SUPERVISED ADAPTIVE PREDICTIVE CONTROL USING DUAL MODELS

Eduardo J. Dozal-Mejorada, Priyesh Thakker and B. Erik Ydstie

Chemical Engineering Department
Carnegie Mellon University
Pittsburgh, PA USA 15213
{edozal,pthakker,ydstie}@andrew.cmu.edu

Abstract: A new adaptive control method based on supervision is presented. The algorithm only updates the control estimates when new relevant information is obtained. The technique is based on a dual-model approach. The first model acts as the control model and the second model is the supervisor which determines when the control model should update its estimated parameters. Simulation results using a CSTR illustrate the set point tracking capabilities of the algorithm. Experimental studies on a heat exchanger showcase how the algorithm successfully suppresses output bursting.

Keywords: adaptive control; self tuning control; supervised control; stability; robustness

1. INTRODUCTION

Adaptive control is a method to optimize control parameters in uncertain and time-varying systems. In a typical application a linear transfer function model is estimated using input output data and recursive least squares. Once the model has been obtained it can be used to design a number of different controllers, such as predictive control, extended horizon and pole-placement. Descriptions of these and many other effective adaptive control methods are presented by Goodwin and Sin (1984), Åström and Wittenmark (1995) and Mareels and Polderman (1996). Many different approaches and applications of adaptive control have been reported in the literature. Furthermore, commercial software systems for adaptive process control have been developed as well.

Adaptive control has not won widespread acceptance in the chemical process industries, however. The main reason for this is that many of the proposed adaptive control algorithms are not robust with respect to un-modeled dynamics and noise. The problem was recognized in the early adaptive control literature (Rohrs *et al.*, 1985) and many methods were subsequently proposed to make the adaptive controllers more stable. One of the most effective techniques is the deadzone method. In this approach the estimation algorithm stops when the prediction error becomes smaller than a preset value (Egardt, 1979), (Peterson and Narendra, 1982) and (Middleton *et al.*, 1988). One problem with this approach is the difficulty of choosing an appropriate value for the magnitude of the deadzone. Another method called leakage ensures that the parameter estimates stay close to the initial conditions (Ioannou and Sun, 1996). Persistent excitation ensures stability by manipulating the input signals so that data remain informative (Mareels and Polderman, 1996). Finally, parameter projection can be used to ensure that the parameter estimates do not exceed pre-specified bounds. The main disadvantage of these methods

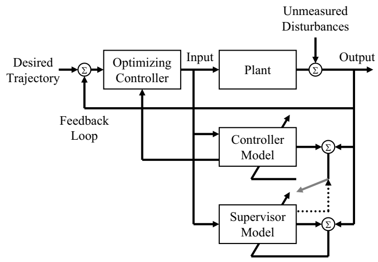

Fig. 1. Block diagram showing the supervised dual-model algorithm approach.

is that they make unrealistic assumptions about the process and the disturbances (Hill and Ydstie, 2004).

The objective of the present paper is to describe a new method to improve robustness of adaptive process control. The proposed method is closely related to the deadzone method since it relies on the idea of turning estimation off once the prediction is smaller than a given threshold. However, in this case we adapt the threshold by using a second adaptive controller (Ydstie, 2005). Adaptive prediction theory can be used to show that the prediction error of the second model converges to an optimal estimate of the plant disturbances. The deadzone is now optimally tuned and it is possible to show that the parameter estimates and controller converge close to optimal performance. In a companion paper we show the theoretical stability analysis to support this claim for the special case of direct adaptive predictive control (Dozal-Mejorada and Ydstie, 2006).

The paper is organized as follows. In the next section we describe the new method for supervisory adaptive control. We then introduce a predictive controller which is suitable for adaptive control of open loop stable systems. Lastly, we present simulation and experimental studies that support the proposed method.

2. SUPERVISED ADAPTIVE CONTROL

In our novel approach to adaptive control we monitor the performance of the estimated control with a *supervisor*. Figure 1 shows that the method uses a dual-model concept where one model is used by the controller to regulate the plant to constant or time-varying set points while the second model is used by a supervisor to decide when to update the control model. Ideally the first model should only be updated when the data are excited so that the estimated parameters converge to better values.

In order to highlight the technique, consider a single-input single-output discrete time plant given by

$$A(q^{-1})y(t) = B(q^{-1})u(t) + \gamma(t) \tag{1}$$

The signals $\{y(t), u(t), \gamma(t)\}$ are the measured output, manipulated input and unmeasured disturbances respectively. $A(q^{-1})$ and $B(q^{-1})$ are polynomials in the backward shift operator, q^{-1} so that

$$A(q^{-1}) = 1 + a_1 q^{-1} + \cdots + a_n q^{-n} \tag{2}$$

$$B(q^{-1}) = b_0 + b_1 q^{-1} + \cdots + b_m q^{-m} \tag{3}$$

System (1) is written in shorthand notation

$$y(t) = \varphi(t-1)^T \theta^\star + \gamma(t) \tag{4}$$

with the regression vector $\varphi(t)$ defined by

$$\varphi(t)^T = (-y(t), \ldots, -y(t-n+1), \quad u(t), \ldots, u(t-m+1)) \tag{5}$$

The parameters of the polynomials $\{A(q^{-1}), B(q^{-1})\}$ are used to define the vector

$$\theta^\star = (a_1, \ldots, a_n, b_0, \ldots, b_m)^T \tag{6}$$

The certainty equivalence principle applied to adaptive control gives a method for control which involves fitting a linear model to plant data and calculating the control law as if the estimated parameters gave a perfect representation of the plant dynamics (Ydstie, 1997). Thus, replacing $\theta^\star$ with estimates

$$\hat{\theta}(t)^T = (\hat{a}_1(t), \ldots, a_n(t), \hat{b}_0(t), \ldots, \hat{b}_m(t)) \tag{7}$$

allows the online development of feedback and feed forward controllers. The controllers are updated whenever the parameter vector $\hat{\theta}(t)$ changes. Then, the *adaptive control* problem is to develop a self-tuning controller for system (4) while satisfying stability and robustness conditions.

The schematic depicted in Figure 1 shows that the supervisory approach involves two models being estimated in parallel with a switch determining when to update the control model. The controller and supervisor models are expressed in terms of the regression vector and their respective estimated parameters

$$\hat{y}_C(t) = \varphi(t-1)^T \hat{\theta}_C(t-1) \tag{8}$$

$$\hat{y}_S(t) = \varphi(t-1)^T \hat{\theta}_S(t-1) \tag{9}$$

where the subscripts C and S denote controller and supervisor respectively. The corresponding model prediction errors are given by

$$e_i(t) = y(t) - \varphi(t-1)^T \hat{\theta}_i(t-1) \tag{10}$$

where $i \in [C, S]$ represents the Controller and Supervisor model respectively. In addition, the variance of the disturbance sequence for each model is approximated by

$$r_i(t) = \sigma^2 r_i(t-1) + (1-\sigma^2) e_i(t)^2 \tag{11}$$

with σ defined in terms of $M_0 \geq 1$, the estimation memory length,

$$\sigma = 1 - \frac{1}{M_0} \tag{12}$$

Typically $M_0 \in [100, 10^5]$. Lastly, define the normalization sequences

$$n_i(t) = \lambda_i(t) r_i(t) + \varphi(t-1)^T P_i(t-1) \varphi(t-1) \tag{13}$$

The least squares algorithm for parameter estimation can be written in its recursive form (Goodwin and Sin, 1984)

$$\hat{\theta}(t) = \hat{\theta}(t-1) + \frac{P(t-1)\varphi(t-1)e(t)}{\lambda(t)r(t) + \varphi(t-1)^T P(t-1)\varphi(t-1)} \tag{14}$$

$$P(t) = C(t) + \{P(t-1)\}\lambda(t)^{-1} - \left\{ \frac{P(t-1)\varphi(t-1)\varphi(t-1)^T P(t-1)}{\lambda(t)r(t) + \varphi(t-1)^T P(t-1)\varphi(t-1)} \right\} \lambda(t)^{-1} \tag{15}$$

Here $P(t)$ is the inverse of the Fisher information matrix called the covariance matrix and $\lambda(t)$ is the forgetting factor which may be constant or time-varying (Goodwin and Sin, 1984). The matrix $C(t)$ is chosen so that $P_{min} \leq P(t) \leq P_{max}$.

Two main modifications to (14) are made in order to improve control model stability. First, introduce parameter projection following the work by (Ydstie, 1992). Here, the parameters of (14) are projected onto a convex set yielding the update law for the models

$$\hat{\theta}_i(t) = F_{\Theta^\star} \left\{ \hat{\theta}_i(t-1) + \frac{P_i(t-1)\varphi(t-1)e_i(t)}{n_i(t)} \right\} \tag{16}$$

Second, leakage is introduced into the supervisor model to change its fixed point so that the closed loop convergence to the leakage parameters is stable. This effectively adds an extra term in the parameter update law and favorably drives the estimates towards (or away from) a particular choice of values (Hovd and Bitmead, 2006) (Ioannou and Kokotovic, 1983). Incorporating leakage into equation (14) gives the supervisor parameter update expression

$$\hat{\theta}_S(t) = F_{\Theta^\star} \left\{ \hat{\theta}_S(t-1) + \frac{P_S(t-1)\varphi(t-1)e_S(t)}{n_S(t)} \right\} + \sigma^2(\hat{\theta}_S(t-1) - \theta_{S,0}) \tag{17}$$

The last component of the algorithm is the switching condition. This condition determines when and how to pass information from the supervisor model to the control model. The metric used to evaluate model performance is based on the corresponding model prediction errors. The switch obeys

$$Switch = \begin{cases} 1(ON) & \text{if } ||e_C(t)||^2 \geq \varepsilon ||e_S(t)||^2 \\ 0(OFF) & \text{else} \end{cases} \tag{18}$$

Expression (18) states that as long as the magnitude of the prediction error from the control model is smaller than the one from the supervisor model then the control model provides a better approximation to the plant and should *not* update its parameters. Otherwise, parameter estimation should be implemented in the control model.

The following **Supervised Adaptive Control** algorithm results

Step 0	Initialize all estimated parameters, signals and covariance matrices $\hat{\theta}_C(0)$, $\hat{\theta}_S(0)$, $u(0)$, $y(0)$, $P_C(0)$ and $P_S(0)$
Step 1	Calculate prediction errors and normalization sequences $e_C(t)$, $e_S(t)$, $n_C(t)$, $n_S(t)$, $r_C(t)$, and $r_S(t)$ Equations (10), (13) and (11)
Step 2	Update the supervisor parameters and the supervisor covariance matrix Equations (17) and (15)
Step 3	Check Switch Condition, if satisfied update the control parameters and the control covariance matrix Equations (18), (16) and (15)
Step 4	Design a stabilizing control law according to the estimated control parameters
Step 5	Set $t = t + 1$ and GOTO 1

3. ADAPTIVE PREDICTIVE CONTROL WITH SUPERVISION

One powerful characteristic of adaptive supervision is the ability to couple the algorithm to other well-established control concepts. In this section, the supervisory algorithm introduced in section 2 is coupled with Model Predictive Control (MPC) to yield an algorithm with both adaptive and predictive capabilities. The predictive control approach includes concepts from Self Tuning Control (STC) and Extended Horizon Control (EHC). The former provides control weighting parameters and the latter provides the ability to handle unknown delays (Goodwin and Sin, 1984)

There are two main approaches to achieve adaptive control. Namely,

(1) Direct Adaptive Control. The control parameters are estimated directly therefore removing the control law design calculations.
(2) Indirect Adaptive Control. The estimation is performed on the plant and the estimates are used to possibly design a family of controllers. This method adds an extra calculation step as compared to the direct approach.

The supervision concept will be applied to a direct adaptive controller. In order to derive the predictive controller, first start with the system given by equation (1) and assume there is no external disturbance or noise.

$$A(q^{-1})y(t) = B(q^{-1})u(t) \tag{19}$$

Introduce the Diophantine equation

$$FA\Delta + q^{-T}G\Delta = (1 - q^{-T}) \tag{20}$$

where $\Delta = 1 - q^{-1}$, T is the prediction horizon and for simplicity the (q^{-1}) terms have been suppressed. Multiply (19) through with $F\Delta$

$$FA\Delta y(t) = FB\Delta u(t) \tag{21}$$

Substituting (20) into (21)

$$\left[(1 - q^{-T}) - q^{-T}G\Delta\right] y(t) = FB\Delta u(t) \tag{22}$$

It follows that

$$y(t) = y(t-T) + \varphi(t-1)^T \hat{\hat{\theta}}(t-1) \tag{23}$$

with

$$\varphi(t-1)^T = (\Delta y(t), \Delta u(t))$$
$$\hat{\hat{\theta}}(t-1) = (G, FB)^T$$

Multiply equation (23) by q^T to get the T-step ahead predictor form

$$y(t+T) = y(t) + \varphi(t+T-1)^T \hat{\hat{\theta}}(t+T-1) \tag{24}$$

The final controller design is posed as an optimization problem. The objective function is to minimize the difference between the observed and predicted outputs while imposing a penalty if the input magnitude becomes "too large". Namely,

$$\min_{\Delta u(t)} J = [y(t+T)^\star - y(t+T)]^2 + rT\Delta u(t)^2 \tag{25}$$

In order to get an optimizing controller, equation (25) is solved subject to equation (24) and $\Delta u(t+i) = u(t+i) - u(t)$ for all $i \geq 1$. Solving the optimization problem for the manipulated input yields

$$\begin{aligned}\Delta u(t) = {} & \frac{\sum_{i=1}^{T} \beta_i}{rT + \sum_{i=1}^{T} \beta_i} [y(t+T)^\star - y(t) \\ & - \alpha_1 \Delta y(t) - \cdots - \alpha_n \Delta y(t-n) \\ & - \beta_1 \Delta u(t-1) - \cdots - \beta_m \Delta u(t-m)]\end{aligned} \tag{26}$$

where $\hat{\hat{\theta}}(t-1) = (\alpha' s, \beta' s)^T$ has replaced $\hat{\hat{\theta}}(t-1) = (G, FB)^T$.

4. CASE STUDIES FOR DIRECT ADAPTIVE CONTROL WITH SUPERVISION

A series of simulations and experiments have been conducted to test and validate the adaptive scheme. Here, two case studies are presented. The first case is simulation based and involves investigating the behavior of the adaptive algorithm applied to a chemical reactor. The second case is experimental in nature and involves regulation of a shell and tube heat exchanger.

4.1 Chemical Reactor

Simulations were carried out on an isothermal CSTR in order to highlight the capabilities of the supervisory algorithm (Douglas, 1972). The reactions taking place in the reactor are

$$A + R \rightarrow B$$
$$B + R \rightarrow C$$
$$C + R \leftrightarrow D$$
$$D + R \rightarrow E$$

Assume that component R is present in excess so that all the reaction rates can be approximated by first order expressions of the form

$$k_1 = k_A R \tag{27}$$
$$k_2 = k_B R \tag{28}$$
$$k_3 = k_C R \tag{29}$$
$$k_4 = k_D R \tag{30}$$

The system equations are developed by performing material balances on the species under consideration around the reactor.

$$V\frac{dA}{dt} = QA_f - qA - k_1 VA \tag{31}$$
$$V\frac{dB}{dt} = -qB + k_1 VA - k_2 VB \tag{32}$$
$$V\frac{dC}{dt} = -qC + k_2 VB - k_3 VC + k_3' VD \tag{33}$$
$$V\frac{dD}{dt} = -qD + k_3 VC - k_3' VD - k_4 VD \tag{34}$$

where V is the vessel volume, Q and q are flow rates, $\{A_f, A, B, C, D, E\}$ are component compositions and the feed compositions of B, C, D, and E have been assumed to be zero. A complete description of the system would necessitate a material balance equation for component R. However, because this component is in large excess and remains relatively unchanged the choice to neglect it is appropriate.

Consider the design of a control scheme which will maintain the composition of component C as close to as possible to the steady state design value despite disturbances entering the system. The controller measures the actual composition of C and uses the difference between the desired and measured values to manipulate Q, the inlet flow rate. In this study, it is assumed that the system disturbances are due to changes in the composition of R in the vessel. The system expressions are linearized for small deviations from steady state conditions. All system parameters are specified in (Douglas, 1972).

The CSTR was subjected to set point changes for the concentration of C. Figure 2 shows the set point, measured output and manipulated input signals for the system. The adaptive controller

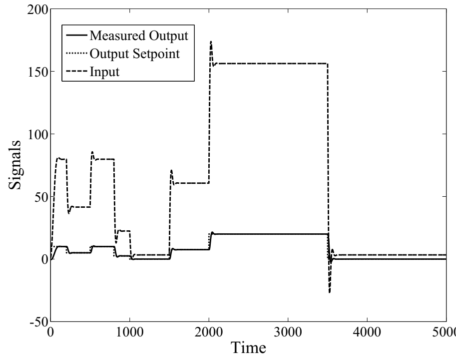

Fig. 2. CSTR simulation showing the reactor signals while undergoing set point changes.

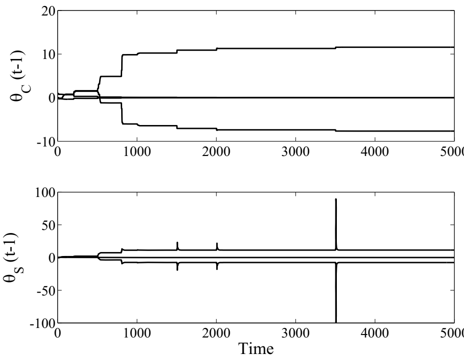

Fig. 3. Controller and supervisor estimated parameters for the CSTR simulation during set point changes.

succesfully achieves the desired set points with minimum overshoot. Figure 3 shows the parameter estimates for both the control (top) and supervisor (bottom) models. Note that the control model parameters are more stable while a noticeable spike or burst occurs in the supervisor around $t = 3500$. The estimated control parameter converge slowly.

4.2 Heat Exchanger

Experimental studies were carried out on a pilot plant scale standard shell and tube heat exchanger. The operational conditions placed hot water flow through the shell side of the exchanger and cold water flow in the tube side. The supervised adaptive predictive control algorithm was implemented in LabVIEW® with adjacent field point boxes holding the A/D and D/A converters. A thermocouple was used to obtain temperature measurements. The sampling time used was $T_s = 2$ sec.

The control objective was to regulate the hot water outlet temperature to specific set points. The cold water flow rate was used as the manipulated variable and disturbances were introduced to the system by varying the hot water flow rate through the heat exchanger.

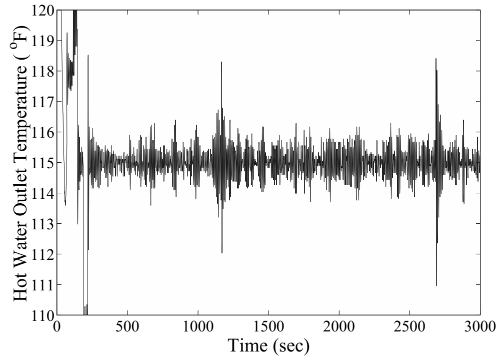

Fig. 4. Heat exchanger experiment showing the bursting output behavior observed for regular adaptive control scheme *without* supervision.

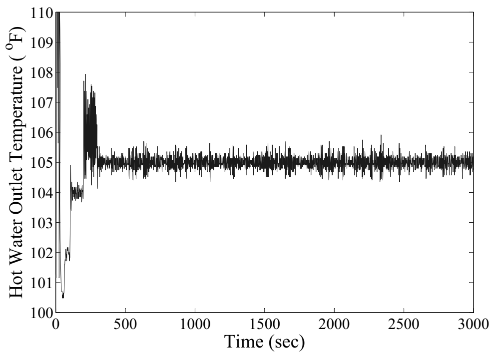

Fig. 5. Heat exchanger experiment under supervised adaptive control.

The supervisory algorithm was compared to an adaptive control set-up without supervision. Figure 4 shows the hot water temperature outlet controlled by an adaptive regulator *without* supervision. Here the typical output bursting reviewed in (Hill and Ydstie, 2004) is observed. In this case, the estimated parameters cross the linear stability boundary and bursts are seen first around $t = 1100$ and then again around $t = 2600$. The heat exchanger was then put under supervisory adaptive control. The bursting behavior has been eliminated with the supervisor algorithm as can been seen in Figure 5. The parameters involved in the heat exchanger controller converge as seen in Figure 6. Lastly, Figure 7 shows the drifting parameters in the supervisor estimated model. The control performance does not seem to be affected by the supervisor's slowly converging estimated parameters. The control model only updates its parameters when the predicted error is worse than the one obtained from the supervisor.

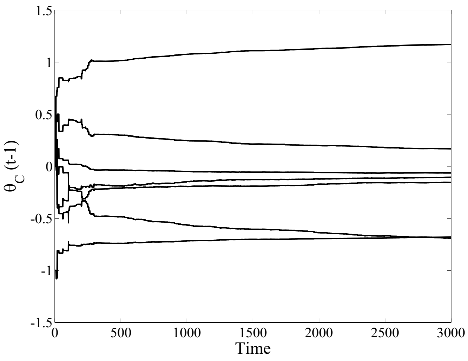

Fig. 6. Heat exchanger control model parameter estimates for the supervised adaptive control case.

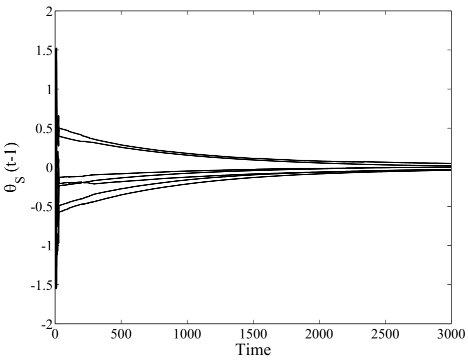

Fig. 7. The supervisor parameter estimates during adaptive control of heat exchanger.

5. CONCLUSIONS

Un-modelled dynamics and unknown disturbances are known to cause problems in adaptive controllers. The supervised adaptive control algorithm presented here addresses these issues by coupling together two adaptive controllers so that the control model only updates its parameters when new and relevant information enters the system. The supervisor model never stops estimating parameters. The control model, however, stops the estimation process according to a given metric. The proposed metric compares the prediction errors from the supervisor and the control models. Estimation is based on recursive least squares with appropriate projection and leakage modifications to ensure stability and convergence. Simulation and experimental studies show the potential of the supervision approach to adaptive control.

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