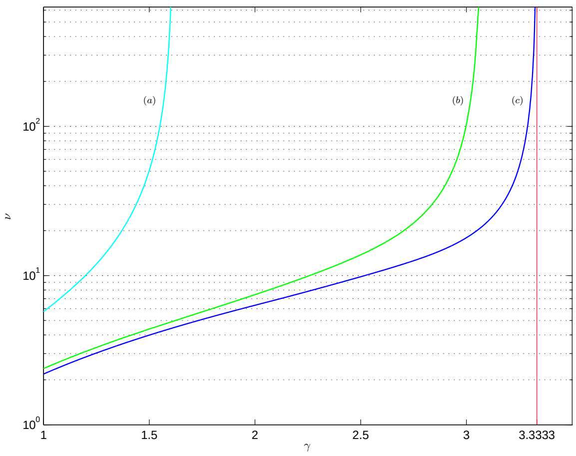

Fig. 1. (a) Geromel et al. Geromel (1999); Geromel and de Oliveira (2001). (b) Tuan et al. Tuan et al. (2001). (c) The proposed LMI solution.

4. CONCLUSIONS

A linear matrix inequality based approach has been studied for robust H_2 optimal filtering of continuous-time stochastic systems with polytopic type uncertainty. A new robust H_2 performance condition has been developed, which enables us to use a linear parameter dependent Lyapunov function in robust design. Using the robust H_2 performance condition presented in this paper, a new LMI solution for continuous-time robust H_2 optimal filtering has been proposed that guarantees asymptotic stability and an upper bound on the the variance of the estimation error for all uncertain parameters. The applicability of the proposed LMI method has been illustrated through numerical examples. Consequently, the proposed method can deal with a wider class of uncertainties for real physical systems.

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