8th International IFAC Symposium on
Dynamics and Control of Process Systems
Preprints Vol.3, June 6-8, 2007, Cancún, Mexico

OPTIMAL MEASUREMENT COMBINATIONS AS CONTROLLED VARIABLES

Vinay Kariwala [*,1] **Yi Cao** [**] **S. Janardhanan** [*]

[*] *Division of Chemical & Biomolecular Engineering, Nanyang Technological University, Singapore 637722*
[**] *School of Engineering, Cranfield University, Cranfield, Bedford MK43 0AL, UK*

Abstract: Self-optimizing control is a promising method for finding appropriate controlled variables. Recently, locally optimal methods were introduced for finding controlled variables by minimizing the worst-case loss. In this paper, we extend these local methods for average-case loss minimization. Furthermore, we present a method for finding optimal combinations of measurements for local self-optimizing control, for both of worst- and average-case loss minimization. The proposed results find the optimal solution efficiently, as compared to the available techniques like non-linear optimization or null space method. The usefulness of the results is demonstrated using an evaporator case study.

Keywords: Control structure design, Controlled variables, Optimal operation, Self-optimizing control.

1. INTRODUCTION

A key step in the design of control systems for physical processes is to find the appropriate set of controlled variables (CVs). A number of methods dealing with the selection of CVs have appeared in process control literature over the past few decades; see *e.g.* Van de Wal and de Jager (2001). Recently, Skogestad (2000) introduced the concept of self-optimizing control, which is useful for selecting CVs. This method involves minimization of the economic loss incurred in indirectly optimizing the operation by holding the selected controlled variables constant, as compared to frequent online optimization.

The choice of CVs based on the general non-linear formulation of self-optimizing control can be time-consuming. To quickly pre-screen the alternatives, Halvorsen *et al.* (2003) presented an exact local method for worst-case loss minimization. This method is useful for finding the locally optimal subset or linear combinations of measurements as CVs. Halvorsen *et al.* (2003) suggested the use of non-linear optimization for finding the optimal linear combinations. Similar to any non-convex optimization problem, the non-linear optimization method can be time-consuming, and more importantly can converge to local optima. Later, Alstad and Skogestad (2007) proposed the use of the null space method, which is computationally more efficient, but suffers from the following drawbacks:

- It ignores the implementation error and thus can only provide a sub-optimal solution; see Section 2 for details.
- It holds only when the number of measurements exceeds the sum of the number of manipulated variables and disturbances.

[1] Corresponding Author: Tel: +65-6316-8746; Fax: +65-6794-7553 ; E-mail:vinay@ntu.edu.sg

Recognizing that the minimization of worst-case loss can be conservative, we extend the ideas of Halvorsen *et al.* (2003) to average-case loss minimization. We present an exact and computationally efficient solution to the problem of finding the optimal linear combinations of measurements for locally optimal self-optimizing control, both for worst- and average-case loss minimization. The results presented in this paper overcome the limitation of the null space method regarding the number of available measurements. Whenever the number of measurements is greater than the number of manipulated variables, the optimal combination matrix can be found using the proposed results. When this condition is not satisfied, the proposed results still hold, but the solution is trivial and the combination matrix can be selected to be any non-singular matrix. We demonstrate the usefulness of the results using the realistic case study of an evaporator. Due to space limitations, the proofs of the different results are omitted.

2. LOCAL SELF-OPTIMIZING CONTROL

In this section, we briefly introduce the exact local method for self-optimizing control. We denote the inputs or manipulated variables and disturbances by u and d, respectively. Let us assume that the economics of the plant are characterized by the scalar objective functional J. When the disturbances change from their nominal value, the optimal operation policy is to update $u_{\text{opt}}(d)$ according to d using an online optimizer.

A simpler strategy results when u can be indirectly adjusted using a feedback controller. In this case, the feedback controller manipulates u to hold the CVs c, close to their specified setpoints. Here, in addition to u and d, J is also affected by the error n in implementing constant setpoint policy, which results due to uncertainty and measurement noise. Thus, $J = J(u, d, n)$. The simpler strategy results in a loss and self-optimizing control is said to occur, when the loss is acceptable (Skogestad, 2000). Based on this concept, the CVs can be selected by comparing the losses for different alternatives.

Finding the best CVs based on the general nonlinear formulation of self-optimizing control can be very time-consuming. To quickly pre-screen the alternatives, Halvorsen *et al.* (2003) presented a local method. This method assumes that the set of active constraints for the nonlinear optimization problem does not change with d. The case when the set of active constraints change with disturbances is considered in (Cao, 2004; Cao, 2005).

To present the local method, let the linearized model of the process, obtained around the nominally optimal operating point, be given as

$$y = G^y\, u + G^y_d\, W_d\, d + W_n\, n \tag{1}$$

where y denotes the process measurements and the diagonal matrices W_d and W_n contain the magnitudes of expected disturbances and implementation errors associated with the individual measurements, respectively. We have $y, n \in \mathbb{R}^{n_y}$, $u \in \mathbb{R}^{n_u}$ and $d \in \mathbb{R}^{n_d}$. The CVs c are given as

$$c = H\, y = \underbrace{H\, G^y}_{G}\, u + \underbrace{H\, G^y_d}_{G_d}\, W_d\, d + H\, W_n\, n \tag{2}$$

It is assumed that the dimension of c is same as u and $G = H\, G^y$ is invertible. The second assumption is necessary for integral control.

Worst-case loss. Halvorsen *et al.* (2003) have shown that the worst-case loss is given as

$$L_{\text{worst}} = \frac{1}{2}\bar{\sigma}^2 \left(\begin{bmatrix} M_d & M_n \end{bmatrix} \right) \tag{3}$$

where $\bar{\sigma}(\cdot)$ is the maximum singular value and

$$M_d = J_{uu}^{1/2} \left(J_{uu}^{-1} J_{ud} - G^{-1} G_d \right) W_d \tag{4}$$

$$M_n = J_{uu}^{1/2} G^{-1} H W_n \tag{5}$$

Here, $J_{uu} = \frac{\partial^2 J}{\partial u^2}$ and $J_{ud} = \frac{\partial^2 J}{\partial u\, \partial d}$, evaluated at the nominally optimal operating point, respectively.

Average-case loss. The minimization of worst-case loss can be conservative, as it does not occur frequently in practice. It is more appropriate to minimize the average-case loss. Using the same steps used for finding the expression for the worst-case loss, it can be shown that the average-case loss is given as

$$L_{\text{average}} = \frac{1}{6\, n_u} \| \begin{bmatrix} M_d & M_n \end{bmatrix} \|_F^2 \tag{6}$$

where $\|\cdot\|_F$ denotes the Frobenius norm. Note that the worst-case loss in (3) is based on the maximum singular value of $[M_d \quad M_n]$. In comparison, the average-case loss in (6) is based on the average of the squared singular values of $[M_d \quad M_n]$, as $\|[M_d \quad M_n]\|_F^2 = \sum_{i=1}^{n_u} \sigma_i^2([M_d \quad M_n])$. Here, $\sigma_i(\cdot)$ denotes the i^{th} singular value of the matrix.

2.1 Selection of controlled variables

Note that the losses in (3) and (6) depend on H and the CVs are selected by minimizing the losses with respect to H. Next, we briefly discuss different approaches for selecting H.

Individual measurements. When individual measurements are selected as CVs, the optimization problem involves finding the best n_u measurements such that the loss in (3) or (6) is minimized. In this case, the elements of H are restricted to be 0 or 1 and

$$HH^T = I, \text{ where } H \in \mathbb{R}^{n_u \times n_y}. \tag{7}$$

Under minor assumptions, the exact local method minimizing the worst-case loss in (3) can be simplified to provide the minimum singular value (MSV) rule (Halvorsen *et al.*, 2003). To select the optimal subset of measurements based on the MSV rule, branch and bound based search methods have been proposed in (Cao *et al.*, 1998; Kariwala and Skogestad, 2006), which avoid enumeration of all possible alternatives. Finding efficient search methods for selection of CVs based on (3) and (6) is currently under research.

Measurement combinations. Instead of using individual measurements, it is possible to use combinations of measurements as CVs. In this case, the integer restriction of H is relaxed but the condition $\text{rank}(H) = n_u$ is still imposed. Halvorsen *et al.* (2003) used non-linear optimization for finding H, which can be very time consuming, and more importantly can converge to local optima. As an alternative, Alstad and Skogestad (2007) proposed the use of the *null space* method. In this method, the implementation error is ignored and H is selected such that

$$H \left(G^y J_{uu}^{-1} J_{ud} - G_d^y\right) = 0 \tag{8}$$

or H is in the null space of $G_y J_{uu}^{-1} J_{ud} - G_d$. It can be verified that when (8) holds, $\bar{\sigma}(M_d) = 0$. Clearly, the assumption of ignoring the implementation error is limiting and can only provide a sub-optimal solution. More importantly, for (8) to hold, it is necessary that $n_y \geq n_u + n_d$. When this condition is not satisfied, the null space method cannot be applied, which limits its application.

3. WORST-CASE LOSS MINIMIZATION

In this section, we present an exact and computationally efficient method for finding optimal linear combinations of measurements by minimizing the worst-case loss. In the following discussion, as a shorthand notation, we denote

$$Y = \left[\left(G^y J_{uu}^{-1} J_{ud} - G_d^y\right) W_d \; W_n \right] \tag{9}$$

The following lemma expresses the loss in (3) in terms of matrix inequalities and establishes the basis for finding the optimal combinations of measurements.

Lemma 1. The matrix H minimizing the loss in (3) can be found by solving

$$\min_H 0.5\,\gamma^2$$
$$\text{s.t. } H\, M_\gamma\, H^T \succeq 0 \tag{10}$$
$$\text{rank}(H) = n_u \tag{11}$$

where $M_\gamma = \gamma^2 G^y J_{uu}^{-1} (G^y)^T - YY^T$

The optimization problem posed in Lemma 1 is bilinear in H and thus is difficult to solve. The rank constraint on H further complicates solving the optimization problem. In the following proposition, using eigenvalue decomposition, we establish necessary and sufficient conditions for existence of γ and H such that (10) and (11) hold.

Proposition 2. Let $\lambda_1, \lambda_2, \cdots, \lambda_{n_y}$ be the eigenvalues of M_γ arranged in descending order. Then, there exists a non-singular matrix H such that (10) holds, if and only if (iff) γ is selected such that

$$\lambda_{n_u}(M_\gamma) \geq 0. \tag{12}$$

Intuitively, if H is a square matrix, (10) holds iff $M_\gamma \succeq 0$ or all the eigenvalues of M_γ are non-negative. Then, for $H \in \mathbb{R}^{n_u \times n_y}$, Proposition 2 requires that only the largest n_u eigenvalues of M_γ be non-negative. Furthermore, based on Proposition 2, it can be readily inferred that the γ which provides minimal worst-case loss satisfies

$$\lambda_{n_u}(M_\gamma) = 0. \tag{13}$$

This happens as γ satisfying (13) represents the minimal value of γ such that (12) holds. The matrix H can be selected as

$$H = \left[\nu_1 \; \nu_2 \; \cdots \; \nu_{n_u} \right]^T \tag{14}$$

when (13) holds. Here, $\nu_1, \nu_2, \cdots, \nu_{n_u}$ denote the n_u mutually orthogonal eigenvectors corresponding to the largest n_u eigenvalues of M_γ.

The question remains: how to find γ? The solution to this problem is proposed next, where the optimal value of γ is expressed directly in terms of G^y, J_{uu} and Y.

Proposition 3. The γ that solves (13) is given as

$$\gamma = \underline{\lambda}^{-1/2} \left(J_{uu}^{-0.5} \, (G^y)^T \, (Y\, Y^T)^{-1} \, G^y \, J_{uu}^{-0.5} \right) \tag{15}$$

where $\underline{\lambda}(\cdot)$ denotes the smallest eigenvalue.

The expression in (15) is derived using Cholesky factorization. The findings of this section are summarized in the following algorithm.

Algorithm 1. The optimal combinations of measurements which minimize worst-case loss can be found by using the following steps:

(1) Find $\gamma \geq 0$ using Proposition 3, such that (13) holds.
(2) Perform an eigenvalue decomposition of $\left(\gamma^2 G^y J_{uu}^{-1} (G^y)^T - Y Y^T\right)$ and find the eigenvectors corresponding to the largest n_u eigenvalues.
(3) Choose H as given in (14).

When the implementation error is ignored, Algorithm 1 gives same results as the null space method (Alstad and Skogestad, 2007). Note that the optimal combination matrix found using Algorithm 1 is non-unique and pre-multiplying H by a non-singular matrix also provides an optimal solution. The non-singular matrix can be selected, for example, to make HG^y diagonal.

4. AVERAGE-CASE LOSS MINIMIZATION

In this section, we derive results for finding the optimal combinations of measurements for average-case loss minimization.

Lemma 4. The matrix H minimizing the loss in (6) can be found by solving

$$\begin{aligned} \min_H \ & \frac{1}{6\, n_u} \operatorname{trace}(X) \\ \text{s.t. } & X \succeq 0 \\ & \operatorname{rank}(H) = n_u \end{aligned} \tag{16}$$

$$H \left(G^y J_{uu}^{-0.5} X J_{uu}^{-0.5} (G^y)^T - Y Y^T\right) H^T \succeq 0 \tag{17}$$

The optimization problem in Lemma 4 is a restatement of (6) in terms of matrix inequalities. Here, the similarity between the optimization problems posed in Lemmas 1 and 4 is noteworthy. When $X = \gamma \cdot I$, the two optimization problems are equivalent. Similar to worst-case loss minimization, the optimization problem for average-case loss minimization is also bilinear in H and the solution is difficult. The following proposition provides the optimal value of X, which is subsequently used to find the combination matrix that minimizes average-case loss.

Proposition 5. For the optimization problem posed in Lemma 4, it is optimal to select X as

$$X = \left(J_{uu}^{-0.5} (G^y)^T (Y Y^T)^{-1} G^y J_{uu}^{-0.5}\right)^{-1} \tag{18}$$

Similar to (15), (18) is derived using Cholesky factorization. Let $\tilde{\nu}_1, \tilde{\nu}_2 \cdots \tilde{\nu}_{n_u}$ be the mutually orthogonal eigenvectors corresponding to the largest n_u eigenvalues of $\left(G^y J_{uu}^{-0.5} X J_{uu}^{-0.5} (G^y)^T - Y Y^T\right)$, where X is given by (18). Then the optimal combination matrix can be selected as

$$H = \begin{bmatrix} \tilde{\nu}_1 & \tilde{\nu}_2 & \cdots & \tilde{\nu}_{n_u} \end{bmatrix}^T. \tag{19}$$

Similar to worst-case loss minimization, the choice of H in (19) is not unique and pre-multiplication by a non-singular matrix also provides a valid solution. The procedure outlined in Algorithm 1 can also be used for finding optimal measurement combinations of measurements with minor modifications. We next present an attractive property of minimizing average-case loss.

Corollary 6. The matrix H in (19) also minimizes the worst-case loss.

Based on Corollary 6, the matrix H in (19) is super-optimal. Note that the converse is not true, *i.e.* the matrix H minimizing the worst-case loss does not necessarily minimizes the average-case loss. In this sense, selection of matrix H by minimizing average-case loss is advantageous.

5. EVAPORATOR CASE STUDY

The optimal measurement combination design approach is applied to the evaporation process of Newell and Lee (1989). This is a "forced-circulation" evaporator, where the concentration of dilute liquor is increased by evaporating solvent from the feed stream through a vertical heat exchanger with circulated liquor. The process variables are listed in Table 1 and model equations are given in (Cao, 2005).

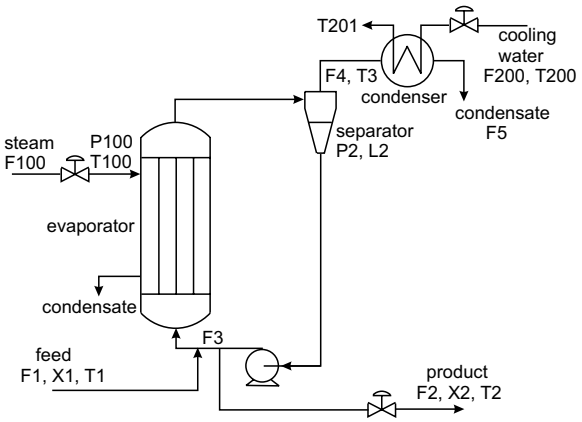

Fig. 1. Evaporator System

The economic objective is to maximize the operational profit [$/h], formulated as a minimization problem of the negative profit (20). The first three terms of (20) are operational costs relating to steam, water and pumping; see *e.g.* (Heath *et*

al., 2000; Wang and Cameron, 1994). The fourth term is the raw material cost whilst the last term is the product value, both of which provide an extra degree of freedom for self-optimizing study.

$$J = 600F_{100} + 0.6F_{200} + 1.009(F_2 + F_3) + 0.2F_1 - 4800F_2 \quad (20)$$

The process has the following constraints related to product specification, safety and design limits:

$$X_2 \geq 35 + 0.5\% \quad (21)$$

$$40 \text{ kPa} \leq P_2 \leq 80 \text{ kPa} \quad (22)$$

$$P_{100} \leq 400 \text{ kPa} \quad (23)$$

$$0 \text{ kg/min} \leq F_{200} \leq 400 \text{ kg/min} \quad (24)$$

$$0 \text{ kg/min} \leq F_1 \leq 20 \text{ kg/min} \quad (25)$$

$$0 \text{ kg/min} \leq F_3 \leq 100 \text{ kg/min} \quad (26)$$

Note that a 0.5% back-off has been enforced on X_2 to ensure that the variable remains feasible for all possible disturbances. The process model has three state variables, L_2, X_2 and P_2 with eight degrees of freedom. Three of them are disturbances, X_1, T_1 and T_{200}. The rest five degrees of freedom are manipulable variables, F_1, F_2, P_{100}, F_3 and F_{200}. The optimization problem in (20) with process constraints (21)-(26) is solved for the following nominal disturbances:

$$d = \begin{bmatrix} X_1 & T_1 & T_{200} \end{bmatrix}^T = \begin{bmatrix} 5 & 40 & 25 \end{bmatrix}^T. \quad (27)$$

The minimum negative profit obtained is -582.23 [\$/h] and corresponding values of process variables are shown in Table 1. At the optimal point, there are two active constraints, $X_2 = 35.50\%$ and $P_{100} = 400$ [kPa]. These two constraints remain active within the whole disturbance region, which is defined as $\pm 20\%$ of the nominal disturbances. The reader is referred to (Cao, 2005) for physical explanation of these two active constraints.

These two active constraints plus the separator level, which has no steady-state effect on the plant operation, but must be stabilized at its nominal setpoint, consume three degrees of freedom. Therefore, there are two degrees of freedom left for self-optimizing control. We select $u = [F_{200}, F_1]^T$ and $y = [P_2, T_2, T_3, F_2, F_{100}, T_{201}, F_3, F_{200}, F_1]^T$. Due to the high cost involved in the measurement of vapor flowrate, F_4 is not considered for measurement here. Using MATLAB® 2006a symbolic toolbox, the following Hessian and gain matrices are obtained at the nominally optimal operating point:

$$J_{uu} = \begin{bmatrix} 0.006 & -0.133 \\ -0.133 & 16.737 \end{bmatrix}, J_{ud} = \begin{bmatrix} 0.023 & 0 & -0.001 \\ -158.373 & -1.161 & 1.484 \end{bmatrix}$$

$$G^y = \begin{bmatrix} -0.093 & 11.678 \\ -0.052 & 6.559 \\ -0.047 & 5.921 \\ 0 & 0.141 \\ -0.001 & 1.115 \\ -0.094 & 2.170 \\ -0.032 & 6.594 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}, G_d^y = \begin{bmatrix} -3.626 & 0 & 1.972 \\ -2.036 & 0 & 1.108 \\ -1.838 & 0 & 1 \\ 0.267 & 0 & 0 \\ -0.317 & -0.018 & 0.020 \\ -0.674 & 0 & 1 \\ -2.253 & -0.066 & 0.673 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Table 1. Variables and Optimal Values

Var.	Description	Value	Units
F_1	Feed flowrate	9.47	kg/min
F_2	Product flowrate	1.33	kg/min
F_3	Circulating flowrate	24.72	kg/min
F_4	Vapor flowrate	8.14	kg/min
F_5	Condensate flowrate	8.14	kg/min
X_1	Feed composition	5	%
X_2	Product composition	35.50	%
T_1	Feed temperature	40	°C
T_2	Product temperature	88.40	°C
T_3	Vapor temperature	81.07	°C
L_2	Separator level	1	meter
P_2	Operating pressure	51.41	kPa
F_{100}	Steam flowrate	9.43	kg/min
T_{100}	Steam temperature	151.52	°C
P_{100}	Steam pressure	400	kPa
Q_{100}	Heat duty	345.29	kW
F_{200}	Cooling water flowrate	217.73	kg/min
T_{200}	Inlet C.W. temperature	25	°C
T_{201}	Outlet C.W. temperature	45.55	°C
Q_{200}	Condenser duty	313.21	kW

The disturbance weighting matrix is based on $\pm 20\%$ of nominal disturbance, *i.e.* $W_d = \text{diag}(1, 8, 5)$. The implementation error in measurement leads to $W_n = \text{diag}(0.1,\ 0.1,\ 0.1,\ 0.1,\ 0.01,\ 0.1,\ 0.01,\ 0.01,\ 0.01)$.

For this model, the best individual measurements were found to be P_2 and F_3, for which the local loss is 863.05 [\$/h]. In comparison, when the optimal combinations of all the measurements is used, the local worst-case loss decreases to 5.673 [\$/h]. In practice, use of combinations of all available measurements is often not necessary. We use a branch and bound method to find the best n out of 9 measurements, $n < 9$, whose combinations can be used as CV. The lowest worst-case losses for the best combinations of 2 to 8 measurements were found to be 863.05, 80.38, 12.26, 6.73, 6.14, 5.76 and 5.71 [\$/h] respectively. It is clear that having combinations of 4 measurements gives the best trade off between complexity and incurred worst case loss. The optimal 4-measurement set consists of P_2, T_{201}, F_{200} and F_1 . The optimal worst-case (H_w) and average-case (H_a) combination matrices are

$$H_w = \begin{bmatrix} 113.599 & -225.518 & -9.71 & -837.243 \\ 4.991 & -9.73 & -0.454 & -36.169 \end{bmatrix}$$

$$H_a = \begin{bmatrix} 117.954 & -230.113 & -9.739 & -878.13 \\ 4.991 & -9.73 & -0.454 & -36.172 \end{bmatrix}$$

which are scaled versions of combination matrices obtained using (14) and (19) such that HG^y is diagonal. Though these combination matrices seem promising based on local analysis, the nonlinear simulation shows that the use of measurement combinations is only marginally advantageous than controlling the individual measurements; see Table 2 for details.

The difference between the local analysis and nonlinear simulation arises as for $\pm 20\%$ disturbance variations, the linearized model incurs significant

model errors. The model errors are estimated through Monte Carlo simulation with feasible disturbance scenarios and assigned to the measurement error matrix, where W_{n1} = diag(57.5393, 32.3579, 29.2217, 0.2052, 0.5771, 59.5280, 17.1361, 0.1, 0.1).

For the revised model, the best individual measurements are found to be F_{200} and F_{100} with the worst-case local loss being 880.85 [$/h]. The branch and bound method shows that the local worst-case loss for the best four measurement set consisting of F_4, F_2, F_{200} and F_1 is 305.89 [$/h], which is close to the loss incurred using combinations of all the measurements, 304.68 [$/h]. Using these measurements, the following combination matrices for minimizing the worst-case (H_{w1}) and average-case (H_{a1}) losses are calculated:

$$H_{w1} = \begin{bmatrix} -37.861 & 5.859 & -0.182 & 1.0 \\ -24.348 & 3.791 & 0.031 & 0.005 \end{bmatrix}$$
$$H_{a1} = \begin{bmatrix} -598.381 & 71.926 & 0.618 & 1.088 \\ -24.43 & 3.8 & 0.031 & 0.005 \end{bmatrix}$$

Using H_{w1}, the worst-case and average losses are 305.897 and 71.65 [$/h], whilst applying H_{a1}, the worst-case and average losses are 305.897 and 51.05 [$/h], respectively. Therefore, the local analysis shows that using H_{a1} is slightly better than using H_{w1} in the sense of average loss minimization. Also note that the use of H_{a1} minimizes worst-case loss as well.

To verify above results, the six designs are tested through nonlinear dynamic simulation. The average hourly loss values based on 50 hours simulated operation are calculated in Table 2. The results clearly show that model errors caused by linearization have a significant effect on self-optimizing control design. By including model errors in the error weighting matrix, the linear combination matrices effectively reduce the control loss.

Table 2. Average loss [$/h] based on 50-hour simulated operation

Error Model	individual measurements	combination average	combination worst-case
W_n	279.6	266.381	273.361
W_{n1}	267.485	**156.188**	167.305

6. CONCLUSIONS

In addition to the worst-case loss minimization, controlled variables can also be selected for self-optimizing control through average-case loss minimization. Efficient algorithms are developed for both minimization problems to design the optimal measurement combinations. The combinations obtained through average loss minimization also minimize the worst-case loss, hence are super-optimal. These design approaches are demonstrated via the evaporation case study. The case study shows that both designs are able to achieve self-optimizing control. The results also reveal that modelling error has a significant effect on self-optimizing control performance.

REFERENCES

Alstad, V. and S. Skogestad (2007). Null space method for selecting optimal measurement combinations as controlled variables. *Ind. Eng. Chem. Res.* **46**(3), 846–853.

Cao, Y. (2004). Constrained self-optimizaing control via differenratiation. In: *Proc. 7th International symposium on ADCHEM*. Hong Kong, P. R. China. pp. 63–70.

Cao, Y. (2005). Direct and indirect gradient control for static optimisation. *Internationl J. of Automation and Computing* **2**(1), 60–66.

Cao, Y., D. Rossiter and D. H. Owens (1998). Globally optimal control structure selection using branch and bound method. In: *Proc. IFAC symposium on DYCOPS 5*. Corfu, Greece. pp. 183–188.

Halvorsen, I. J., S. Skogestad, J. C. Morud and V. Alstad (2003). Optimal selection of controlled variables. *Ind. Eng. Chem. Res.* **42**(14), 3273–3284.

Heath, J. A., I. K. Kookos and J. D. Perkins (2000). Process control structure selection based on economics. *AICHE J.* **46**(10), 1998–2016.

Kariwala, V. and S. Skogestad (2006). Branch and bound methods for control structure design. In: *Proc. 16th ESCAPE and 9th International Symposium on PSE*. Garmisch-Partenkirchen, Germany.

Newell, R. B. and P. L. Lee (1989). *Applied Process Control – A Case Study*. Prentice Hall. Englewood Cliffs, NJ.

Skogestad, S. (2000). Plantwide control: The search for the self-optimizing control structure.. *J. Proc. Control* **10**(5), 487–507.

Skogestad, S. and I. Postlethwaite (2005). *Multivariable Feedback Control: Analysis and Design*. 2nd ed.. John Wiley & sons. Chichester, UK.

Van de Wal, M. and B. de Jager (2001). A review of methods for input/output selection. *Automatica* **37**(4), 487–510.

Wang, F. Y. and I. T. Cameron (1994). Control studies on a model evaporation process – constrainted state driving with conventional and higher relative degree systems. *J. Process Control* **4**(2), 59–75.