NONLINEAR INTERNAL MODEL CONTROL OF PEM FUEL CELL

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Abstract: Proton exchange membrane fuel cells (PEMFCs) are known to exhibit strongly nonlinear dynamics and input multiplicities. This paper presents an approach for identifying GOBF-Wiener models towards representing the nonlinear dynamics. The identified model is used to synthesis a MIMO nonlinear internal model controller (NIMC). The average power density and solid temperature of the PEMFC are controlled using (a)cell voltage and inlet coolant temperatures (b) inlet molar flow rates of hydrogen and coolant. The proposed NIMC scheme is able to operate the PEMFC at the optimum power density point where the steady state gain reduces to zero and changes its sign.

Keywords: Orthonormal Basis Filters, Wiener Models, Fuel Cell, Nonlinear Internal Model Control, Peak Seeking Control

INTRODUCTION

The need for an efficient, non-polluting power source for stationary and mobile applications has resulted in increased attention towards fuel cells. The proton exchange membrane fuel cell (PEMFC) has received much attention as a low temperature fuel cell alternative for these applications. There are many difficulties in the design of controllers for the PEMFC. The major challenges are sign changes in the gain, severe nonlinearities and possible starvation of reactants. Most of the research literature on PEMFCs has focused on steady-state operation and the study of transport through the various layers in the PEMFC; there are relatively few articles on the development of effective control strategies. Pukrushpan *et al.*, (2002) used a feedback and feedforward control strategy to maintain an excess oxygen ratio and obtain the required power density from the PEMFC. Na *et al.*, (2005) linearized a nonlinear model and developed a linear controller for the PEMFC. However in the presence of nonlinearities, there are significant constraints on the achievable closed loop performance. Nonlinear controller could there fore be necessary to achieve tight control performance.

The strategy of using a low-order identified model with a nonlinear controller has many advantages for fuel cell control. Since fuel cells have fast dynamic response (of the order of seconds) and the servo control requirements in load following applications are therefore stringent, the controller must be able to decide on the manipulated variable action within a small sampling period. Fuel cells are complex systems, and therefore the use of first-principles models (typically involving PDEs) for model based control of these systems are difficult to solve in the time period available for control.

In this article, we address some of the above problems towards the design of nonlinear controllers for the PEMFC. The approach we propose is the use of low-order, nonlinear empirically identified models for the controller design, which obviates the difficulty of solving a first principles model online. With the identified model and the controller being nonlinear, the strategy proposed here is shown to provide good servo control for the nonlinear PEMFC. Furthermore, the controller is also shown to be able to control the system at the peak power density when there is an input multiplicity with respect to voltage. The particular control strategy adopted in our work is based on the principle of NIMC (nonlinear internal model control). While designing the controller, the PEMFC is considered as a MIMO (multi input and multi output) system where the average power density and the average solid temperature are the two controlled outputs. Two sets of manipulated variables are considered. In the first, the cell voltage and inlet coolant temperature are manipulated. The system exhibits input multiplicity and change in sign of steady state gains for this combination. In the second, the inlet flow rates of hydrogen and coolant are manipulated, which have been shown to be the best combination of inputs for decentralized control (Methekar *et al.,* 2007). A Wiener-type model is identified from simulation data using a first-principles model (the identification can be performed using experimental data, too), and is then used to design a nonlinear internal model control based on the analytical approach suggested by Patwardhan and Madhavan (1998). Orthonormal basis functions (OBF) are used to parametrize the linear dynamic part of the model (Srinivasarao *et al.*, (2006)). The resulting discrete nonlinear state space model, referred to as OBF-Wiener model in the rest of the text, is then used to synthesize a nonlinear IMC controller based on the analytical approach suggested by Patwardhan and Madhavan (1998). Unlike most of the NIMC formulations available in the literature, the proposed NIMC control law is derived using multi-step predictions. The efficacy of the proposed modelling and control scheme is demonstrated by simulating servo control problems involving operation of the PEMFC at its optimum (singular) operating point.

This paper is organized in four main sections. The method of identification using OBF-Wiener models and design of NIMC are presented in the next section. Section 3 deals with the implementation of NIMC on the reduced order PEMFC model. The main conclusions are presented in the last sections.

1. METHOD OF IDENTIFICATION USING OBF-WIENER MODEL

Consider a process represented as a set of nonlinear ODEs

$$\frac{d\mathbf{z}}{dt} = \mathbf{F}\left[\mathbf{z}, \mathbf{u}(t), \mathbf{d}, \mathbf{p}\right] \tag{1}$$

$$\mathbf{y}(t) = \mathbf{G}\left[\mathbf{z}\right] + \boldsymbol{\upsilon}_y(t) \tag{2}$$

where $\mathbf{z} \in R^s$ represents the state vector, $\mathbf{u} \in R^m$ represents the true value of manipulated inputs, $\mathbf{d} \in \mathbf{R}^d$ represents unmeasured disturbances, $\mathbf{y} \in R^r$ represents the vector of measured outputs corrupted with measurement noise $\boldsymbol{\upsilon}_y(t)$ and $\mathbf{p} \in R^\nu$ represents the parameter vector. The information available from the plant is the sampled sequence of input and output vectors $\Sigma_N = \{(\mathbf{y}(k), \mathbf{u}(k)) : k = 1, 2, ...N\}$. Given input and output data set Σ_N collected from a plant, the problem of identifying a nonlinear time series model can be stated as finding a nonlinear operator $\Xi[.]$

$$\mathbf{y}(k) = \Xi\left[\boldsymbol{\varphi}(k), \boldsymbol{\theta}\right] + \mathbf{e}(k) \tag{3}$$

$$\boldsymbol{\varphi}(k) = \boldsymbol{\varphi}\left[\mathbf{u}(k-1), ..., \mathbf{u}(1), \mathbf{y}(k-1), ..., \mathbf{y}(1)\right] \tag{4}$$

such that a suitable norm of model residuals $\{\mathbf{e}(k) : k = 1, ...N\}$ is minimized with respect to parameter vector $\boldsymbol{\theta}$. The modeling problem can be further decomposed as (a) choosing a suitable regressor $\boldsymbol{\varphi}\,[.]$ and (b) selecting a suitable nonlinear mapping $\Xi[.]$ from regressor space to the output space. In this work we propose to develop MISO Wiener type state models of the form

$$\mathbf{X}^{(i)}(k+1) = \boldsymbol{\Phi}^{(i)}\mathbf{X}^{(i)}(k) + \boldsymbol{\Gamma}^{(i)}\mathbf{u}(k) \tag{5}$$

$$\mathbf{y}_i(k) = \Omega^{(i)}\left[\mathbf{X}^{(i)}(k)\right] + \mathbf{v}_i(k) \tag{6}$$

where $\mathbf{X}^{(i)}(k) \in R^{n_i}$ represents the state vector and $\Omega^{(i)}\,[.]$ represents some nonlinear static map relating states with the outputs for the i^{th} MISO model. Here we propose to composite this MISO models to generate an overall MIMO model.

1.1 Parameterization of Linear Dynamic Component

In this work, we choose to parametrize these matrices using Generalized Orthonormal Basis Filters, which represent an orthonormal basis for the set of strictly proper stable transfer functions (denoted as $\mathcal{H}_2$). Ninness and Gustafson (1997) have shown that a complete orthogonal set in $\mathcal{H}_2$ can be constructed as follows

$$F_k(z, \xi) = \frac{\sqrt{(1 - |\xi_k|^2)}}{(z - \xi_k)} \prod_{i=1}^{k-1} \frac{(1 - \xi_i^* z)}{(z - \xi_i)} \tag{7}$$

where $\{\xi_k : k = 1, 2, ...\}$ is an arbitrary sequence of poles inside the unit circle appearing in complex conjugate pairs. Classical FIR models, Laguerre filters or Kautz filters based models are well known examples of the class of models parameterized using GOBF. The main advantage of using orthogonal basis filters instead of $\left\{z^{-j}\right\}$ in the classical FIR models is that the transfer function $G(z)$ can be approximated by only a small number of coefficients in the expansion, i.e. a *parsimonious in parameters* model is generated. If the system under consideration has scattered poles, GOBF is a better choice of orthonormal basis filters than Laguerre or Kautz filters. Given a set of real poles inside the unit circle, a method of parameterization of matrices $(\mathbf{\Phi}, \mathbf{\Gamma}, \mathbf{K})$ using a GOBF network for a multi-input model is given in Patwardhan and Shah (2005).

1.2 Parameterization of State Output Map

The nonlinear state output map $\Omega_i[.] : R^{n_i} \rightarrow R$ can be parameterized as a function space expansion

$$\mathbf{y}_i(k) = \Omega_i \left[\mathbf{X}^{(i)}(k)\right] + \mathbf{v}_i(k) \quad (8)$$

$$\Omega_i \left[\mathbf{X}^{(i)}(k)\right] = \sum c_{ij} \boldsymbol{\omega}_{ij} \left[\mathbf{X}^{(i)}(k)\right] \quad (9)$$

where $\boldsymbol{\omega}_{ij}[.]$ represent some basis functions. In the present work, these static non-linear maps are chosen to be simple multi-dimensional polynomial functions of finite order mainly with the intention of simplifying the resulting parameter estimation problem. For example, a quadratic polynomial function can be expressed as

$$\Omega_i [.] = \mathbf{C}^{(i)} \mathbf{X}^{(i)}(k) + \left(\mathbf{X}^{(i)}(k)\right)^T \mathbf{D}^{(i)} \left(\mathbf{X}^{(i)}(k)\right) \quad (10)$$

for $i = 1, ..., m$, where $\mathbf{C}^{(i)}$ represents a $(1 \times n_i)$ vector and $D^{(i)}$ represents a $n_i \times n_i$ symmetrical matrix. Thus, with state-output map given by equation (10), the output can be expressed as

$$\mathbf{y}_i(k) = \left(\mathbf{\Theta}^{(i)}\right)^T \mathbf{Z}^{(i)}(k) + \mathbf{v}_i(k) \quad (11)$$

where

$$\mathbf{Z}^{(i)}(k) = \left[\left[\mathbf{X}^{(i)}(k)\right]^T \left[\boldsymbol{\chi}^{(i)}(k)\right]^T \right]^T \quad (12)$$

$$\boldsymbol{\chi}^{(i)}(k) = \left[\left(\mathbf{X}_1^{(i)}(k)\right)^2 \; 2\mathbf{X}_1^{(i)}(k)\mathbf{X}_2^{(i)}(k) \; .... \right]^T \quad (13)$$

and

$$\mathbf{\Theta}^{(i)}(k) = \left[\mathbf{C}^{(i)} \; \mathbf{D}_{11}^{(i)} \; \mathbf{D}_{12}^{(i)} \; .... \; \mathbf{D}_{N_i,N_i}^{(i)} \right]^T \quad (14)$$

Here, $\mathbf{\Theta}^{(i)}$ is a $N_i \times 1$ vector with $N_i = n_i \times (n_i + 3)/2$, $\mathbf{X}_l^{(i)}(k)$ represents l'th element of vector $\mathbf{X}^{(i)}(k)$ and $\mathbf{D}_{jl}^{(i)}$ represents (j, l)'th element of matrix $\mathbf{D}^{(i)}$. Thus, the main advantage of choosing polynomial functions is that the resulting state-output map is linear in parameters.

1.3 Model Parameter Estimation

The key step in the development of the GOBF based models is the selection of filter poles and number of basis filters (filter order) necessary to develop a reasonably good approximation of the system dynamics. Given an input-output data set Σ_N, the least squares estimate of the parameters can be obtained by solving the following minimization problem

$$(\widehat{\Theta}^{(i)}, \widehat{\boldsymbol{\xi}}^{(i)}) = \underset{\Theta^{(i)}, \boldsymbol{\xi}^{(i)}}{\arg\min} \frac{1}{N} \sum_{k=1}^{N} \left[\widehat{\mathbf{v}}_i(k, \Theta^{(i)}, \boldsymbol{\xi}^{(i)})\right]^2 \quad (15)$$

subject to

$$\boldsymbol{\xi}_j^{(i)} < 0 \text{ for } j = 1, 2....n_i \quad (16)$$

where $\boldsymbol{\xi}^{(i)}$ represents the vector of GOBF poles. To keep the variance errors low, we propose to fix filter orders *a-priori* and perform a search in the set of filter poles as proposed in Patwardhan and Shah (2005). Thus, the parameter estimation problem is formulated in terms of two nested optimization problems as follows:

$$(\widehat{\Theta}^{(i)}, \widehat{\boldsymbol{\xi}}^{(i)}) = \underset{\boldsymbol{\xi}^{(i)}}{\arg\min} \frac{1}{N} \sum_{k=1}^{N} \left[\widehat{\mathbf{v}}_i(k, \widehat{\Theta}^{(i)}, \boldsymbol{\xi}^{(i)})\right]^2 \quad (17)$$

subject to constraint (16). Here, given a guess of pole vector $\widetilde{\boldsymbol{\xi}}^{(i)}$, the parameter vector $\widehat{\Theta}^{(i)}$ is estimated by solving another optimization problem

$$\widehat{\Theta}^{(i)} \left[\widetilde{\boldsymbol{\xi}}^{(i)}\right] = \underset{\Theta^{(i)}}{\arg\min} \frac{1}{N} \sum_{k=1}^{N} \left[\widehat{\mathbf{v}}_i(k, \Theta^{(i)}, \widetilde{\boldsymbol{\xi}}^{(i)})\right]^2 \quad (18)$$

Since the state-output map is linear in parameters, we can exploit the fact that the parameter vector $\Theta^{(i)}$ can be estimated analytically by a simple linear regression scheme. Thus, the parameter vector $\widehat{\Theta}^{(i)}$ can be estimated as follows

$$\widehat{\Theta}^{(i)} \left[\widetilde{\boldsymbol{\xi}}^{(i)}\right] = \left[\overline{E}\left(\mathbf{Z}_u(k_l)\mathbf{Z}_u(k_l)^T\right)\right]^{-1} \overline{E}\left(\mathbf{Z}_u(k_l)\mathbf{Y}\right) \quad (19)$$

$$\mathbf{Y} = \left[\, \mathbf{y}_i(1) \; \mathbf{y}_i(2) \; \cdots \; \mathbf{y}_i(N_s) \,\right]^T$$

and $\overline{E}(.)$ represents the expected value operator.

2. DESIGN OF NONLINEAR INTERNAL MODEL CONTROLLER

For development of NIMC, it is assumed that the system under consideration is a square system. Consider the p-steps ahead prediction obtained using equations (5-6) under the constraints

$$\mathbf{u}(k + i|k) = \mathbf{u}(k|k) \quad \text{for } i = 1, 2, ...p - 1$$

The p-steps ahead output prediction generated can be expressed as

$$\mathbf{X}^{(i)}(k + p|k) = \left(\mathbf{\Phi}^{(i)}\right)^p \mathbf{X}^{(i)}(k) + \Omega^{(i)}\mathbf{u}(k) \quad (20)$$

$$\Omega^{(i)} = \left[\left(\mathbf{\Phi}^{(i)}\right)^{p-1} + \left(\mathbf{\Phi}^{(i)}\right)^{p-2} + ... + I\right] \Gamma^{(i)}$$

$$\mathbf{y}_i(k + p|k) = \overline{\mathbf{y}}_i(k + p|k) + \mathbf{u}^T(k) \left[\left(\Omega^{(i)}\right)^T \mathbf{D}^{(i)}\Omega^{(i)}\right] \mathbf{u}(k) + \left[\mathbf{C}^{(i)}\Omega^{(i)} + 2\left[\left(\Phi^{(i)}\right)^p \mathbf{X}^{(i)}(k)\right]^T \mathbf{D}^{(i)}\Omega^{(i)}\right] \mathbf{u}(k) \quad (21)$$

for $i = 1, 2, ...r$ where vector $\overline{\mathbf{y}}(k + p|k)$ is defined as

$$\overline{\mathbf{y}}_i(k + p|k) = \mathbf{C}^{(i)} \left(\mathbf{\Phi}^{(i)}\right)^p \mathbf{X}^{(i)}(k) + \left[\left[\left(\Phi^{(i)}\right)^p \mathbf{X}^{(i)}(k)\right]^T \mathbf{D}^{(i)} \left[\left(\Phi^{(i)}\right)^p \mathbf{X}^{(i)}(k)\right]\right]$$

Defining matrix $\Lambda^{(k)}$ as

$$\mathbf{\Lambda}^{(k)} = \begin{bmatrix} \mathbf{C}^{(1)}\Omega^{(1)} + 2\left[\left(\Phi^{(i)}\right)^p \mathbf{X}^{(i)}(k)\right]^T \mathbf{D}^{(i)}\Omega^{(i)} \\ \cdots\cdots\cdots\cdots \\ \mathbf{C}^{(r)}\Omega^{(r)} + 2\left[\left(\Phi^{(r)}\right)^p \mathbf{X}^{(r)}(k)\right]^T \mathbf{D}^{(r)}\Omega^{(r)} \end{bmatrix}$$

and $\{\Psi\}$ as the bilinear matrix representation ((Patwardhan and Madhavan (1998), for details) of the following three dimensional array

$$\{\mathbf{\Psi}\} \equiv \begin{bmatrix} \dfrac{\left[\left(\Omega^{(1)}\right)^T \mathbf{D}^{(1)}\Omega^{(1)}\right]}{\dfrac{\cdots\cdots\cdots}{\left[\left(\Omega^{(r)}\right)^T \mathbf{D}^{(r)}\Omega^{(r)}\right]}} \end{bmatrix}$$

the above r quadratic output prediction equations can be represented as the following multi-dimensional quadratic equation (Patwardhan and Madhavan (1998)).

$$\widehat{\mathbf{y}}(k+p|k) = \overline{\mathbf{y}}(k+p|k) + [\Lambda(k)]\,\mathbf{u}(k) + \{\mathbf{\Psi}\}\,[\mathbf{u}(k), \mathbf{u}(k)] \tag{22}$$

To account for plant-model mismatch and unmeasured disturbance mismatch, it is proposed to use an *open loop* state observer with a dead beat disturbance estimator. By this approach, the output predictions are corrected as follows

$$\mathbf{y}_c(k+p|k) = \widehat{\mathbf{y}}(k+p|k) + \widehat{\mathbf{d}}(k+p|k) \tag{23}$$

where the estimation of future disturbances is generated as

$$\widehat{\mathbf{d}}(k+j+1|k) = \widehat{\mathbf{d}}(k+j|k) \quad \text{for } j = 0, \ldots p-1 \tag{24}$$

$$\widehat{\mathbf{d}}(k|k) = \mathbf{y}(k) - \widehat{\mathbf{y}}(k) \tag{25}$$

Here, $\mathbf{y}(k)$ represents the measurement vector and $\widehat{\mathbf{y}}(k)$ represents the output prediction generated using MISO models in open loop as follows

$$\widehat{\mathbf{X}}^{(i)}(k) = \mathbf{\Phi}^{(i)}\widehat{\mathbf{X}}^{(i)}(k-1) + \mathbf{\Gamma}^{(i)}\mathbf{u}(k-1) \tag{26}$$

$$\widehat{\mathbf{y}}_i(k) = \Omega^{(i)}\left[\widehat{\mathbf{X}}^{(i)}(k)\right] \tag{27}$$

If $\mathbf{y}_r(k)$ represent the desired setpoint for the process, then imposing the constraint that the setpoint should be reached after p-steps in the future, i.e., $\mathbf{y}_c(k+p|k) = \mathbf{y}_r$, gives the following controller design equation $Q(\mathbf{u}(k)) = \overline{0}$, where

$$\begin{aligned} Q(\mathbf{u}(k)) &= \{\mathbf{\Psi}\}\,[\mathbf{u}(k), \mathbf{u}(k)] \\ &+ \left[\mathbf{\Lambda}^{(k)}\right]\mathbf{u}(k) + \overline{\mathbf{y}}(k+p|k) - \boldsymbol{\epsilon}(k) \end{aligned} \tag{28}$$

where $\boldsymbol{\epsilon}(k) = \mathbf{y}_r(k) - \mathbf{d}(k)$. The above equation is a multi-dimensional quadratic operator polynomial, which can be solved analytically. Using the approach suggested by Patwardhan and Madhavan (1998), a quadratic control law can be derived as follows. Let $\widetilde{\mathbf{u}}$ denote some input vector such that the matrix

$$\nabla_U\left[Q(\widetilde{\mathbf{u}})\right] = 2\left[\{\mathbf{\Psi}\}(\widetilde{\mathbf{u}})\right] + \mathbf{\Lambda}^{(k)} \tag{29}$$

is nonsingular. Then, the equation (28) can be transformed as

$$\left\{\widetilde{\mathbf{\Psi}}(k)\right\}(\mathbf{u}(k) - \widetilde{\mathbf{u}}, \mathbf{u}(k) - \widetilde{\mathbf{u}}) + (\mathbf{u}(k) - \widetilde{\mathbf{u}}) + \mathbf{E}_0(k) = \overline{0} \tag{30}$$

where

$$\left\{\widetilde{\mathbf{\Psi}}(k)\right\} = \left(\nabla_U\left[Q(\widetilde{\mathbf{u}})\right]\right)^{-1} \bullet \{\Psi\} \tag{31}$$

$$\mathbf{E}_0(k) = \left(\nabla_U\left[Q(\widetilde{\mathbf{u}})\right]\right)^{-1}\left[Q(\widetilde{\mathbf{u}})\right] \tag{32}$$

Here, symbol ($\bullet$) denotes the 'left dot product' between matrix $(\nabla_U[Q(\widetilde{\mathbf{u}})])^{-1}$ and the bilinear matrix $\{\Psi\}$, (Patwardhan and Madhavan (1998)). The solution of the above transformed multidimensional quadratic equation can be written as

$$(\mathbf{u}(k) - \widetilde{\mathbf{u}}) = -\left[\frac{1}{2}\left\{I + (\mathbf{\Delta}(k))^{\frac{1}{2}}\right\}\right]^{-1} E_0(k) \tag{33}$$

$$\text{where } \Delta(k) = \left[\mathbf{I} - 4\left\{\widetilde{\mathbf{\Psi}}(k)\right\}(\mathbf{E}_0(k))\right] \tag{34}$$

Note that, in general, a matrix has multiple square roots and consequently different values of $\mathbf{u}(k)$ will be obtained for every choice of the square root of matrix $\Delta(k)$. Also, even though the original matrix has all real elements, the square root can have complex elements and consequently the resulting $\mathbf{u}(k)$ can be complex. Patwardhan and Madhavan (1998) have suggested the following remedies to alleviating these difficulties:

- The matrix square root $(\Delta(k))^{\frac{1}{2}}$ should be selected such that all its eigenvalues have non-negative real parts. Specifically, when matrix $\Delta(k)$ is a positive definite matrix, the positive definite square root of the matrix should be used for control law computations.
- When the solution vector becomes complex, the real part of the complex solution vector can be used for manipulation.

They have also shown that the situation where the solution becomes complex arises when the specified setpoint is unattainable due to system nonlinearity. A detailed discussion regarding the rationale behind these recommendations and related theoretical results can be found in Patwardhan and Madhavan (1998). Thus, incorporating the above suggestions, the **quadratic control law** becomes

$$\mathbf{u}(k) = \widetilde{\mathbf{u}} - REAL\left\{\left[\frac{1}{2}\left\{I + (\Delta(k))^{\frac{1}{2}}\right\}\right]^{-1}\mathbf{E}_0(k)\right\} \tag{35}$$

Note that if the specified setpoint is attainable at steady state and the prediction horizon is selected sufficiently large, the complex solutions are not expected to arise during control law implementation.

The robustness of the control law in the presence of plant model mismatch can be increased by

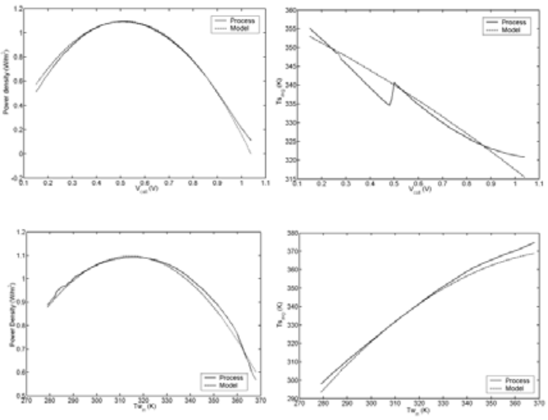

Fig. 1. Model Validation (Configuration I) : Comparison of steady state behavior of process and model

introducing a diagonal filter $F_{IMC}(z)$ in the feedback path. This can be achieved by replacing $\boldsymbol{\epsilon}(k)$ in equation (28) by $\boldsymbol{\epsilon}_f(k)$ where $\boldsymbol{\epsilon}_f(k)$ represents the filtered feedback signal computed as

$$\boldsymbol{\epsilon}_f(z) = F_{IMC}(z)\,\boldsymbol{\epsilon}(z) \qquad (36)$$

The IMC filter $F_{IMC}(z)$ is typically selected as a diagonal matrix with first or higher order transfer functions appearing on the main diagonal.

3. IMPLEMENTATION OF NIMC ON PEMFC

The process dynamics of the PEMFC is simulated using the reduced order model given by Golbert and Lewin (2004). The primary aim of any control scheme for PEMFC is control of the power density, which is defined as $P = I_{avg}V_{cell}$. Apart from power density, the other important controlled output is the average solid/stack temperature (Ts_{avg}). While choosing manipulated inputs, we consider following two possible configurations

(a) Configuration I: Cell voltage (V_{cell}) and coolant inlet temperature (Tw_{in})

(b) Configuration II: Hydrogen flow rate($M_{H2,in}$) and Coolant flow rate($M_{cool,in}$)

While the inputs for Configuration I can be used to operate the system over a wider range, the control problem for this choice is difficult to handle as the system exhibits input multiplicity behavior. Figure (1) represents the steady state behaviour of the system outputs with respect to the manipulated inputs of Configuration I. In order to obtain smaller, lighter and cheaper fuel cells, it is desired to operate them at the operating point where power density attains a maximum (corresponding to $V_{cell} = 0.53\ V$; $T_{w,in} = 317\ K$; $P = 1.09\ W/m^2$; $Ts_{avg} = 338.8\ K$ in the present case). As can be observed from this figure, the power density exhibits a change in the sign of steady state gain across the optimum operating point with respect to V_{cell} as well as Tw_{in}. Thus, controlling the system at the peak power density

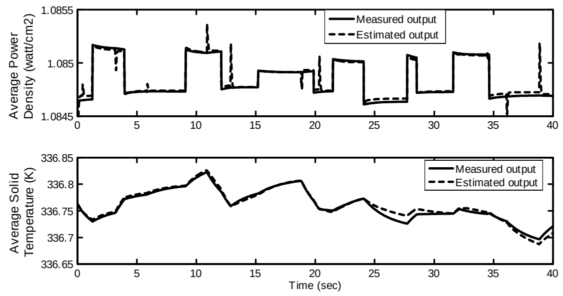

Fig. 2. Model Validation (Configuration II): Comparison of measured and estimated outputs

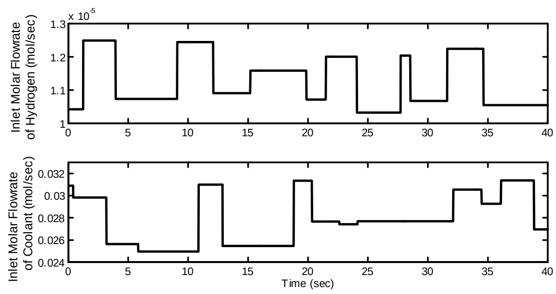

Fig. 3. Model Validation (Configuration II): Variation of manipulated inputs

point is a challenging control problem as this is a singular operating point.

The data required for parameter estimation of the OBF-Weiner model was obtained by perturbing the reduced order fuel cell model in open loop by simultaneously introducing square pulse sequences of random magnitude in manipulated inputs. Two MISO OBF-Weiner models were developed in each case. Figure (1) presents steady state model validation results for Configutaion I. It can be observed that the identified OBF-Weiner models capture the nonlinear steady state characteristics of the process over a wide operating range. The model validation results using an independent dynamic data set for Configuration II are shown in Figure (2). The corresponding variation of manipulated inputs is shown in Figure (3). As is evident from this figure, the identified OBF-Weiner models are able to capture process dynamics reasonably accurately. These models are used to formulate the NIMC controller together with a first order IMC filter of the form

$$\boldsymbol{\epsilon}_f(k) = \alpha \mathbf{I}\boldsymbol{\epsilon}_f(k-1) + (1-\alpha)\mathbf{I}\boldsymbol{\epsilon}_f(k-1)$$

where $0 < \alpha < 1$.

For Configuration I, the control problem is formulated as shifting the fuel cell operating point from a given suboptimal initial steady state to the optimum operating point. The closed loop response obtained using the NIMC controller ($p = 20$, $\alpha = 0.9$) is presented in Figures (4) and (5). The NIMC controller quickly moves the system to the singular operating point without excessive vari-

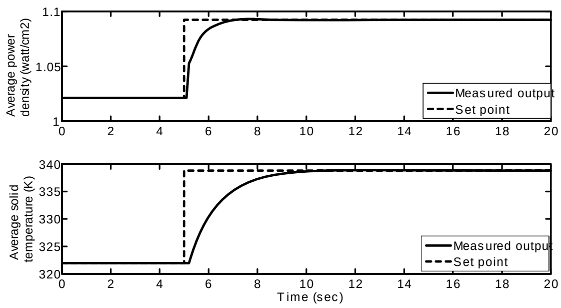

Fig. 4. Configuration I: Control of PEMFC at optimum operating point - variation of controlled outputs.

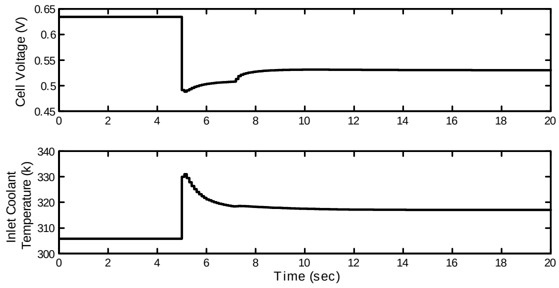

Fig. 5. Configuration I: Control of PEMFC at optimum operating point - variation of manipulated inputs.

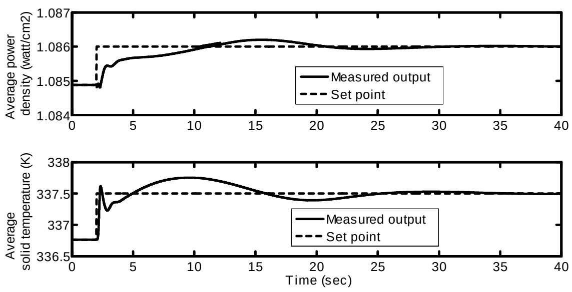

Fig. 6. Configuration II: Servo control of PEMFC - variation of controlled outputs.

ations in manipulated inputs and maintains the operation at the optimum. The results of the servo control problem for Configuration II are shown in Figures (6) and (7). The NIMC controller tuning parameters in this case are $p = 20$ and $\alpha = 0.98$. While the performance of the NIMC controller is satisfactory, the settling time is somewhat longer for the second configuration.

4. CONCLUSIONS

In this paper, a Wiener type of model has been developed for capturing the nonlinear dynamics of a proton exchange membrane fuel cell. The linear part is parametrized using ortho-normal basis filters (OBF). Using multi-step predictions generated by the OBF-Wiener model together with theory of solutions of multi-dimensional quadratic equations, a closed form control law is derived in

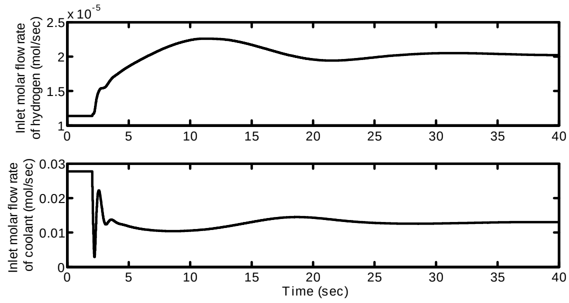

Fig. 7. Configuration II: Servo control of PEMFC - variation of manipulated inputs.

the nonlinear IMC framework. The efficacy of the proposed modeling and NIMC control scheme is demonstrated by simulating servo control problems associated with PEMFC for two different control configurations. The NIMC controller is able to shift the operation of the PEMFC to the operating point corresponding to maximum power density, which happens to be a singular operating point of the system in the first configuration. The NIMC controller also generates satisfactory servo performance for the second control configuration.

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