8th International IFAC Symposium on
Dynamics and Control of Process Systems
Preprints Vol.3, June 6-8, 2007, Cancún, Mexico

CONTROL STRUCTURE ANALYSIS AND DESIGN FOR NONLINEAR MULTIVARIABLE SYSTEMS

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Abstract: A new approach to control structure analysis and design for nonlinear multivariable systems is proposed in this paper. By integrating the concept of gap metric, factorial design and multivariate projection methods, a systematic analysis of control structures, operating conditions and operating regime based control strategies can be established. The application to an extractive fermentation process shows how the proposed method systematically guides the designer to determine the best control structure and strategy for the process.

Keywords: control structure design, nonlinear systems, extractive fermentation, gap metric, multivariate projection methods, factorial design.

1. INTRODUCTION

Most of biochemical processes are inherently nonlinear and multivariable in nature. Their dynamics unavoidably presents an extraordinary challenge for process control. Unlike the linear multivariable systems, it is rather difficult to quantify process controllability for the nonlinear cases because the operating regime would affect their controllability. This means that a control structure that works well in one particular operating regime may perform poorly in different operating regimes. Hence this has a very important implication particularly on the design of the operating regime-based controller for the nonlinear systems.

Another key challenge in the control structure design for a nonlinear MIMO process is that the number of manipulated variables is often smaller than that of controlled variables. In this case, the concept of partial control is frequently adopted where the dominant outputs are identified and selected for controlling the overall objectives (quality variables) within the desired ranges i.e. it is difficult to control the quality variables precisely at desired values. Hence, the problems pertinent to the plant controllability are vital and it has become one of the key issues in the process design other than profitability.

The synthesis of control structures for a multivariable system can be performed in two steps: (1) generation of feasible control configurations, and (2) evaluation and selection of a control structure/configuration (Daoutidis and Kravaris, 1992). With regard to the second step, the application of Bristol relative gain array (RGA) technique has been most widely used for control structure design due to its simplicity i.e. only requires steady-state gains. The RGA has also been shown to relate to many other fundamental closed-loop system properties such as (1) robustness, (2) decentralized integral controllability, and (3) fault tolerance (see Chen and Seborg, 2002).

Despite its capability to measure process interactions, the RGA application has been limited to linear multivariable systems. Currently, very few measures of interactions have been proposed for nonlinear MIMO processes (Guay, *et al,* 1997). Arbel et al (1996) showed that the choice of control structure and variables in the complex nonlinear processes have far more reaching impact on the overall performance of control system than that of multivariable algorithms to be used. Nevertheless, the issues pertaining to the control structure decisions are in most cases made in ad hoc manner, which are based on experience and engineering insights, without considering the details of the problems (Skogestad, 2004).

Over the last three decades, vast research efforts have been dedicated to exploring and studying the control structure analysis particularly for the multivariable chemical processes. Some of the leading works in this area are the systematic procedure for regulatory control structure selection (Hovd and Skogestad, 1993), control structure selection based on linear dynamic economics coupled with MILP method (Narraway and Perkins, 1993), mathematical measure to generate alternative control structure coupled with decomposition technique (Morari *et al*, 1980) and examination of control-law nonlinearity by optimal control structure approach (Stack and Doyle, 1997). In general, an advantage of these generic approaches is that the ability to address the detailed problems related with the control structures, but still at the expense of requiring extensive process knowledge and experiences. Thus, the exploitation of extensive process knowledge and experiences although within the context of systematic methodology and together with the associated computational burden (i.e. problem formulation) could become a prohibitive factor, particularly to those designers who are relatively lack of experiences and detailed knowledge about the process.

Recently, the concept of gap metric has been widely adopted in various aspects related to controllability issues. Some examples of its applications are the design of decentralized control (Samyudia, *et al,* 1995), multilininear model-based controller design (Galan, *et al,* 2003), control design for recycled multi unit processes (Samyudia and Kadiman, 2003) and nonlinearity analysis of boiler-turbine unit (Tan, *et al,* 2005). Additionally, the gap-metric could also be used to quantify the closed-loop process nonlinearity and provide the basis for designing the operating-regime based controller of nonlinear processes (Arslan, *et al,* 2004). However unlike RGA and its many variants such as the most recently reported of effective RGA (Xiong *et al*, 2006), the adoption of v-gap metric as a tool to analyze control structure issues is still scarcely reported in the open literature. The key advantage of gap-metric over the other controllability measures such as RGA and condition number is its inherent ability to systematically handle plant uncertainties arising from the situations such as plant/model mismatch, parameters changes and unmodelled dynamics.

The aim of this paper is to propose a new method for control structure analysis of nonlinear multivariable systems based on the integration of the concepts of v-gap metric, multivariate projection technique and factorial design experiment. The key advantage of the proposed methodology rests on its simplicity where the extensive process knowledge and experiences are not required, provided the nonlinear process model is available. Also unlike simple RGA analysis that is based only on the static gains, the proposed method is able to take into consideration of both process dynamic and uncertainties.

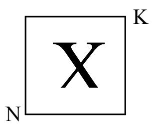
2. PRELIMINARIES

2.1 v-Gap Metric

For the details about the concept and definition of v–gap metric $\delta_v(P_1, P_2)$, the interested readers could refer to (Vidyasagar, 1985; Vinnicombe, 2001). An important property of v-gap is its relationship with the robustness-performance indicator ($b_{P,C}$) of an optimal loop-shaping controller (McFarlane and Glover, 1992). If $\delta_v(P_1, P_2) < b_{P_1,C}$, the controller C designed based on the model P_1 is guaranteed to be stable for P_2. Furthermore, if the value of δ_v is small compared to $b_{P_1,C}$ the performance of the controller will be comparable for both plant models. However, if $\delta_v(P_1, P_2) > b_{P_1,C}$ then the stability is not guaranteed but this does not imply that the system is unstable when the controller C is used to control P_2. Nevertheless this normally leads to performance degradation of the controller.

2.2 Multivariate Projection Methods

Multivariate projection methods of Principal Component Analysis (PCA) form the basis for multivariate data analysis. Let us consider a large set of measurement data, ***X***, which are collected from the N observations and for K variables as follows:



In industrial applications, one often finds that the variables in the data set, ***X***, are highly correlated, and their covariance matrix $\boldsymbol{\Lambda}$ is nearly singular (or not full rank). PCA provides an approximation of the large data set, ***X***, in terms of product of two small matrices ***T*** and $\boldsymbol{P^T}$ that capture the essential data patterns of ***X*** (Wold, *et al,* 1987, Kourti and MacGregor, 1995).

$$X = TP^T + E = \sum_{i=1}^{A} t_i p_i^T + E \qquad (1)$$

Where E is the residual (or error) matrix and $A \le K$. If A=K, then E = 0 as all the variability directions are described. By representing the data set as in Eq. (1), the principal components (***T***) are orthogonal to each other. For example, the first principal component (PC) of x is defined as a linear combination $t_1 = xp_1$ that has maximum variance subject to $|p_1| = 1$. The second PC is that a linear combination $t_2 = xp_2$. It has the next greatest variance subject to $|p_2| = 1$, and subject to the condition that it is orthogonal (or uncorrelated) to the first PC. Plotting the columns of $\boldsymbol{T}$ gives a picture of the dominant "observation patterns" of $\boldsymbol{X}$, while plotting the rows of $\boldsymbol{P}^T$ shows the complementary "variable patterns".

3. CONTROL STRUCTURE ANALYSIS AND DESIGN

The proposed control structure analysis and design is summarized in a systematic procedure as follows:

Step 1 – Generation of operating regimes. Select a set of nominal operating levels, and for each operating level apply step inputs perturbations using a factorial design concept. For example in this paper, the number of inputs perturbations is generated based on three inputs leading to eight perturbed operating levels ($P_1, P_2, P_3, ...P_8$).
Step 2 – Generation of linear model sets. Linearize the nonlinear process model at each nominal operating level and its corresponding perturbed levels. In the case of three inputs, this would lead to nine linear models for each nominal operating conditions i.e. one at nominal conditions (P_0) and the other 8 at perturbed levels ($P_1, P_2, ...P_8$).
Step 3 – Quality variable data generation. Determine the quality variables such as productivity, yield, etc corresponding to each perturbed level. Also, compute the v-gap between the nominal and perturbed levels.
Step 4 – PCA Analysis. Gather and combine all the generated data on the process parameters, input-output variables and the computed quality variables in Step 3 to form a data matrix $\boldsymbol{X}$ as in Section 2.2. Scale and normalize the data and then apply the PCA analysis. The key objective is to reduce the dimensionality and hence facilitates the identification of the sets of data, which have strong correlations.
Step 5 – Control structure design. Based on the PCA analysis, decisions would be made with regards to control structure, selection of operating conditions and operating regime-based control strategies.

3. EXTRACTIVE FERMENTATION – A CASE STUDY

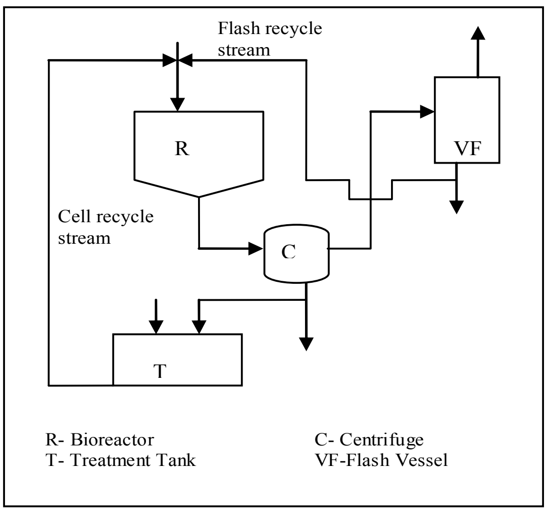


Fig. 1. The schematic diagram of the extractive alcoholic fermentation (Silva, *et al,* 1999).

The conventional alcoholic fermentation is a typical product inhibitory process, where the accumulation of ethanol concentration above ca. 12 %(v/v) would drastically reduce the cell growth and ethanol production rates (Minier and Goma, 1982). The simultaneous removal of ethanol and fermentation through extractive fermentation techniques is a well-accepted method to increase productivity and yield in alcoholic fermentation. Some examples of extractive alcoholic fermentation are: fermentation with membrane distillation (Gryta, *et al*, 2000), liquid-liquid extraction (Gyamerah and Glover, 1996) and vacuum flash vessel (Silva, *et al,* 1999). Silva *et al* (1999) showed that the application of fermentation coupled with vacuum flash vessel (see Fig. 1) has a number of advantages such as: (1) ease of operation, (2) low cost, (3) heat exchanger is not required, (4) low inhibitory conditions for yeast cells, and (5) high inhibitory conditions for contaminants.

The performance of a biochemical plant depends strongly on the dynamic operability of fermentation process. Kuhlmann *et al* (1997) showed that by using the concept of 'perfect control' the linear controllability analysis is a valuable tool in evaluating and comparing the controllability properties of different configuration at an early stage of the fermentation design. By using the concept of factorial design, Costa *et al* (2001) studied the control structure of the extractive fermentation originally proposed by Silva *et al* (1999). It was shown that the operating conditions have a strong influence on the performance of the control algorithm. Besides, Nandong *et al* (2006) extended the study of the single-stage design by Silva *et al* (1999) to two-stage design where the performance of the two designs was compared based on the steady-state economic and dynamic operability measures. It was shown that by using the two-stage design, the total bioreactor volume that accomplished comparable yield and productivity as the single-stage design could be reduced by about 30%. However, based on the dynamic controllability analysis, the single-stage design showed more favourable dynamic operability than the two-stage design.

4. RESULTS AND DISCUSSION

Table 1 Variables used in the multiprojection analysis

v-gap	*v-gap metric*	Yield	*Ethanol Yield*
Prod	*Productivity*	X_v	*Cell density*
R	*Cell recycle ratio*	T	*Temperature*
r	*Flash recycle ratio*	S	*Substrate concentration*
Fo	*Feed substrate flowrate*	L	*Fermenter holdup level*
S_0	*Feed substrate concentration*	Rp	*Ethanol production rate*
EtOH	*Ethanol concentration*	Rx	*Cell growth rate*
D	*Dilution rate*	Rs	*Substrate utilization rate*

Table 1 shows the 16 variables consisting of process parameters (Rx, Rp, Rs), inputs (R, r, F_0, S_0, D), outputs (X_v, S, EtOH, T, L) and quality variables (v-gap, productivity, yield) used in the multiprojection analysis. There are 18 observations (P0, P1,…P17) in total and due to space limitation only 6 numbers of observations for some variables are shown in Table 2. Fig. 2 shows the boxplot of the data set with P0 as the reference. Two sets of multiprojection analyses were performed; one with the operating level P0 as the nominal model and another with P16.

Table 2 Values of some variables for a number of observations P

P	v-gap P0	v-gap P16	X_v (kg/m^3)	EtOH (kg/m^3)	T (^0C)
P0	0	0.8353	32.8	36.8	31.0
P1	0.7023	0.9569	31.7	29.4	30.1
P9	0.7336	0.9363	22.4	21.9	28.9
P10	0.7059	**0.9119**	32.0	38.7	31.1
P16	**0.8353**	0	**10.6**	**22.6**	**29.8**
P17	0.2748	0.7369	28.5	40.0	31.1

Case 1 – P0 as the nominal model

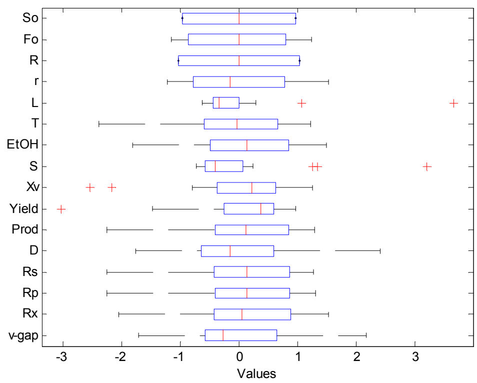


Fig. 2 Boxplot of scaled data set with P0 as reference.

PCA analysis for each set of data yielded two matrices i.e. scores ***T*** and loading $\boldsymbol{P}^T$ matrices. The plot of score and loading matrices for the first principal component PC1 and second principal component PC2 is shown in Fig. 3. In this case since PC1 and PC2 explained more than 75% of variances in data set ***X***, plot of score and loading for these two components is sufficient to explain the behaviour of the data set.

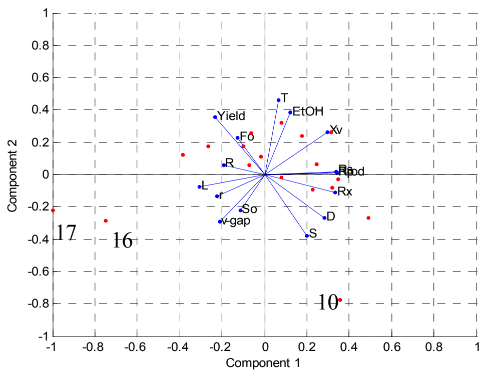


Fig. 3. Plot of PC score and loading for two principal components PC1 and PC2: P0 as the reference.

Fig. 3 shows three outliers i.e. observations 10, 16 and 17. The outliers 16 and 17 are due to 9 variables (v-gap, S_0, L, T, EtOH, X_v, Rp, Rs and Productivity). To identify the variable/s responsible for trends shown by observations P16 and P17, the original data set is reduced to only 9 variables from originally 16. The other 7 variables (Yield, F_0, R, Rx, D and S) are probably responsible for the *abnormal* observation P10. In this case, we only focus our analysis on outliers 16 and 17 because the v-gap has some contributions to their trends.

On the reduced data set, another PCA analysis was performed where the plot of score and loading matrices for PC1 and PC2 is shown in Fig. 4. From the figure it can be concluded that there are four variables (v-gap, EtOH, X_v and T) responsible for the outlier 16. Note that the outlier 17 is due to the other five variables (r, S_0, L, Prod, Rs and Rp). Furthermore, the outlier 16 is solely positively correlated to v-gap, and negatively correlated to X_v, EtOH and T.

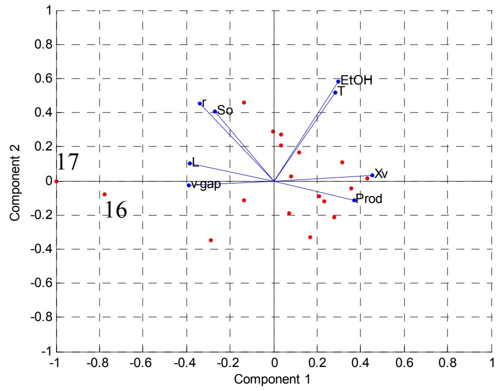


Fig. 4. Plot of PC score and loading for two principal components PC1 and PC2 for the reduced data with P0 as reference.

From the inspection of the original data set, the outlier 16 is due to the abnormally large value of v-gap (see Table 2), which is the largest value within the observations. Therefore based on the analysis results shown in Fig. 4 and inspection of the original data set, the value of v-gap is largely influenced by the bioreactor cell concentration X_v, ethanol concentration EtOH and temperature. This implies that to reduce the value of this v-gap it is necessary to increase the values of X_v, EtOH and T.

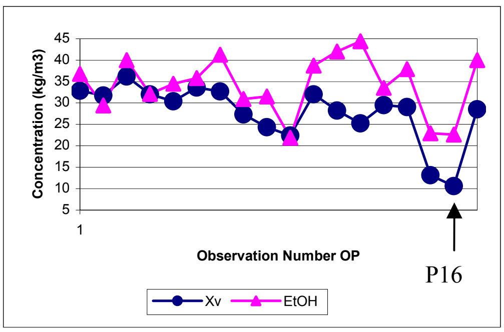


Fig. 5. Plot of Xv and EtOH against the observation number P.

From Fig. 5, it can be shown that at P16 among the variables the value of X_v is abnormally low compared to its average values. Hence this is the key deviation that causes the large deviation in v-gap at P16. Consequently, to reduce the v-gap or improve the controllability of the process, it is therefore very important to ensure that the cell concentration inside the fermenter is above a certain threshold value (i.e. lower bound on X_v exists).

Case 2 – P16 as the nominal model

Another set of PCA analysis was repeated with the second data set using P16 as the nominal model. Only v-gap values change while other variables remain the same. The result of score and loading plot is shown in Fig. 6.

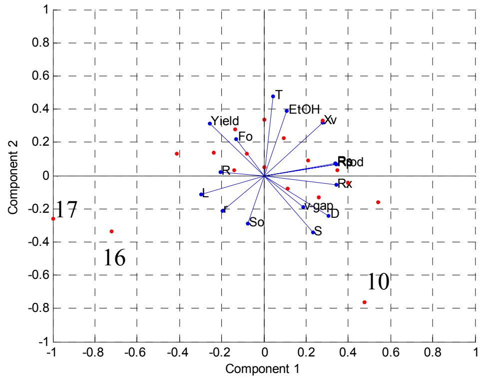


Fig. 6. Plot of PC score and loading for two principal components PC1 and PC2: P16 as the reference.

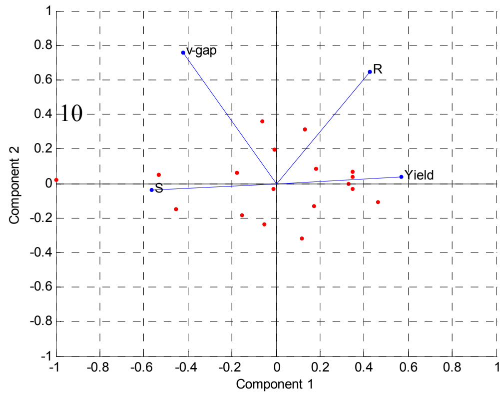


Fig. 7. Plot of PC score and loading of reduced data for two principal components PC1 and PC2: P16

As seen, there are three outliers at 10, 16 and 17 where in this case the v-gap is in the same quadrant as the outlier at 10. Following the same procedure as for the first data set, a subsequent PCA analysis on the reduced data set was performed and Fig. 7 shows the result. The v-gap is the only variable responsible for the outlier 10. It is also important to note that the outlier 10 is not correlated to any other variable. Since the v-gap observed at 10 is very large (but not the maximum one), unlike in the first case (where v-gap could be reduced effectively by increasing X_v, EtOH and T) the v-gap seems to show a lack of effective correlation with the other variables i.e. measured output. Another point to note is that overall the v-gap values when using P16 as the nominal model are mostly larger than those using P0 as the nominal model.

Control Structure and Operating Points

There are two important implications that can be drawn from the proposed analysis on the controllability measures of the extractive fermentation process. The first implication is on the existence of lower bound values for EtOH, X_v and T where below which the dynamic operability of the extractive fermentation would be unfavorable. This is indicated by a large v-gap value when X_v is small. As a result, if the nominal operating conditions selected were based on the low values of X_v, EtOH and T then the dynamic behaviour of the system would be more difficult to control as indicated by large value of v-gap.

The second important implication is on the choice of control structure i.e. what measured output variables should be controlled as to ensure favourable dynamic operability. As for this extractive fermentation case, there are other 3 output variables, which are dead cell concentration, holdup level and substrate concentration. Among these variables it is intuitively clear that the level control is essential to establish materials balance. However, there are other 5 variables to control in the face of smaller number of possible manipulated variables. In this case there are only three potential manipulated variables (MVs); cell recycle ratio, flash stream recycle ratio and feed flowrate. In view of the limited number of MVs available, we have to choose the most important variables (other than holdup level) to control. Our analysis using the proposed framework reveals that when operating at P0, because the v-gap (indicator of dynamic operability) has been shown to have relationships to EtOH, X_v, and T, it is important to ensure that these variables are given a high priority to control. Otherwise, the drift in one or more of these variables (if uncontrolled) due to disturbances occurrences would severely change the dynamic behaviour of the overall process. Consequently, this would severely penalize the performance of the linear controller used.

Furthermore, the proposed analysis framework also indicates that the effective control structure is rather strongly dependent on the choice of nominal operating levels. Take for an example, when the nominal operating condition is chosen at P16, there is no particular structure that can ensure efficient dynamic operability. Hence choosing such operating conditions would inevitably lead to difficult control problems (i.e. absence of effective control structure). In addition to the results obtained via multiprojection analysis, the data set gathered points out the difficulty of controlling the extractive fermentation using P16 as nominal operating level i.e. most of the v-gap average value (for 18 observations) of 0.73 compare to only 0.45 when using P0 as the nominal operating level.

For each P0 and P16 a robust loop-shaping controller as proposed by McFarlane and Glover (1992) is designed with similar optimal indicator b_{opt} of 0.3. The controller designed based on P16 is then used to control the operating conditions at P0 (denoted as

K16-P0) and that designed at P0 to control P16 (K0-P16). The closed-loop performances subject to disturbance in S_o (25% change) are compared for both cases as shown in Fig.8. The configuration of K16-P0 is unstable while that of K0-P16 shows relatively superior performance. This result confirms the importance of control structure selection, which in this case it is strongly dependent on the nominal operating conditions. Surprisingly, analysis of RGA for the two nominal operating levels shows a little change in its RGA values implying no change of control structure for both operating points. Hence, the change in control structure due to the shift in operating level cannot be adequately captured by the simple RGA analysis.

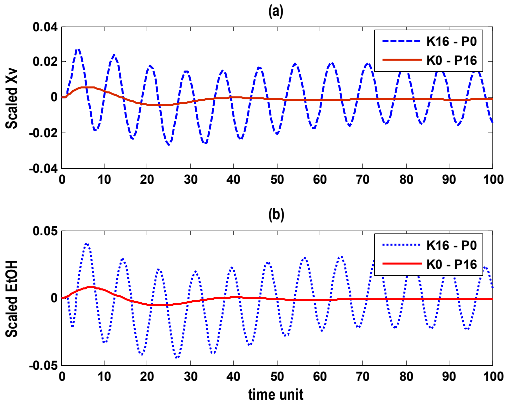


Fig. 8. Closed-loop performance: (a) scaled X_v and (b) scaled EtOH

5. CONCLUSIONS

In this paper, a new method for control structure analysis and design has been proposed and applied to a case study of extractive fermentation process. The proposed analysis framework could be used to answer some of the critical questions pertinent to controllability issues such as: (1) choice of control structure, (2) operating conditions and (3) identification of process constraints, which should not be violated for a good dynamic operability.

ACKNOWLEDGEMENTS

We acknowledge Dr. Chua Han Bing for his comments and CSRF for financial support.

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