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ROBUSTNESS ANALYSIS OF WIENER SYSTEMS

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Abstract: As reported in the literature, Wiener models have emerge as an appealing proposal for nonlinear processes representation due to their simplicity and the property of being valid over a larger operating region than a LTI model.
In this article, we propose a methodology to analyzed the robustness of a typical control scheme. To perform this analysis, we use a parametric description for the Wiener system. This model allows to describe the uncertainty as a set of parameters for the linear and the nonlinear blocks. Then, the linear block uncertainty is considered as a parameter-affine-dependent model and the nonlinear block uncertainty is studied as a conic-sector. The robustness analysis is then performed using μ-theory.

Keywords: Wiener Models, Process Control, Uncertainty, Robustness

1. INTRODUCTION

Model-based control of block oriented process has been widely diffused among the chemical engineering community (Pottmann and Pearson, 1998). Wiener systems are included in this king of models Pearson and Pottmann (2000). These models consist in a dynamic linear time invariant (LTI) submodel $H(z)$ in cascade with a static (i.e. memoryless) nonlinear block $N(.)$.

The use of these models has been treated in literature in various contexts such as chemical and biological processes (Kalafatis *et al.*, 1995; Pajunen, 1999; Zhu, 1999; Tian and Fujii, 2005; Norquay *et al.*, 1998; Gerkšič *et al.*, 2000; Lussón *et al.*, 2003; Biagiola *et al.*, 2004). The advantage of using Wiener models are twofold: the low computational effort associated to identification and the suitability for control design.

In general, the linear dynamics and the static nonlinearity cannot be identified in an independent way due to the cascade structure of the Wiener model. In this sense, several identification algorithms had been presented (Greblicki, 1994; Narendra and Gallman, 1966) in the literature. In particular, an approach that allows the simultaneous identification of the linear and nonlinear block, was introduced by Kalafakis *et al* (1995,1997) and extended to more general block-oriented models by Bai (1998) and Gómez and Baeyens (2004).

In most of the control applications of Wiener Models, the underlying strategy involves the inverse of the nonlinearity. However, to the best of the authors knowledge, a systematic robustness analysis for this scheme under uncertainty has not been developed in the literature.

In this paper, we present a novel characterization approach as well as an identification algorithm

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for uncertain Wiener Models and its application to the robustness analysis of the system. The goal is to obtain a nominal model of the process plus a parametric description of the uncertainty, and to use this model to analyze the closed-loop robustness of the system. These are the main contributions of this work.

For this purpose, Laguerre polynomials are used to model the linear dynamic block, and a piece-wise linear (PWL) representation of the nonlinear static block is provided. This modeling approach shows to be advantageous due to its simplicity, easy implementation and good application results. Moreover, it happens to be a convenient formulation in order to model the uncertainty which, in this way, can be easily mapped on the model parameters.

Then, an analysis of the robustness of closed loop Wiener process can be performed. The uncertainty in the Wiener model is treated as a partitioned problem. The uncertainty in the linear block is considered as a parameter-affine-dependent model and the uncertainty in the nonlinear block is treated as a conic-sector. The robustness analysis is performed using μ-theory. Although the study will be developed in the context of a pH neutralization reactor control, the conclusions can be directly extended to any other application.

The paper is organized as follows. In Section 2, the description and identification technique of Wiener process is reviewed, and the proposed uncertainty model is presented. In Section 3 the basis for robustness analysis are presented. In Section 4, the results are evaluated on the basis of a simulation of a pH neutralization process. Final remarks are addressed in Section 5.

2. WIENER MODEL IDENTIFICATION

2.1 Model Description

The Wiener model is shown in Figure 1. It consists of a LTI system $H(z)$ followed by a static nonlinearity $N(.)$. That is, the linear model $H(z)$ maps the input sequence $\{u(k)\}$ into the intermediate sequence $\{v(k)\}$, and the overall model output is $y(k) = N(v(k))$.

In this paper, the linear block of order N_l is described as (Wahlberg, 1991).

$$H(z) = \sum_{i=0}^{N_l} h_i L_i(z,a) \tag{1}$$

$$L_i(z,a) = \frac{\sqrt{1-a^2}}{z-a}\left(\frac{1-az}{z-a}\right)^{i-1} \tag{2}$$

where the parameters of the model are the coefficients h_i, the functions $L_i(z,a)$ are the Laguerre

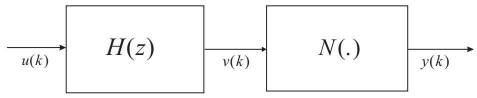

Fig. 1. The Wiener model structure.

basis for LTI models and $a \in \Re$ is a filter coefficient chosen a priori. The advantage of the use of Laguerre basis in comparison with other representations is that they need a lower number of parameters to describe a system with a slow impulse response or a damped system. Moreover, they allow the use of prior knowledge about the dominant pole (a). The nonlinear block $N(.)$ is, in general, a real-value function of one variable, i.e. $y = N(v)$. We describe the nonlinear function as

$$y = \sum_{i=0}^{N_n} \tilde{f}_i \tilde{\Lambda}(v, \beta_i) \tag{3}$$

where the basis functions $\tilde{\Lambda}(v, \beta_i)$ are predetermined PWL functions (Julián *et al.*, 1999), the values $\tilde{f}_i$ are the parameters that should be computed and N_n will be referred to as "order" of the nonlinearity. The use of fixed basis functions $\tilde{\Lambda}(v, \beta_i)$ makes the output to be a linear function of the parameters. This allows us to use a linear regression to estimate the parameters. The two basic advantages of this approach are the low complexity and the uniqueness of the solution. In this description, β_i are given parameters that define the partition of the domain of v, and $\tilde{\Lambda}$ are functions that involve nested absolute values (Julián *et al.*, 1999).

Since a scale factor can be arbitrarily distributed between the linear block and the memoryless one without affecting the input-output characteristics of the model, the gain can be fixed in one of them. Therefore, in the following we assume $h_0 = 1$ (Pearson and Pottmann, 2000).

2.2 Nominal Model Identification

Let us assume that an input-output data set is available, noted as u_k and y_k, respectively [2]. From Figure 1, the signal v_k can be written as the output of the linear block ($v_k = H(z) \bullet u_k$) and it can also be obtained from the output y_k and the inverse $N^{-1}(.)$ as $v_k = N^{-1}(y_k) = \sum_{i=0}^{N_n} f_i \Lambda(y_k, \beta_i)$ (Kalafakis *et al.*, 1997). Equating both sides of these equations (with the inclusion of an error function $\epsilon(k)$ to allow for modeling error) the following linear regression is obtained

$$\epsilon(k) = \theta^T \phi - l_0(u_k) \tag{4}$$

[2] To obtain these data sets, several aspects should be taken into account. For example, the process should be persistently excited in the whole domain of the nonlinear block, such that all the relevant dynamics is captured.

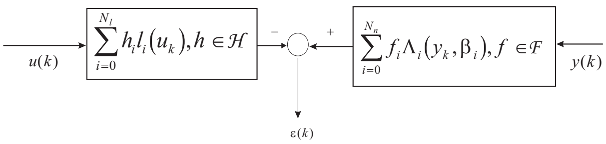

Fig. 2. Uncertainties in Wiener Model

where

$$\theta = [f_0, f_1, \cdots, f_{N_n}, h_1, h_2, \cdots, h_{N_l}]^T \quad (5)$$

and

$$\phi = [\Lambda(y_k, \beta_0), \Lambda(y_k, \beta_1), \cdots, \Lambda(y_k, \beta_{N_n}), \\ -l_1(u_k), -l_2(u_k), \cdots, -l_{N_l}(u_k)]^T. \quad (6)$$

Now, an least squares estimate $\hat{\theta}$ of θ can be computed by minimizing a quadratic criterion on the prediction errors $\epsilon(k)$:

$$\hat{\theta} = \left(\Phi_N \Phi_N^T\right)^{-1} \Phi_N \Gamma \quad (7)$$

where $\Gamma = [-l_0(u_1), \cdots, -l_0(u_N)]^T$ and $\Phi_N = [\phi(1), \cdots, \phi(N)]$ are formed using the set of the N data available from the process.

Now, estimates of the parameters $\hat{f}$ and $\hat{h}$ can be computed by partitioning the estimate $\hat{\theta}$, according to the definition of θ in (5). It is important to remark that we are identifying the inverse of the nonlinearity, which is frequently required in many control applications.

2.3 Uncertainty Characterization

In this section we describe an algorithm to characterize the uncertainties of the model obtained above (Figueroa *et al.*, 2006). We introduce a set of parameters $\mathcal{H}$ for the linear dynamic block and a set $\mathcal{F}$ for the parameters of the inverse of the nonlinear block (see Fig. 2):

$$\mathcal{H} = \left\{h : h = \hat{h} + \delta^h, h_i^l \leq \delta_i^h \leq h_i^u \; 1 \leq i \leq N_l\right\} \quad (8)$$

$$\mathcal{F} = \left\{f : f = \hat{f} + \delta^f, f_i^l \leq \delta_i^f \leq f_i^u \; 1 \leq i \leq N_n\right\} \quad (9)$$

To determine these parameter sets, let us define the sets $\mathcal{V}_u$ and $\mathcal{V}_y$. Given the input data u_k, the linear uncertain system defined by $\mathcal{H}$, maps the input at some specific time k over the set $\mathcal{V}_u$ (see Fig. 3). Then:

$$\mathcal{V}_u = \left\{v : v = \sum_{i=0}^{N_l} h_i l_i(u_k), h \in \mathcal{H}\right\} \quad (10)$$

Therefore, the Laguerre term of order i, i.e. $l_i(u_k)$, is a real number and the set $\mathcal{V}_u$ takes the form of $\mathcal{V}_u = \{v : v_l \leq v \leq v_u\}$.

On the other hand, if we consider the uncertain description of the parameters in $\mathcal{F}$, a given output y_k is mapped at some specific time k over a set

$$\mathcal{V}_y = \left\{v : v = \sum_{i=0}^{N_n} f_i \Lambda(y_k, \beta_i), f \in \mathcal{F}\right\} \quad (11)$$

This situation is showed in Fig. 3. From this picture it is clear that the parameters set will match the uncertainties description if $\mathcal{V}_y \cap \mathcal{V}_u \neq \emptyset$. In this way, the point u_k is mapped onto $\mathcal{V}_u$ through $\mathcal{H}$. Then, since $\mathcal{V}_y \cap \mathcal{V}_u \neq \emptyset$, this point will be mapped in y_k through the inverse of $\mathcal{F}$. Then, it is only necessary to compute the parameters bounds to satisfy this condition. The nominal linear model parameters $\hat{h}_i$ can be written as a vector, by considering that the Laguerre basis $l_i(u_k)$ are a set of real numbers for each input u_k. Let $l(u_k)$ be the vector whose i^{th} entry is the Laguerre basis $l_i(u_k)$. Then, the expression of the linear model is

$$\hat{v}_u(k) = \hat{h}^T l(u_k). \quad (12)$$

Note that since the entries of $l(u_k)$ could be positive or negative, it is possible to split the vector $l(u_k)$ by defining $l^+(u_k) = max(l(u_k), 0)$ and $l^-(u_k) = min(l(u_k), 0)$ and to form the vector $\gamma = \left[-(l^-(u_k))^T, (l^+(u_k))^T\right]^T$.

In a similar way, the PWL basis $\Lambda(y_k, \beta_i)$ are a set of positive real numbers for each output y_k. $\Lambda(y_k)$ is the vector whose i^{th} entry is the PWL basis $\Lambda(y_k, \beta_i)$. Then, the nonlinear model expression is:

$$\tilde{v}_y(k) = \hat{f}^T \Lambda(y_k). \quad (13)$$

Let us consider how to compute the bounds on the parameters.

Now, in order to described the uncertainty, the intersection of the uncertainties associated to the linear and nonlinear models should be non empty. This can be solved as:

$$\begin{aligned} &\min_{h^l, h^u, f^l, f^u} \left(\alpha \sum_{i=0}^{N_l} (h_i^l + h_i^u) + \right. \\ &\qquad \left. (1-\alpha) \sum_{i=0}^{N_n} (f_i^l + f_i^u)\right) \\ &\text{s.t.} \\ &\left[(h^l)^T, (h^u)^T, (f^u)^T\right] \begin{bmatrix} \gamma \\ \Lambda(y_k) \end{bmatrix} \geq e(k), \\ &\qquad \text{if } e(k) \geq 0; k = 1, \cdots, N \\ &\left[-(h^l)^T, -(h^u)^T, -(f^l)^T\right] \begin{bmatrix} \gamma \\ \Lambda(y_k) \end{bmatrix} \leq e(k), \\ &\qquad \text{if } e(k) < 0; k = 1, \cdots, N \end{aligned} \quad (14)$$

where the parameter $\alpha \in (0, 1)$ is a selected factor which allows to weight the uncertainty on the linear or nonlinear block and where

$$e(k) = \hat{f}^T \Lambda(y_k) - \hat{h}^T l(u_k). \quad (15)$$

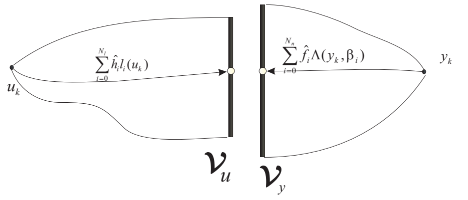

Fig. 3. Uncertainty sets in Wiener Model

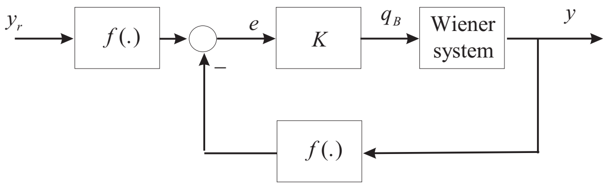

Fig. 4. The closed loop scheme for Wiener Model.

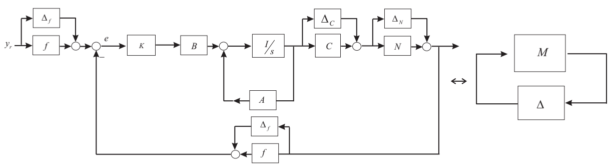

Fig. 5. The closed loop scheme for robustness analysis.

3. ROBUSTNESS ANALYSIS FOR A WIENER PROCESS

Wigren (1990) presents a structure for the control of Wiener process. In this scheme (see Fig. 4) two static nonlinearities are included in the loop. Under the hypothesis that $N(.)$ is invertible, the natural selection for the controller nonlinear functions is $f(.) \equiv N^{-1}(.)$, as was identified in the previous section. Then, the controller design involves two steps: a) the use of the inverse of the nonlinear gain and, b) compute a LTI controller that compensate the linear block model of the process. In Figueroa *et al.* (2007) a linear controller K for this structure is designed by using the H_∞ methodology (Gabinet *et al.*,1995).

To analyze the robustness of this controller it is necessary to consider all the uncertainty sources: the linear model parameters variation, the uncertainty on the nonlinear block and the variation in the feedback gain due to the nonlinear block in the feedback loop. This is illustrated in Fig. 5, which shows the loop used for robustness analysis.

This analysis is performed using μ-tools (Gabinet *et al.*, 1995). Then, it is necessary to treat each uncertainty block in the appropriate form. In particular, the block Δ_C represents a parametric uncertainty directly associate with the parameter bounds h^u and h^l. The block Δ_f is modeled as a conic sector that covers the nominal nonlinear gain associated with $\hat{f}$ and the block Δ_N is modeled as a conic sector that covers the inverse of the set of Eq. 9 (see example in next section).

Table 1. Neutralization Parameters.

PARAMETER	VALUE
x_{1i}	0.0012 $mol\ HCL/l$
x_{2i}	0.0020 $mol\ NaOH/l$
x_{3i}	0.0025 $mol\ NaHCO_3/l$
K_x	10^{-7} mol/l
K_w	10^{-14} mol^2/l^2
q_A	1 l/m
V	2.5 l

Then, to a perform the analysis for robust stability, the complete uncertainty description[3] is $\Delta = diag\,\{\Delta_C, \Delta_N, \Delta_f\}$.

In next section, the characterization of uncertainties and the robustness analysis will be exemplified in the context of a simulation of a neutralization reactor.

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4. NEUTRALIZATION REACTOR

To illustrate both the robustness analysis procedure, simulation results were obtained. The example consists of the neutralization reaction between a strong acid (HA) and a strong base (BOH) in the presence of a buffer agent (BX) (Galán, 2000). The neutralization takes place in a CSTR with a constant volume V. An acidic solution with a time-varying flow $q_A(t)$ of composition $x_{1i}(t)$ is neutralized using an alkaline solution with flow $q_B(t)$ of known composition made up of base x_{2i} and buffer agent x_{3i}. For this specific case, under some assumptions (Galán, 2000), the dynamic behavior of the process can be described considering the state variables: $x_1 = [A^-]$, $x_2 = [B^+]$ and $x_3 = [X^-]$. Then, the mathematical model of the process is:

$$\dot{x}_1 = q_A/V\ x_{1i} - (q_A + q_B)/V\ x_1 \quad (16)$$

$$\dot{x}_2 = q_B/V\ x_{2i} - (q_A + q_B)/V\ x_2 \quad (17)$$

$$\dot{x}_3 = q_B/V\ x_{3i} - (q_A + q_B)/V\ x_3 \quad (18)$$

$$F(x,\xi) \equiv \xi + x_2 + x_3 - x_1 - K_w/\xi - x_3/[1 + (K_x\,\xi/K_w)] = 0 \quad (19)$$

where $\xi = 10^{-pH}$. The parameters of the system are addressed in Table 1.

Using this model a set of data is generated by simulating 2000 samples with a sample time $T_s = 0.5$. A random signal uniformly distributed in $[0, 1]$ is applied to the manipulated variable q_B, this input changes every five samples. In a first step, compute a nominal Wiener Model is computed.

[3] The uncertainty Δ_f in the forward line does not affect the stability

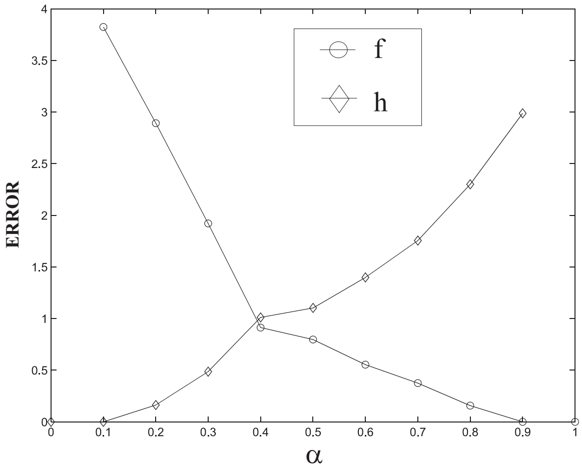

Fig. 6. Uncertainty norm as function of the parameter α

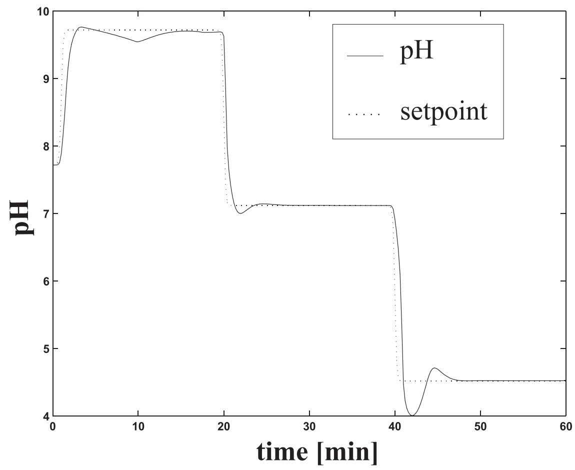

Fig. 7. pH as function of time

In this model three Laguerre polynomials (i.e. $N_l = 3$) with $a = 0.7$ to represent the linear model and a PWL with 8 sections partition to describe the nonlinear static gain are considered.

For the characterization of the uncertainty the Problem 14 is solved for several values of α. Figure 6 shows the functions $error_h = \sum_{i=0}^{N_l}(h_i^l + h_i^u)$ and $error_f = \sum_{i=0}^{N_n}(f_i^l + f_i^u)$ as function of α. The curves are the signal $v(k)$ as the output of the linear block and as the output of the inverse of the nonlinear block $N^{-1}(y(k))$ and the bounds computed using the parametric uncertainties.

As mentioned above, the controller is designed using the H_∞ methodology. Figure 7 shows the simulation results for set point changes when the controller is applied to the nonlinear process. Note that the system follows the set point even when a wide excursion of the reference signal is proposed.

For the robustness analysis, we use the uncertainty characterized previously for the nonlinear block which is modeled as a conic sector. This is showed in Figures 8 and 9 for Δ_f.

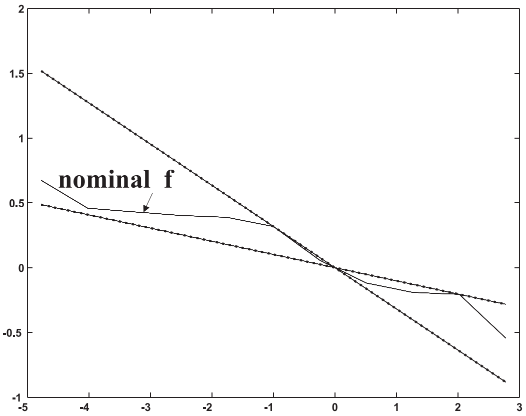

Fig. 8. Conic sector for nominal $\hat{f}$

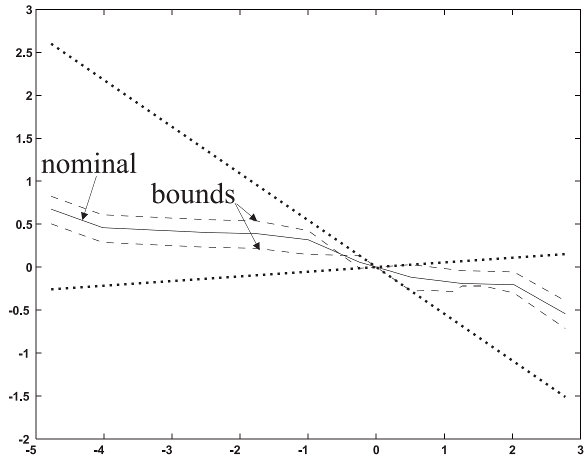

Fig. 9. Conic sector for uncertain $\hat{f}$

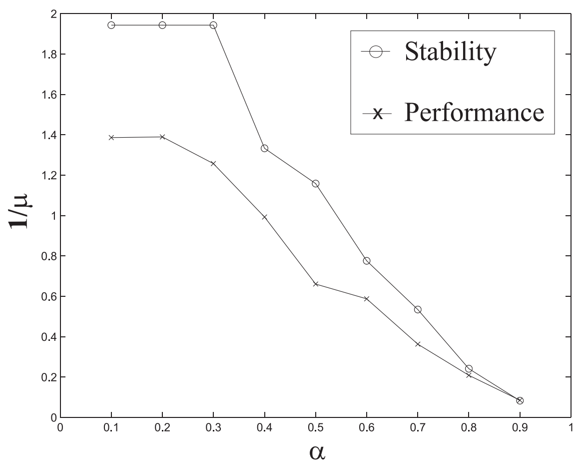

Fig. 10. μ^{-1} as function of α

Figure 10 shows the plot of μ^{-1} as measures of stability and performance margins (as function of α). From this plots it is clear that the conservatism is lower when the uncertainties are concentrated on the linear block. The analysis of performance is done by including additional blocks to add some measures of the gain from inputs to outputs.

5. CONCLUSIONS

This work dealt with robustness analysis of Wiener process. The robustness analysis was accomplished in the context of the more realistic situation in which different sources of uncertainty are present. In order to describe the model uncertainty a convenient parametric approach was followed. This strategy allows to pose the uncertainty identification problem as a standard optimization formulation. Stability aspects of the closed loop system under uncertainty were also dealt with using μ-theory. The different developed topics were tackled altogether in a neutralization reactor, an application example of significant complexity.

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